1989JAVSO..18...97V

EVIDENCE FOR CHAOTIC BEHAVIOR IN R SCUTI?

TODD VELDHUIZEN
JOHN R. PERCY

Department of Astronomy
Erindale Campus
University of Toronto
Mississauga, Ontario L5L 1C6
Canada

Received 7 March 1990

Abstract

Recent discoveries in nonlinear dynamics have made it desirable to investigate various dynamical systems for evidence of chaotic behavior. An analysis of the RV Tauri variable R Scuti is presented, showing possible indications of chaos. The input data consisted of a set of times and magnitudes of maximum and minimum brightness, derived from AAVSO observations of the star.

* * * * *

1. Introduction

Many dynamical systems are seen to demonstrate the features of chaos. These systems possess a finite number of degrees of freedom, yet have the ability to generate complex, aperiodic behavior, exhibiting a sensitive dependence on initial conditions. Analysis of natural phenomena, such as epidemics, cardiac arrhythmias, turbulence, population fluctuations, free-body tumbling, and others have revealed underlying chaotic behavior. The study of chaos as a global organizing principle is quickly becoming a matter of consideration in many disciplines.

It has been recognized for some time that certain variable stars behave in complex ways. Some models describing the behavior of such stars have recently been found to exhibit features of chaos. This paper investigates possible evidence, based on observational data, that R Sct is demonstrating chaos, rather than intrinsically complicated behavior.

2. Observational Data

R Sct is an RV Tauri star with a mean cycle length of 70 days between adjacent minima, and is of particular interest because of the complex behavior which it demonstrates with respect to variations in magnitude and cycle length. RV Tauri variables characteristically alternate between deep and shallow minima, with minor aberrations; R Sct's light curve is predominantly aberrant, with minor lapses into order. Its accepted cycle length of 70 days represents an average; the minima tend to occur anywhere from 50 to 100 days apart.

The data used in the analysis consisted of Julian dates and magnitudes of maximum and minimum brightness, determined by Percy and Alfred (1988) from AAVSO observations. The database represents approximately 60 years of observations, commencing in 1921; it comprises 413 data points of an estimated 626 point-complete set. Figure 1 shows the number of data points present per two-year interval; the expected number is 21. In order to prevent gaps in the data from affecting results, the software written to produce the calculations and figures presented in this paper automatically discarded cycle lengths greater than 105 days.

3. Some Aspects of the Variability of R Sct

Figure 2 shows several aspects of the variability of R Sct. Panel A shows the magnitude at minimum. It is interesting that, during several intervals, R Sct alternates regularly between deep and shallow minima; this behavior is especially noticeable in 1965-73, 1974-78, and 1980--. Panels B and C show the smoothed variations in the mean magnitude and cycle length, respectively. There is some evidence for a correlation between an increase in magnitude and an increase in cycle length: for example, the dip in B around 1941 corresponds to a crest in C; there are several other instances. Panel D shows the cycle length between minima.

4. Other Indications of Aperiodicity

If aperiodic behavior is observed, the corresponding power spectra are expected to be "noisy." Due to the uneven spacing of the data, a conventional Fast Fourier Transform (FFT) was not applicable. Instead, the algorithm given by Deeming (1974) was used. The spectrum shown in Figure 3 has a central peak which corresponds to the accepted cycle length of 70 days. However, this value does not remain constant when evaluated over discrete intervals, as is demonstrated in Figure 2c. Figure 4 shows a series of Fourier transforms showing the changes in the spectra over time. The central peak fluctuates between values of 70 and 71.3 days. During the period 1958-81, the transform decays from a single peak into four smaller peaks, perhaps indicative of an emerging complexity. This decay also corresponds to a deepening of the minima during the same period (see Figure 2b).

It is not clear how this result should be interpreted. A study of the period of R Sct using the (O-C) method (Percy et al. 1990) suggests that the star's period decreased abruptly from 70.95 to 70 days in 1941. There is also considerable scatter in the (O-C) diagram - some random and some systematic.

We have also studied the correlation between the lengths of successive cycles. We find only that a longer-than-average cycle tends to be followed by a shorter-than-average one, and vice versa.

5. Phase Space Reconstruction

We have also done simple experiments using the technique of **phase space reconstruction.** Certain systems exhibiting chaotic behavior are known to possess what are termed **strange attractors** (Ruelle and Takins 1976). In systems of this type, the physical properties move in trajectories about these attractor(s) in the system's phase space, within the attractors' basins of attraction. If clean, complete data are available, detailing the state of the system for a length of time, numerical techniques can be used to determine quantities known as **Lyapunov characterizing exponents** (Froehling et al. 1981), which measure average rates of exponential convergence and divergence towards and away from attractors. However, when dealing with complex natural phenomena, even the **dimension** of the system (the number of variables needed to model its behavior) is a matter of conjecture, and it is only feasible to collect data on one or two parameters. To analyze these systems, techniques have been developed which utilize a phase space reconstruction from a single variable (Packard et al. 1980). Because any component in a system is uniquely determined from the other variables (assuming negligible noise), the progression of a single variable over time contains implicit information about the state of other components. Thus one can reconstruct an equivalent phase space by choosing a time interval appropriate for the system, and considering each value that the variable had at previous intervals a new dimension. If the system does contain strange attractor(s), patterns indicative of their topology will emerge when the data are viewed in D dimensions, where D represents the inherent dimension of the system. The phase space reconstruction thus becomes a very useful technique both for diagnosing chaotic behavior in dynamical systems and for examining their general attributes.

One of the difficulties with the technique is the requirement of evenly spaced data. In the case of R Sct, the data used consisted of times and magnitudes of minima and maxima. Plotting successive magnitudes failed to reveal any indication of structure; indeed, such a failure is to be expected because of the irregular spacing of the data. We therefore applied a simple interpolation technique to the data to generate evenly-spaced data. We then plotted successive interpolated magnitudes, being careful to filter out sections of "patchy" data where missing data points might generate artificial noise. Again, there was no clear indication of structure. This result is not surprising, given the scarcity of the data, its inherent imprecision, and the fact that the observed visual magnitude is a rather complicated function of the actual physical properties of the system (density, pressure, temperature, etc.).

6. Conclusion

Although it is not possible, given the type of data and techniques used, to state that R Sct is definitely exhibiting some form of chaotic behavior, the possibility is not ruled out. Chaos is a viable principle for explaining the apparent complexity in RV Tauri stars and other aperiodic variable stars. Significant research is being done in this area, and an international conference on the topic is to be held in Japan in 1992. AAVSO data may well figure prominently in this work.

7. Acknowledgements

Todd Veldhuizen (Meadowvale Secondary School, Mississauga, Ontario) was a participant in the University of Toronto Mentorship Program, which enables outstanding senior secondary school students to undertake research projects with university faculty. We thank AAVSO observers and Headquarters staff for making the observations of R Sct available to us.

REFERENCES

References in This Paper

Deeming, T. J. 1975, **Astrophys. Space Sci. 36**, 137.
Froehling, H., Crutchfield, J. P., Farmer, D., Packard, N. H. and Shaw, R. S. 1981, **Physica 3D**, 605.
Packard, N. H., Crutchfield, J. P., Farmer, J. D. and Shaw, R. S. 1980, **Phys. Rev. Lett. 45**, #9, 712.
Percy, J. R. and Alfred, A. 1988, **Journ. Amer. Assoc. Var. Star Obs. 17**, 145.
Percy, J. R., Sasselov, D. D., Alfred, A. and Scott, G. 1990, preprint.
Ruelle, D. and Takens, E. 1976, **Commun. Math. Phys. 50**, 69.

General References on Chaos

Crutchfield, J. P., Farmer, J. D., Packard, N. H. and Shaw, R. S. 1986, "Chaos", in **Scientific American 255**, #6, 46.
Gleick, J. 1987, **Chaos: Making a New Science**, New York: Viking.
Buchler, J. R. and Eichhorn, H. (Eds.) 1987, "Chaotic Phenomena in Astrophysics", **Ann. New York Acad. Sci. 497**; includes a paper by Buchler on "Chaotic Behavior in Variable Stars".
Buchler, J. R., Perdang, J.M. and Spiegel, E.A. (Eds.) 1985, **Chaos in Astrophysics**, Dordrecht: D. Reidel Publ. Co.; includes several papers on irregularity and chaos in variable stars.
Whitney, C. A. 1984, "Semiregular Variables: Are They Chaotic or Ringing?", **Journ. Amer. Assoc. Var. Star Obs. 13**, 31.
____________ 1988, "Chaos and Variable Stars: Good News and Bad", **Journ. Amer. Assoc. Var. Star Obs. 17**, 97.

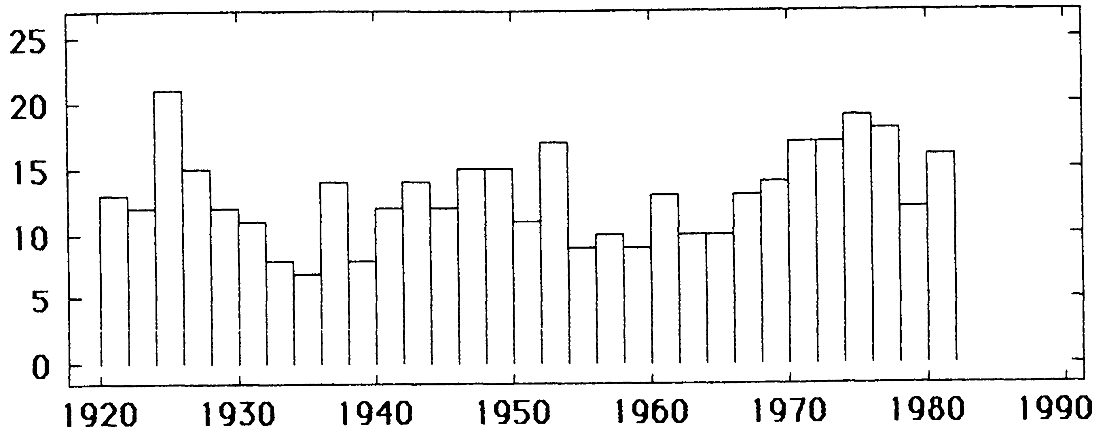

Figure 1. Histogram of the number of data points per two-year interval. The expected number is 21; approximately one third of the data points are unavailable.

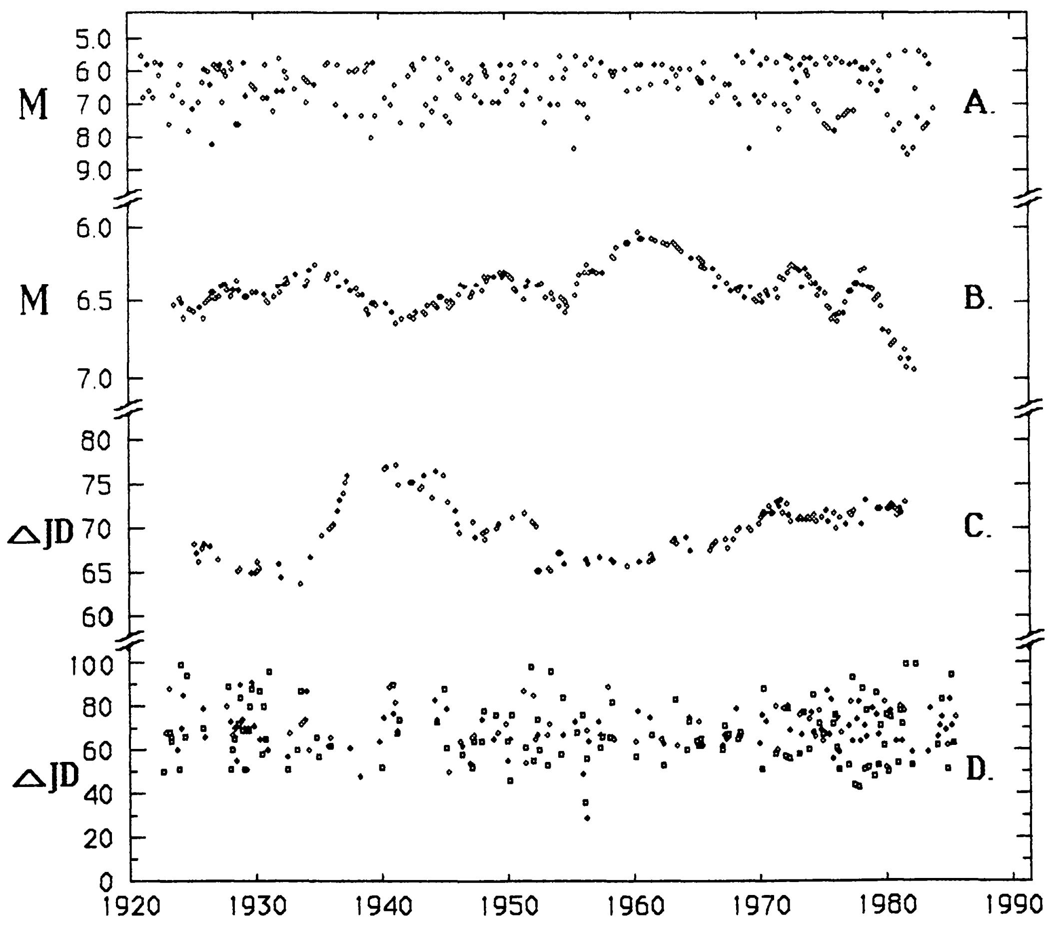

Figure 2a. Magnitude at minima. 2b: Mean Magnitude at minima - average of 8 data points before and after the date. 2c: Mean Cycle Length (between adjacent minima) - average of 8 data points before and after the date. 2d: Cycle Length, taken between adjacent minima (circle) and maxima (square).

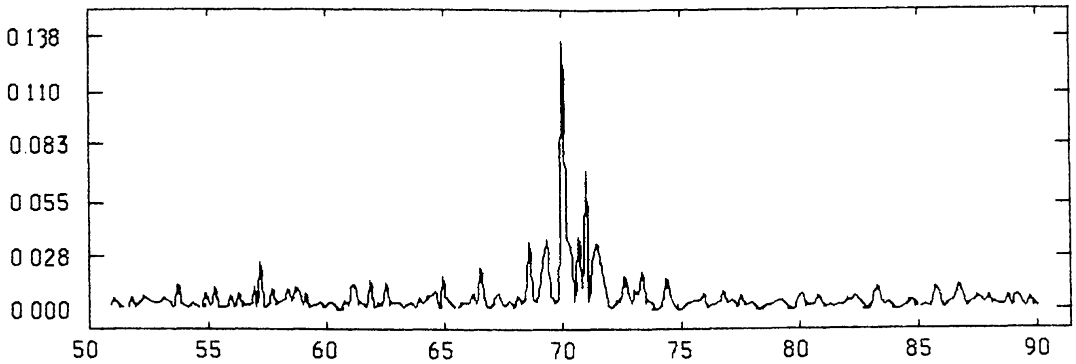

Figure 3. Fourier Transform of times of minima and maxima. Actual data values used were -1 and +1 for minima and maxima, respectively, in order to minimize noise caused by errors inherent in magnitude estimates.

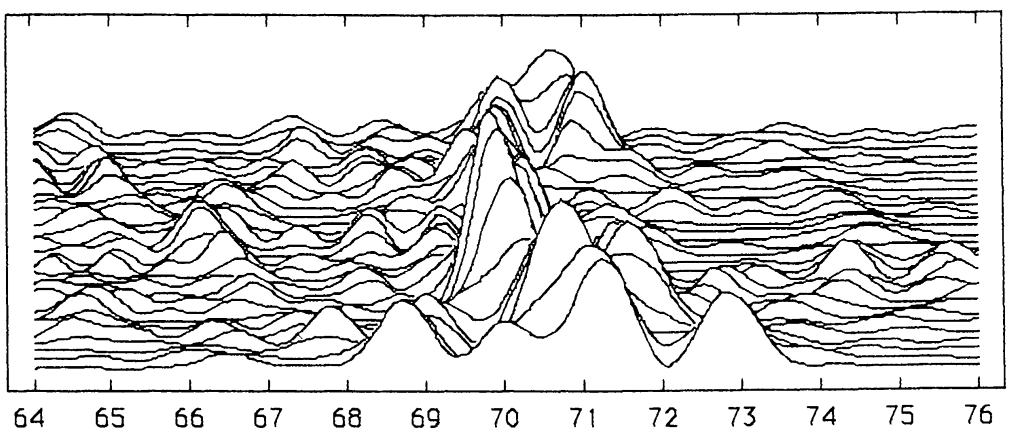

Figure 4. Each of the above spectra are transforms of a set of data with length one-fifth (about twelve years) that of the whole. Sets are spaced at intervals of approximately 1.7 years, with the first at the top of the diagram starting in 1921, and the last, commencing in 1969, at the bottom. The necessity of grouping data into twelve-year units to make a base sufficient for the transform wipes out any short-lived features in the spectra.