CHAOS AND VARIABLE STARS: GOOD NEWS AND BAD

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Abstract

A computer program is used to illustrate several qualitative features of chaotic behavior in a non-linear third-order system: a) The properties of the system depend strongly on the values assigned to the adjustable parameters; b) In the chaotic domain, the trajectory of the system is very sensitive to the starting point, which is the bad news, as it means long-range prediction is impossible; c) Simple systems can display very complex behavior, which is the good news, as it implies that irregular variables may be explainable by simple physics.

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The study of chaos has recently become a fad in many areas of science and technology. It is not really a new area of physics, but it has offered many new insights into the behavior of physics systems. I think the study of chaos may be viewed as an experimental geometry, or as the application of geometry to the behavioral patterns of physical and chemical systems. It is a delight to the mathematically minded, but many physicists are not quite sure what to make of it. I would like very briefly to describe how I see it and to explain why it seems to be bringing bad news to weather forecasters and good news to those of us who study variable stars.

First, the bad news. Today's rain was not forecast 5 days ago, and the news from chaos is that it could not have been forecast 5 days ago. This is not merely a limitation of our computer programs or our data. There is an inexorable limit to the possible length of an accurate forecast. It appears to be about 4 or 5 days, if you are not too fussy, and much less if you require high precision. Long-range, and rather vague, statistical forecasting up to several months remains possible. But the kind of forecasts that help us plan picnics are limited by the chaotic nature of the atmosphere.

This situation can be summarized in the first tenet of chaos:

The detailed course of events is very sensitive to the starting point.

This is the reason a flipped coin or a pair of tossed dice can give the appearance of randomness. The outcome of a toss depends crucially on the immeasurable and uncontrollable details of their bounce on the table. (The key feature of the dice is their sharp corners. A spherical ball shows no such behavior when it bounces from a flat table.)

In the same way, the weather in a few days is extremely sensitive to the state of the air today - so sensitive that we cannot hope to collect sufficient data.

Now let me state the good news and then give an example of the message chaos may bring to the study of variable stars.

Simple systems can show complicated behavior.

The bouncing of dice is a complicated behavior, but the system is merely a pair of cubes. To show why this is good news, I will borrow from a recent paper on the light variations of a white dwarf star (Goupil et al. 1988). The authors of that paper suggest that the star is behaving chaotically, and I would like to describe what they mean by this.

They adapted a relatively simple equation that was constructed to imitate the behavior of a star, and they find that it behaves in a very complicated way. The equation is simple, but its behavior can be complex if the parameters are chosen correctly. The implication is that if we see a complex pattern of behavior, we do not need to look for a complicated explanation. That is the good news from chaos. The complicated behavior of semiregular and irregular variable stars may be understandable in simple terms.

I have put their equation into my Apple Macintosh computer and have generated a few examples. Here is their original equation. It is a non-linear 3rd-order differential equation for R, the radius of a star as a function of time:

$$R''' + kR'' + R' + kmR(1+bR). \qquad (1)$$

Each prime indicates one differentiation with respect to time, and the letters k, b, and m stand for adjustable constants, or parameters, of the system. The equation is completely deterministic and it does not involve random numbers. The behavior of the solution, that is the curve of R against time, depends on the numbers assigned to the parameters, much the same way the behavior of a star depends on its mass and age and chemical composition. But the analogy must not be taken too literally. This equation is not a good representative of an actual star. It is much too crude, but its behavior is similar, and for that reason it can give us some insights. (Another tenet of chaos, that I can only mention in passing, is that there are universal properties in the behavior of physical systems, so we can hope to transfer ideas from one model to another. This is perhaps the best news of all.)

I rewrote their equation in terms of three variables R, V, and F and obtained the following three equations:

$$R' = V,\ V' = F,\ F' = -kF - V - kmR + kmbR^2. \qquad (2)$$

These are the equations I put into my computer. We need not worry about the interpretation of the R, V, and F. Just suppose they are some properties of a star. All I really care about is showing the way their behavior can become chaotic.

To do this, I will use two types of plots. The first (upper part of Figure 1) is a familiar strip chart recording and it looks very much like a conventional light curve, where the data are plotted as functions of time. (I have plotted only the two variables, R and V.) But if we want to follow the long-term history of the star, this plot runs off the screen, so I will use the type shown in the lower part of Figure 1. Here I use R and V to define the x and y coordinates, and R is plotted against V, rather than against time. As the time goes on, the star moves along a track, and the track can take a variety of shapes.

I will illustrate four types of behavior obtained by adjusting the numerical values of the parameters.

1) **Regular oscillation:** (m = -1, k = 0, b = 0)

In this case (Figure 1, upper), R and V look like sine and cosine curves when plotted against time. They are very regular and highly predictable into the distant future. In order to compress the data, we plot R against V in the lower part of the figure. Now the system traces out a circle, and this is a much more compact way to represent the behavior.

2) **Damping toward a stationary point:** (m = 0, k = 0.5, b = -0.5).

In this case (Figure 2) the motion dies out very quickly, imitating a star, like the sun, that is stable against global oscillations. If we start it from an arbitrary position, it tends to become motionless and then stay that way. Nothing very new thus far. This just imitates a swinging pendulum with some friction.

3) **Period-doubling:** (m = -1.65, k = 0.5, b = -0.5).

For the rest of the figures, we will keep the same values of k and b, and we only change m.

In Figure 3, the behavior is a little more interesting and the difference comes from the squared term in the equation for F′. This term is no longer multiplied by zero, so it can do its thing. (This is like putting the sharp corners onto the dice we were tossing a while back.) On the R vs. V plane, the system now follows a curve that has two loops. The result is an alternation of deep and shallow minima, much like the RV Tauri stars. Its period is twice the time required for a single loop, so this is called period-doubling.

4) **Chaos:** (m = -2.0, k = 0.5, b = -0.5).

Now the system wanders all over the diagram (Figure 4) and has become unpredictable. Two stars starting close together on such a diagram will quickly wander apart. This is chaos, and the remarkable feature is that we have used exactly the same equation.

There are two implications for variable stars. The first is that similar physics can explain a wide range of behavior, from highly regular Cepheids to the semiregular and irregular variables. The second is that the complicated behavior of some stars may be understandable in terms of relatively simple equations.

This is the good news from chaos. It encourages us to look deeper for the source of strange and irregular behavior, rather than attributing it to mere "randomness."

For a fascinating layman's historical introduction to chaos, I recommend James Gleick's book (1988).

REFERENCES

Goupil, M. J., Auvergne, M., and Baglin, A. 1988, **Astron. Astrophys. 196**, L13.
Gleick, J. 1988, **Chaos**, Viking Press.

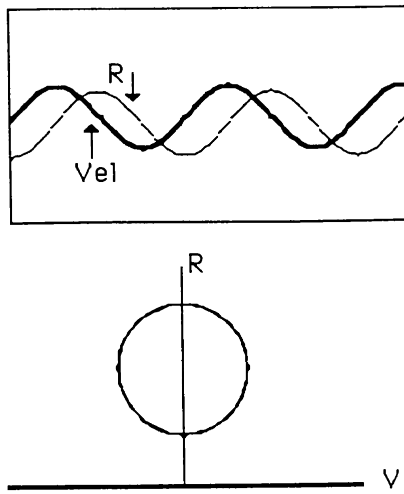

Figure 1. Regular, periodic oscillations (m = -1.0, k = 0, b = 0). The upper diagram shows the two variables, R and V, plotted against time, and they behave as displaced sine curves. In the lower portion, the values of R and V are plotted against each other. That is, each point on the trajectory, represents the values of R and V at a particular time. As time goes by, the system moves along the track, in this case an oval.

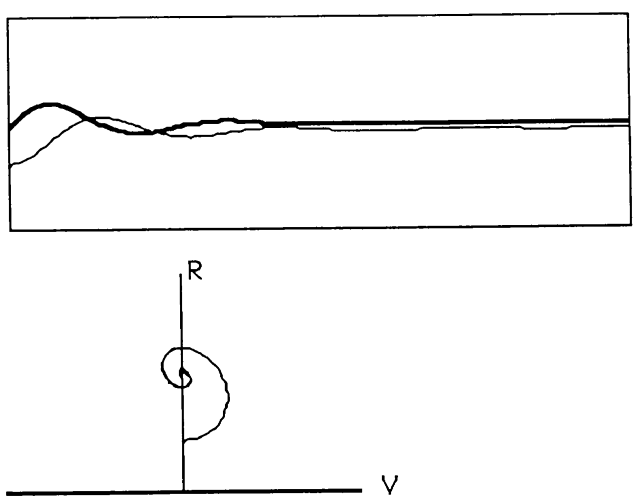

Figure 2. Damped oscillator (m = 0, k = 0.5, b = -0.5). When started from an arbitrary position, the motions of the system quickly die out, imitating pendulum with friction. This behavior is characteristic of stars that are stable against global oscillations.

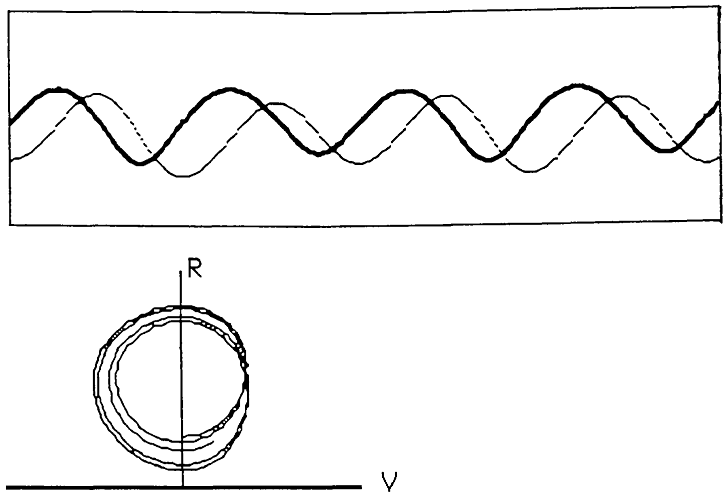

Figure 3. Period-doubling (m = -1.65, k = 0.5, b = -0.5). The trajectory follows a pair of loops (only approximately in the particular calculation) showing alternating extremes. This was the behavior described by Goupil et al. (1988) in the context of the white dwarfs, and perhaps it also applies to RV Tauri stars.

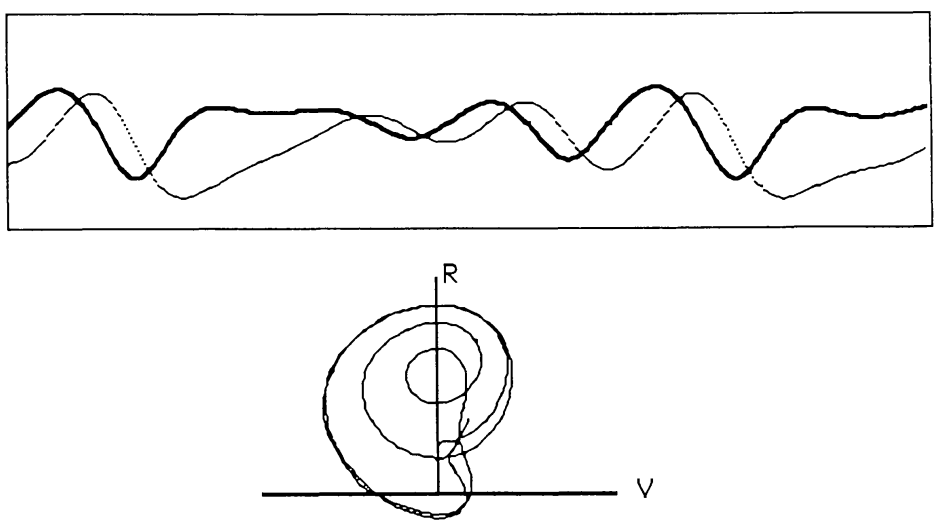

Figure 4. Chaotic behavior (m = -2.00, k = 0.5, b = -0.5). The system wanders apparently aimlessly and the curve shows intervals of relative quiet. Does this imitate the Cepheids that have temporarily stopped pulsating?