DWARF CEPHEID VARIABLE STARS: A STUDY OF CY AQUARII

Chris Dolan
Robin Tripoli
Jim Houck
Department of Astronomy
Cornell University
Ithaca, NY 14853

Received: June 11, 1993

Abstract

We report a period of 87.89534 $\pm$ 0.00015 minutes for the dwarf Cepheid variable star, CY Aquarii. This result is based on observations made during September and October of 1992. This value compares favorably to a published period of about 87.89517792 minutes.

1. Introduction

Dwarf Cepheids are short period, large amplitude, pulsating variable stars. CY Aquarii is a nominal member of this group with a period of roughly an hour-and-a-half and an amplitude variation of roughly 0.6 magnitude. These characteristics made it an ideal choice for a short-term study by a group of enthusiastic Cornell undergraduates. The goal of the project was to determine the period of the star as accurately as possible, realizing that we would not have a great deal of observing time. The observations were made at Cornell's Hartung-Boothroyd Observatory (HBO) with a CCD camera built by undergraduates a few years earlier.

The Thompson CCD chip used had 384 x 576 pixels and an average read noise of 7.12 electrons. Coupled with the 25-inch reflector at HBO and an 0.38 magnification re-imager, it had an angular resolution of about 1.44 arcseconds per pixel. We used the telescope without a filter to maximize the amount of light collected, thus minimizing the necessary exposure time.

Using this equipment, we were able to obtain a high signal-to-noise-ratio image of CY Aqr in a ten-second exposure. After each exposure, we recorded the apparent flux of CY Aqr and a nearby field star about five arcminutes east of the variable star. We also examined a dark background region roughly midway between the stars for calibration purposes. The time each exposure began was also recorded. Our flux measurements were based on a sum of an 11 x 11 pixel box centered on each location. In total, the exposure and data collection averaged a little over a minute, so we were able to collect approximately eighty data points per period.

Between September 24 and October 29, we observed four separate periods (see Table 1). We spaced the observing sessions roughly exponentially so extrapolating the period from one cycle to the next was not too difficult. This was essential in calculating the cycle number, as will be discussed below.

2. Data Analysis

We first subtracted the sky background from the CY Aqr and the field star values to compensate for changing equipment and sky brightness over the course of the night. Then we divided the variable star flux by that of the comparison star to cancel the effects of the atmospheric transmission and camera gain variation on the data.

Table 1. CY Aqr Observation Log

Date	*Cycle Number (n)*	*Time (t) (min)*	*Earth Phase (ϕ)*
Sept. 24, 1992	0	1543.674	21.90°
	1	1631.688	21.96
Sept. 25, 1992	16	2950.070	22.86
Oct. 29, 1992	575	52086.575	56.49

Looking at the data later, however, we realized that since our comparison star was roughly half or a third of the brightness of CY Aqr, its flux values were too noisy for our purposes. This effect, if not removed, would be the dominant noise source in the data. Instead, we used a linear approximation of the comparison star's apparent flux over time, which is effectively a linear fit to the light extinction curve for a given night. Figure 1 shows these data before and after the smoothing. We then divided the CY Aqr flux by this smoothed brightness to obtain our set of flux ratios.

Once we had reduced the data, analyzing them turned out to be more difficult than expected. The main problem was determining the precise time interval between any two peaks of the variable star's light curve. The obvious solution of finding the time at each peak and subtracting had to be abandoned because our data points were over one minute apart. This was much too coarse for our goals and only used one point on the light curve.

On our next attempt, we tried to find times on separate curves where the flux was about the same. If this had worked, we would have been able to use about ten or fifteen data points from each peak to derive a period. However, using absolute flux like this was not possible, since we had no way of accurately normalizing each of the peaks. The absolute flux varied from peak to peak because of various noise properties, such as sky brightness gradients and error in the comparison star data.

The method we chose in the end was to determine where each curve had a certain characteristic width, measured in time. This solved the problem of dealing with the absolute flux of each light curve. Instead, we effectively chose a point when the flux was a constant but unknown value below the point of maximum flux. We chose to examine pulse widths near the middle of the light curve, where the light curve was the most linear, as seen in Figure 2. This linearity was important, as we planned to use a linear interpolation scheme to choose the actual times in the end. The result was a single, accurate time for each peak that we observed.

The next important piece of information that we needed was the exact number of cycles that occurred between each of our observing sessions. The simplest solution would have been to look up a value for the star's period in published material and to extrapolate, but this would have violated the spirit of the project, which was to arrive at a period value independently and to consult the literature only at the end. Instead, we carefully selected the days that we observed so as to aid us best in determining the cycle number. To do this, we started by recording data for two successive peaks on the first night, which enabled us to determine the period to better than a minute. Then, using observations from the following night, we calculated the period to a small number of seconds. Without this progression, we would not have been able to find the cycle number for the data we collected on the third night, over a month later.

Of course, precise data are by nature sensitive data, so we had to be sure that we accounted for as many effects as possible. The most significant effect we discovered was caused by the Earth's orbit around the Sun. As the Earth revolves, the time it takes light to travel from a star to our camera varies by up to 16.6 minutes, depending on where the Earth is in its orbit and on the angle between the observed star and the ecliptic. For our data, the times were distorted significantly, as CY Aqr is at a declination of only 1.50° at the epoch of the observations. To correct for this effect, we assume CY Aqr is precisely in the ecliptic, so

$$t = t_0 + nP + (R_E / c)\cos(\omega_E t + \phi), \quad (1)$$

where t is the predicted time for a maximum, t_0 is the time of the initial maximum, n is the cycle number, P is the period of the variable star, R_E is the radius of the Earth's orbit, ω_E is the angular frequency of the Earth around the Sun, and ϕ is the angle between the Sun-CY Aqr vector and the Sun-Earth vector at $t = t_0$.

The angle ϕ can be determined as follows, assuming the Earth's orbit to be circular. Ignoring very small parallax shifts, the vector from the Earth to the variable star points in the same direction as the one from the Sun to the star. Therefore, ϕ is zero when the Earth is exactly between the Sun and CY Aqr. Then, knowing the Sun's right ascension at a given time, we find that

$$\phi = (\mathrm{RA_{Sun}} - \mathrm{RA_{CY}} + 12 \text{ hours}) \times (15^\circ / 1 \text{ hour}). \quad (2)$$

There is an extra twelve hours because $\mathrm{RA_{Sun}}$ comes from the vector from the Earth to the Sun, whereas we need the vector pointing in the opposite direction. Using data from the 1992 edition of *The Astronomical Almanac* and ignoring the small declination of CY Aqr, we found ϕ to be 20.84°. Some of our observational statistics defined above are listed in Table 1.

Finally, we calculated values for t_0 and P in equation 1 using a linear least-squares fit to the data for various pulse widths. The results of this analysis are presented in the first three columns of Table 2. Averaging these results we report that the period is 87.89534 $\pm$ 0.00015 minutes.

Table 2. CY Aqr Analysis Results

Pulse Width (min)	t_0 *(min)*	*Period (min)*	*Model Radius* (10^{11} *m*)
8.0	1547.004	87.895533	1.593554
9.0	1546.719	87.895411	1.559974
10.0	1546.431	87.895185	1.498064
11.0	1546.188	87.895283	1.524944
12.0	1545.874	87.895502	1.584903
13.0	1545.590	87.895579	1.605925
14.0	1545.358	87.895311	1.532530
15.0	1545.142	87.895287	1.525903
16.0	1544.907	87.895353	1.543975
17.0	1544.636	87.895306	1.531107
18.0	1544.442	87.895160	1.491042
19.0	1544.219	87.895198	1.501497
20.0	1543.971	87.895335	1.538974

3. Discussion

Figure 2 displays the observed relative flux data from CY Aqr as well as a modeled fit overlaid on each peak. This model, obtained from Antonello *et al.* (1986), is a result of Fourier decomposition of data from Zissell (1968). Antonello *et* *al.* used almost 700 nights of observations published by Zissell to derive the coefficients for this model. The period of this model has been published as 0.06104 day, or 87.90 minutes, which matches our results to about 0.3 second. One may note that our observed light curves match the model fairly well also, as seen in Figure 2. A least-squares fit to the model curves would be another way to determine the phase and period of the light curve. We leave this exercise to a future group of students.

By comparison, Rolland *et al.* (1986) state the ephemeris is

$$T = \mathrm{JD}_{(\mathrm{hel.})}\, 2440892.637 + 0.061038318\,\mathrm{E} - 4.58 \times 10^{-13}\mathrm{E}^2, \qquad (3)$$

which translates to a period of 87.89517792 minutes at the mean epoch of our observations. Clearly, our results are in better agreement with Rolland *et al.* than with Antonello *et al.*

Rolland *et al.* also computed an explicit value for the period change of the star's variations. They claim that

$$d(\ln P)/dt = -8.37 \times 10^{-8}\,\mathrm{yr}^{-1}. \qquad (4)$$

This yields a period change on the order of ten milliseconds from the zero date of the ephemeris stated in equation 3 (November 1, 1970) to late September of 1992. This difference is approximately the same order of magnitude as our stated error, so it is too small to be detected with our observations.

As a short extra project, we tried setting the period to be equal to the value published by Rolland *et al.* and varied the Earth's orbital radius along with the initial time, again assuming that the Earth's orbit is effectively circular. The results from this exercise are displayed in the last column of Table 2. From these results, we conclude that the best fit to the data is a radius of $1.5386 \pm 0.0360 \times 10^{11}$ meters.

We also tried varying all three parameters: initial time, period, and radius. Unfortunately, we have too few data points for such an analysis. To successfully determine both the period and the radius from the data, we would need to observe peaks at a few more points in the Earth's orbit, so the numerical routines could distinguish between variations due to the Earth's revolution and ones due to the simple fact that the period is unknown. In the fall of 1993, a new group of students will attempt this same project and, hopefully, the two years of data will enable them to determine these values without the use of published data.

4. Acknowledgements and Dedication

We wish to thank Aparna Venkatesan and David Walend for their assistance in data collection for this project. We are also grateful to Professor James Cordes for his suggestions about the numerical analysis of the data.

This paper is dedicated to Clinton B. Ford, a beloved sponsor and friend of Cornell University and longtime member and officer of the AAVSO. Without Mr. Ford's encouragement and generous support, the Hartung-Boothroyd Observatory might never have come into existence, much to the loss of many aspiring Cornell astronomers. We are saddened by his recent passing and will miss him dearly.

References

Astronomical Almanac, 1992, U. S. Government Printing Office.

Antonello, E., Broglia, P., Conconi, P., and Mantegazza, L. 1986, *Astron. Astrophys.,* **169**, 122.

Houck, J. R. 1989, *Dwarf Cepheid Variables,* unpublished.

Peña, J. H., Peniche, R., and Hobart, M. A. 1985, *Inf. Bull. Var. Stars,* No. 2672, 2.
Roland, A., Peña, J. H., Lopez de Coca, P., Peniche, R., Gonzalez, S. F. 1986, *Astron. Astrophys.,* 168, 125.
Zissell, R. 1968, *Astron. J.,* 73, 696.

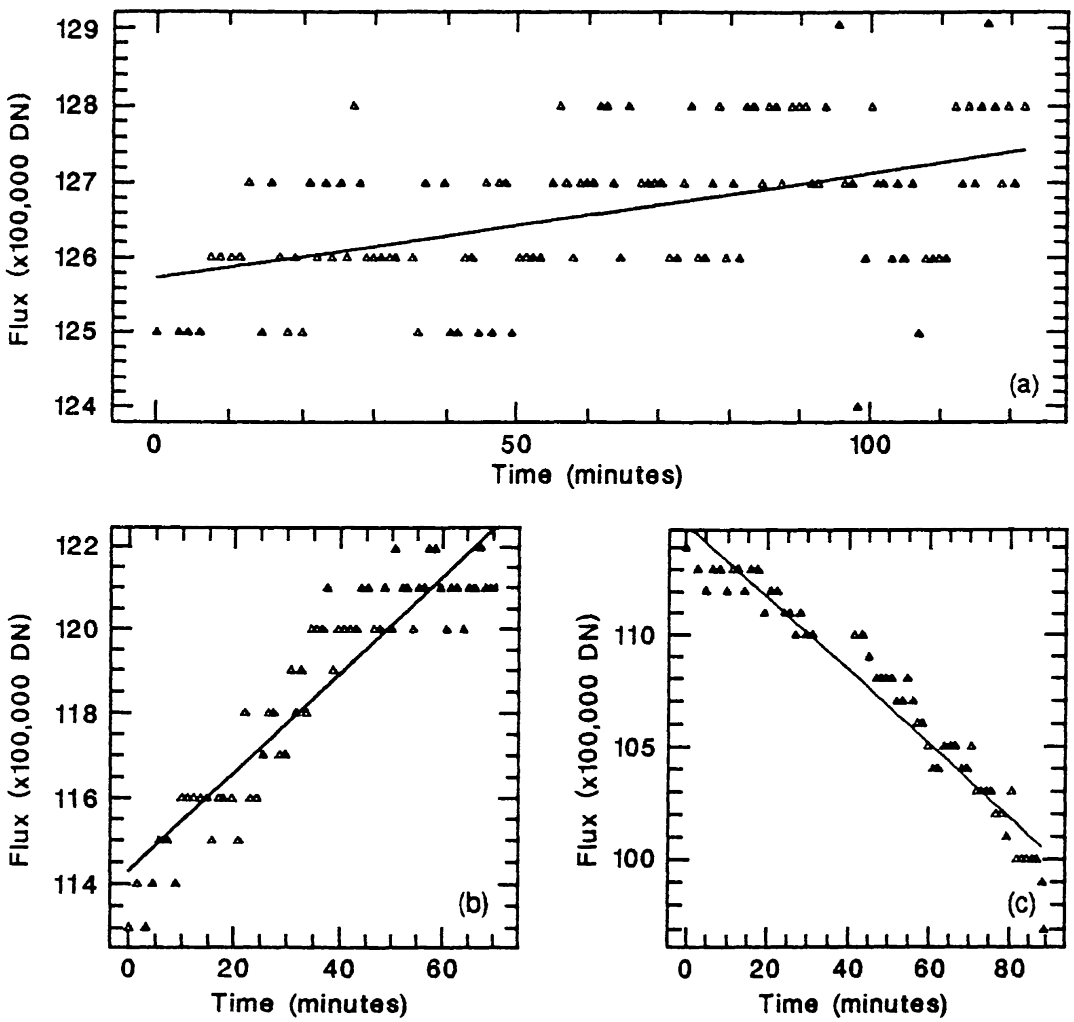

Figure 1. Diagrams of the observed flux of the comparison star (triangles) overlaid with a linear fit to each data set (solid line). Graph (a) is September 24, 1992, peaks 1 and 2; (b) is September 25, peak 16; and (c) is October 29, peak 575.

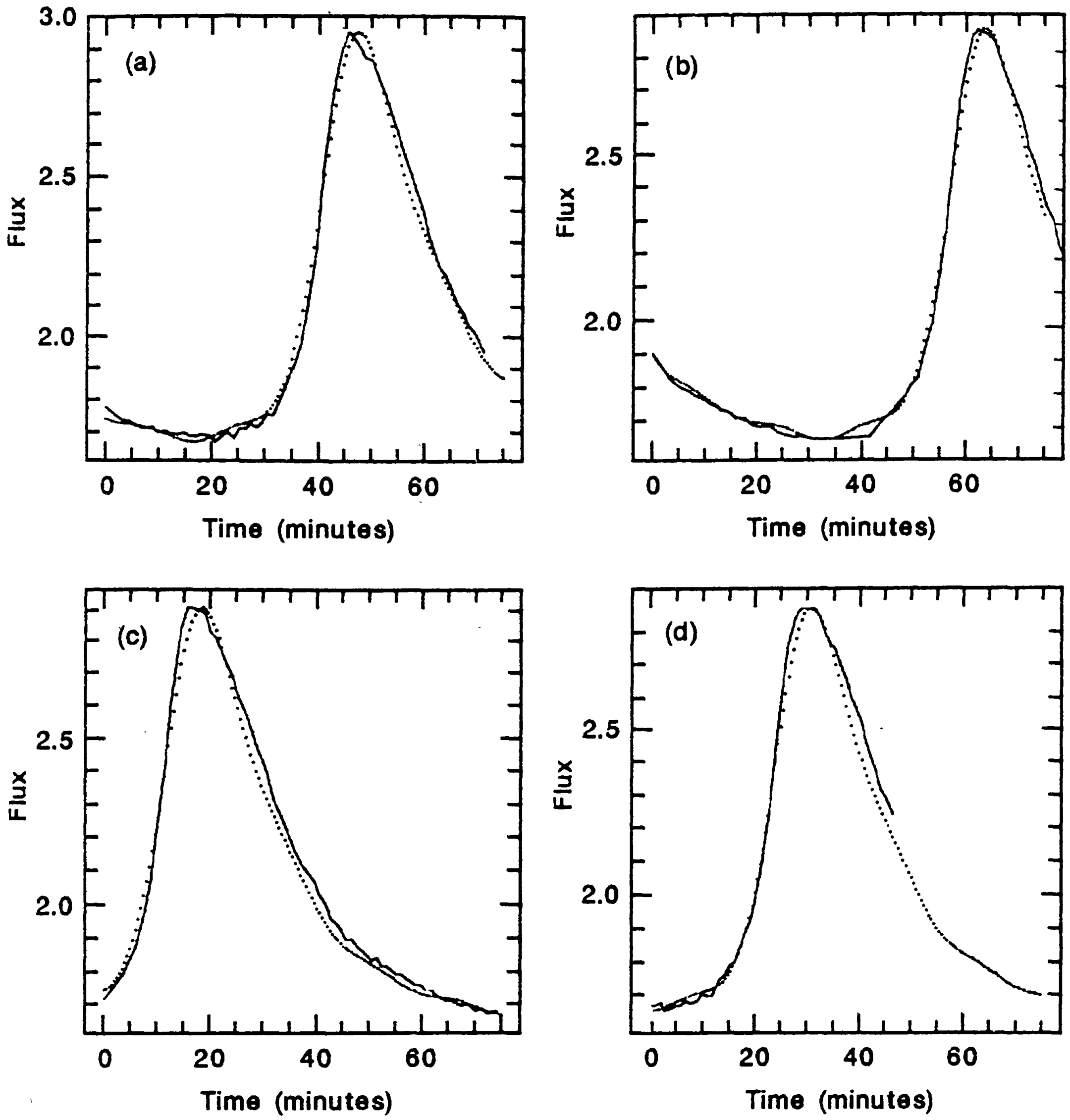

Figure 2. Light curves of CY Aquarii: observed data (solid) overlaid with model (dots). Graph (a) is peak 0; (b) is peak 1; (c) is peak 16; and (d) is peak 575. The flux axis is the ratio of observed CY Aqr flux to the linear fit flux of the comparison star.