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COMPUTER ANIMATIONS AND THE SHAPES OF CEPHEID VELOCITY CURVES

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Abstract

Computer programs are being developed for the simulation of stellar pulsation and stationary stellar atmospheres. This paper describes a developmental version of the pulsation program, with which the velocity curves may be synthesized from linear modes.

1. Introduction

I am developing two interactive programs for personal computers to be used for instruction in astrophysics:

1) A stellar pulsation program will permit users to specify the mass, radius, and luminosity of a star, and the program will construct a static envelope and then evaluate linear radial modes as well as non-linear motions. Pulsational variations of user-selected variables will be displayed by color animation.

2) A stellar atmosphere program will construct a stationary model from specified surface temperature and gravity and then display it in a variety of optional graphs. Three models may be held in memory at once and may be superposed and compared graphically. Limb-darkening and the continuous spectrum are computed, and atomic absorption-line data may be specified for the calculation of individual line profiles. Black body radiation may be imposed on the top of the atmosphere to illustrate the effect of a close companion.

This work is part of the CUPS Project (Consortium for Upper-Level Physics Software) and is sponsored in part by the National Science Foundation, IBM Corporation, George Mason University, and John Wiley and Sons. My participation is supported by the Smithsonian Astrophysical Observatory. The Project consists of 30 physicists who are developing user-friendly software and accompanying texts for use in nine upper-level physics courses. The final materials (including Pascal source code) are expected to be ready in 1995 in both PC and Macintosh versions.

The program demonstrated in this paper is a developmental version of a portion of the stellar pulsation program. It carries out a Fourier synthesis of stellar velocity and radius curves after the user specifies the amplitudes and phases of the first four modes. In its current version, the computer screen provides graphical display of the phases and time-dependent amplitudes of the individual Fourier components, so the user may see how the various components contribute to the shapes of the curves. This program does not represent the physics of the pulsation, merely the motions of linear modes, so it is more properly called an animation.

This paper discusses the use of this program to explore the composition of velocity curves. The light variations of Cepheids are similar in shape to the variations of outward surface velocity generated by this program, so the program can be considered to provide light-curve synthesis as well.

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2. The Hertzsprung Relation

In 1926, the Dutch astronomer Eijnar Hertzsprung pointed out that the shapes of the light curves of classical Cepheids show a progression with period. Stars with periods in the range of 6-10 days tend to have a bump on the descending branch of the light curve and stars with periods of 10-17 days a bump on the ascending branch. Since then, the same pattern has been found among the velocity curves. Before showing how the animation program described in this paper can be used to explore the meaning of this pattern, we must describe the linear modes of pulsation.

2.1 Modes of pulsation

Many types (virtually an infinite number) of spherically symmetric motions are available to a pulsating star. The simplest are called "modes." In a mode, all the stellar layers move with a single period, and they all come to rest at the same instant. (In a periodic motion like this, it is handy to measure time from the start of each cycle, and this is called the "phase").

In the fundamental mode, illustrated in Figure 1, all particles move in synchronism and in the same direction. The amplitude of the motion decreases as we go deeper, due to the higher density of the central regions of the star.

In the first mode (Figure 2), the motions are synchronous but the outer layers move inward while the inner layers move outward. There is an intermediate layer that is stationary; this layer is called a "node." The first mode has a shorter period than the fundamental (zeroth mode), because the gas is working against itself in a smaller space interval, corresponding to a shorter wave length. That causes the period to be shorter. These are "standing" waves because all peaks occur at the same time, and the waves don't seem to be moving anywhere.

2.2 Relationship between radius and velocity changes

Figure 3 shows a typical relationship between velocity and radius changes for the surface layers in a fundamental mode.

2.3 Phasors and an imitation of RV Tauri stars

Time and phase move ahead uniformly, but the stellar layers move up and down with a periodic motion. The "phasor" is a rotating arrow which can represent this oscillatory motion. Figure 4 shows the phasor (or rotating arrows) for two modes; each phasor rotates uniformly like the hand of a clock, and its upward component is the velocity at a particular instant.

What happens when the star moves in two modes at once? The behavior depends on the precise periods and phases of the two modes. In order to predict the behavior, we construct the composite phasor by attaching one arrow to the tip of the other, as in Figure 5.

In the example of Figure 5, I have assumed that the second mode has a period exactly half that of the fundamental. The motion repeats because two cycles of the second mode take exactly the same time as one cycle of the fundamental.

Figure 6 shows the motion inside the star. It is the result of standing waves with phase and amplitude relations that change with depth. It looks like a running wave, because the peak comes progressively later in deeper layers.

2.4 Analyzing the Hertzsprung relation

Simon and Schmidt (1976) have suggested that the Hertzsprung relation is the result of the superposition of the fundamental and second mode in classical Cepheids. The nature of the superposition depends on the period of the star, and the result is the pattern of humps described by Hertzsprung. Simon and Lee (1981) have also

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shown how the light curves of other classical Cepheids can be built up by adding modes (Fourier synthesis).

Let us explore this idea by first building a simple model that shows a velocity hump on the ascending branch. Figure 7 shows the result. It was achieved by adjusting the phase of the second mode until the resulting curve has the appropriate shape.

With a very slight change in the relative phases of the modes it is possible to produce a hump on the descending branch, shown in Figure 8. Looking closely at Figures 7 and 8 reveals an interesting feature of the Hertzsprung relation. The apparent shift of the hump from one side of the primary maximum to the other is an illusion. What happens is that the source of the primary maximum changes. That is, the primary maximum corresponds to one portion of the second mode for shorter period stars and to the other portion of the second mode for longer periods. This is the type of insight that can come from the simple animations possible with this program.

Figure 9 shows the interior displacements and the surface velocity of a star showing a hump. This figure illustrates that the appearance of a running wave - in which the peak is progressively later in adjacent layers - can be generated by the superposition of two standing waves whose amplitudes change with depth. That is, two superposed modes can give the appearance of a running wave and not only give the appearance, but also behave like a running wave. In fact, a running wave is nothing but a superposition of standing waves.

3. Concluding Remarks

This computer program makes the simplifying assumption that the pulsation consists of sinusoidal modes whose phases, periods, and amplitudes may be adjusted arbitrarily. By matching the behavior of a well-observed star with the curves produced by the program in Fourier synthesis, it is possible to determine the modal properties of a pulsation - that is, to measure the phases and amplitudes. This can be achieved in a matter of minutes with this program. But the program cannot explain why these values are found in a particular star. That requires a true simulation. The Fourier synthesis has sharpened the question posed by the Hertzsprung relation, but it has not explained the phenomenon.

This suggests that, although the virtue of this animation program may be primarily educational, the program may also help researchers focus and interpret their work with complex simulation programs.

References

Simon, N., and Schmidt, E. 1976, *Astrophys. J.*, **205**, 162.
Simon, N., and Lee, A. 1981, *Astrophys. J.*, **248**, 291.

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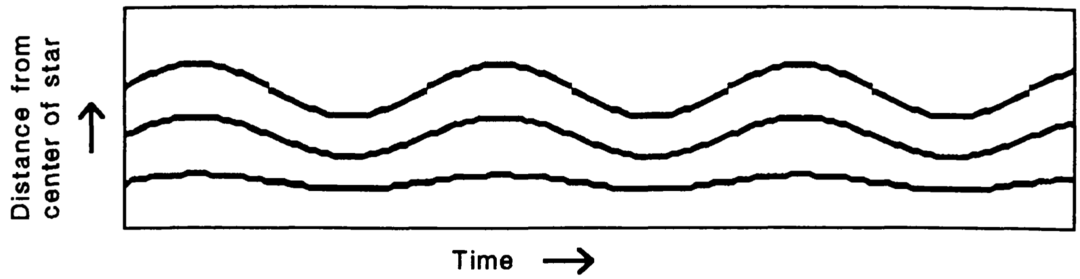

Figure 1. In the fundamental (or zeroth) mode, all layers move in synchronism and in the same direction. The amplitude of motion decreases with depth in the star.

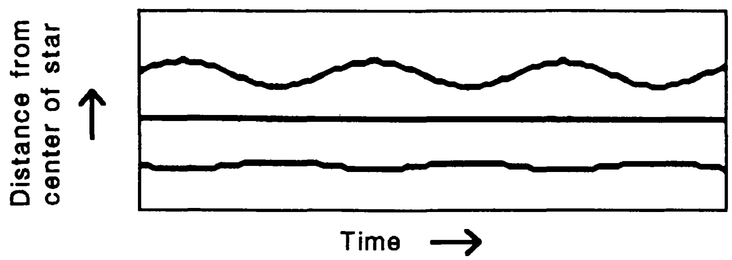

Figure 2. In the first mode, there is an intermediate stationary layer, or node. The inner and outer layers move in opposite directions, and the period of the pulsation is shorter than for the fundamental mode.

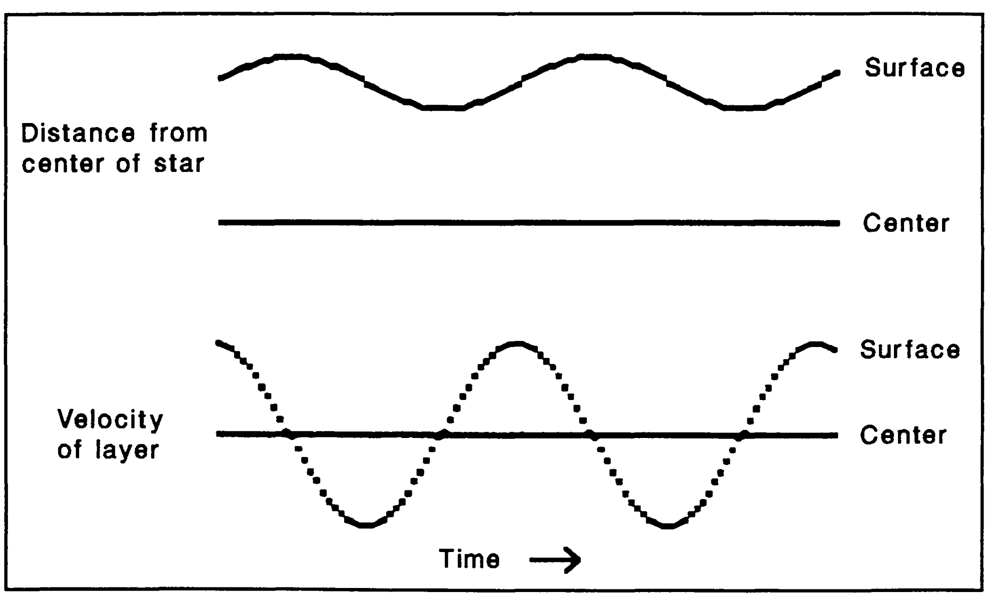

Figure 3. Changes of position (upper) and outward velocity (lower) in the surface layers of a spherically symmetric mode of oscillation. In classical Cepheids, the time of greatest outward velocity typically corresponds to the maximum of the light curve.

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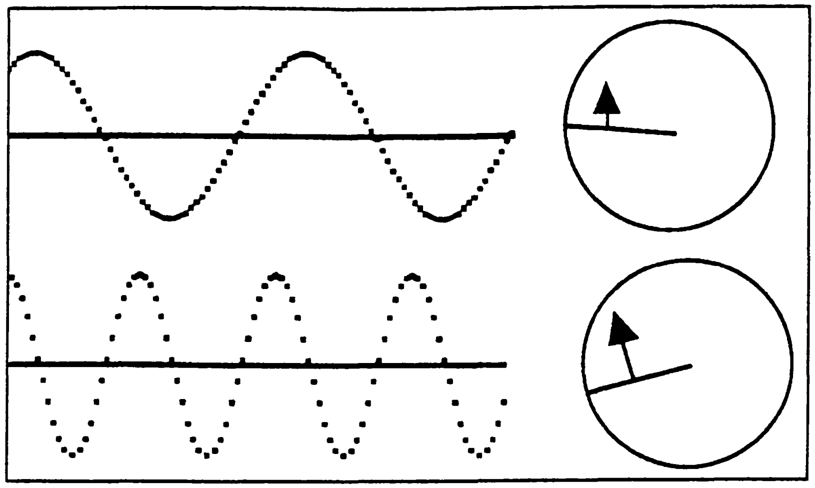

Figure 4. The left hand portion shows the velocity curves for the fundamental (top) and for a mode whose period is half the fundamental period. Each of these sinusoidal motions may be represented by the upward component of a rotating arrow, called a "phasor." The length of the phasor, on the right, shows the amplitude of the motion, and the angle measured clockwise around the circle is the phase of the motion at any instant.

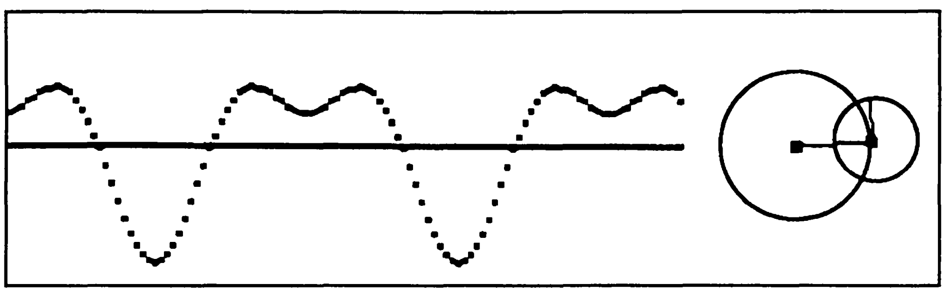

Figure 5. If a star pulsates in two modes simultaneously, its motion can be constructed by attaching one phasor to the tip of the other. This combination acts like an epicycle, and the upward component of the resulting arrow is the velocity at any instant.

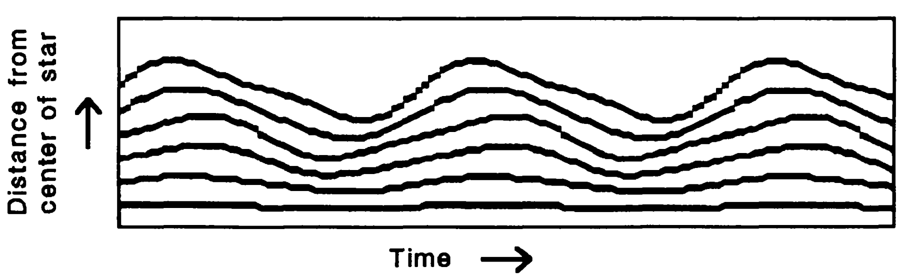

Figure 6. The motions inside a star oscillating in two modes depends on the phases and amplitudes of the modes. This figure corresponds to the velocity curve of Figure 5, and the pattern resembles a running wave, in which the peak comes at progressively later times as we go deeper into the star.

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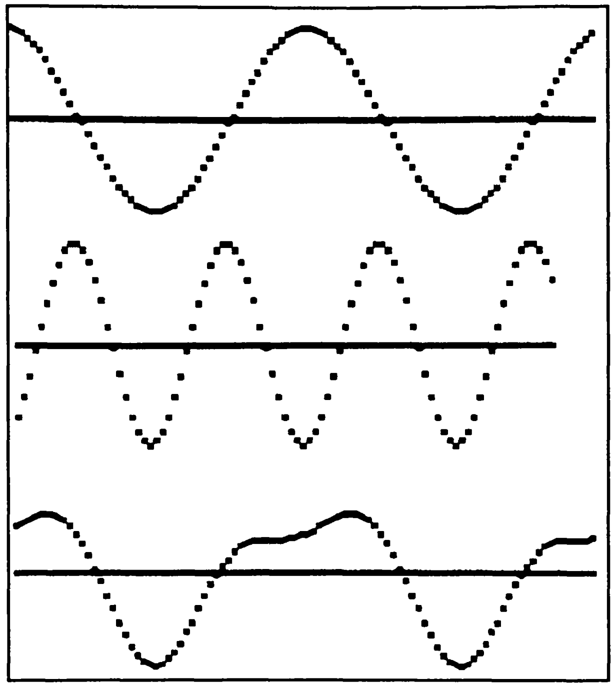

Figure 7. Velocity curve of a star oscillating in two modes whose periods are in the ratio 2:1. The phase of the faster mode was adjusted to give a hump on the ascending branch of the velocity curve.

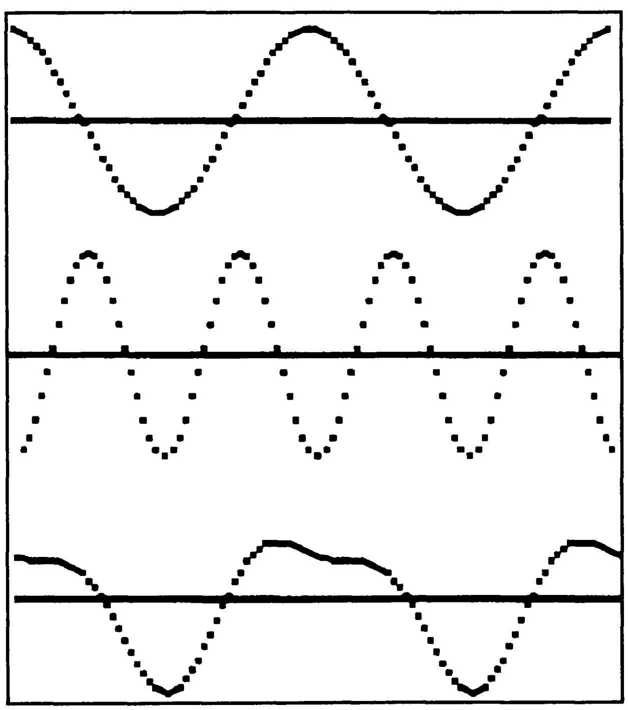

Figure 8. Velocity curve of a star oscillating in two modes whose periods are in the ratio 2:1. The phase of the faster mode was adjusted to give a hump on the descending branch of the velocity curve.

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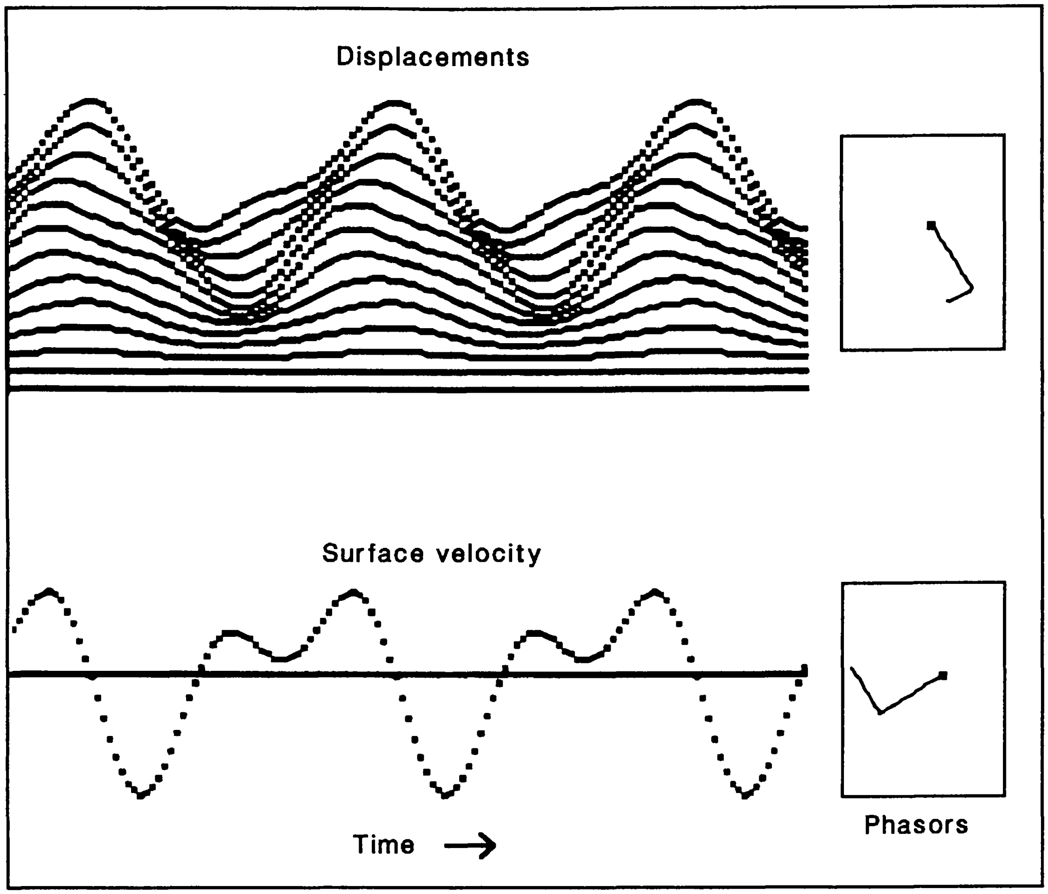

Figure 9. Fourier synthesis of the interior motions (upper) and surface velocity (lower) for a star oscillating in the zeroth and the second mode. In constructing this figure, the ratio of periods was assumed to be 2:1, and the relative phases and amplitudes were arbitrarily adjusted.

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