EXPOSURE NOMOGRAPHS

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Presented at the 84th AAVSO Spring Meeting, May 13, 1995

Abstract

Correct exposure times may be determined from nomographs relating signal-to-noise ratio, exposure time, color, seeing, and magnitude. The equations needed to construct the nomographs are developed. Calibration techniques are discussed.

1. Introduction

When CCD observers gather they are apt to boast of the faintest star they can detect with their systems. Impressive claims of reaching magnitude twenty with six-inch telescopes make one think that one does not need to build giant telescopes any more. Since AAVSO observers are in the business of measuring the brightness of star images, we should be careful to note the difference between the term "detecting" and "measuring" a star.

The following discussion relates to CCD images in particular, but can be applied to photoelectric photometry (PEP) as well. A photoelectric photometer can be thought of as a CCD array of one pixel. By the correct choice of units, the formulas can generate useful nomographs for PEP exposures as well.

2. Noise

Figure 1 contains a 180-second CCD(V) exposure of V1028 Cyg when it was in outburst. The variable and comparison stars are well exposed, and there are traces of fainter stars in the background. The variable can be measured with an uncertainty of about one percent, or +/- 0.01 magnitude. Figures 2 and 3 contain images of the same variable at minimum. Figure 2 is a 1800-second exposure with the same camera as in Figure 1, while Figure 3 is a digital scan of a Palomar Observatory Sky Survey (POSS) photograph in V light (AURA 1993). The variable is definitely detectable at magnitude 20.3 in the 1800-second exposure, and still fainter stars can be found. While the variable is detectable at magnitude 20.3, the uncertainty of the measurement is +/- 1.3 magnitude. This is hardly precision photometry, yet it is a positive observation.

Compare the two images to detect the faintest stars. There are cases of faint dark spots in the CCD image for which there are no POSS stars present. We are seeing examples of fluctuations in the background noise of the system. To claim these fluctuations are detected stars is false or a case of wishful thinking.

Any measurement we make has an uncertainty, or error, associated with it. This uncertainty is a result of the noise in the system. A good measure is one where the signal is much greater than the noise. The ratio of Signal to Noise (S/N) is a useful number for estimating the uncertainty of a measurement. In photometry the signal we measure is proportional to the number of photons detected from our star. The arrival of photons from a particular star has a mean rate depending on its brightness. The longer we expose or integrate a signal, the larger the total signal becomes.

Since photon counting obeys Possion statistics, the error in the signal is given by the square root of the signal. Equation (1) shows that the S/N of a star increases as the

1996JAVSO..24...26Z

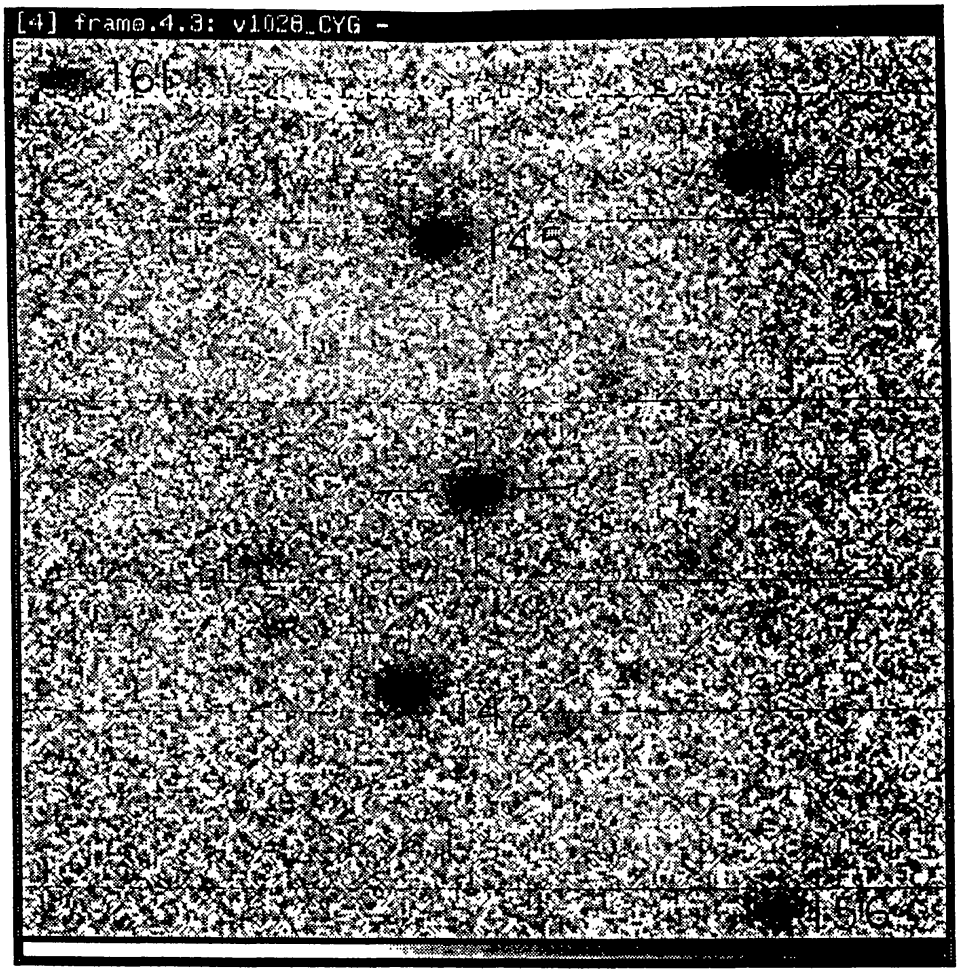

Figure 1. 180-second CCD(V) exposure of V1028 Cyg in outburst. The variable and comparison stars are well exposed. Faint background stars can be detected but have too small a signal for meaningful measurement.

square root of the signal. The total signal is proportional to time, so the S/N is also proportional to the square root of the exposure time. Thus, to double the S/N we must expose four times longer. To add another significant digit to the accuracy of a measure would require an exposure one hundred times longer—if photon noise were the only source of noise.

$$\frac{S_*}{N_*} = \frac{S_*}{S_*^{1/2}} = S_*^{1/2} \qquad (1)$$

There are other sources of noise than just the incoming signal to consider. When we subtract the dark signal and the sky signal from the star signal, the noise contributions from these sources do not subtract out. Also, there is a contribution to the noise level from the readout electronics of the chip.

Equation (2) gives the S/N of an observation with all the noise sources added in quadrature (NOAO 1987). The subscripts *, s, d, and r denote the star, sky, dark, and read source, respectively. Equation (2) requires the total signal and the noise from each source to be entered. It is more useful to rewrite equation (2) to include the exposure time (t) and the number of pixels (p) in a star image.

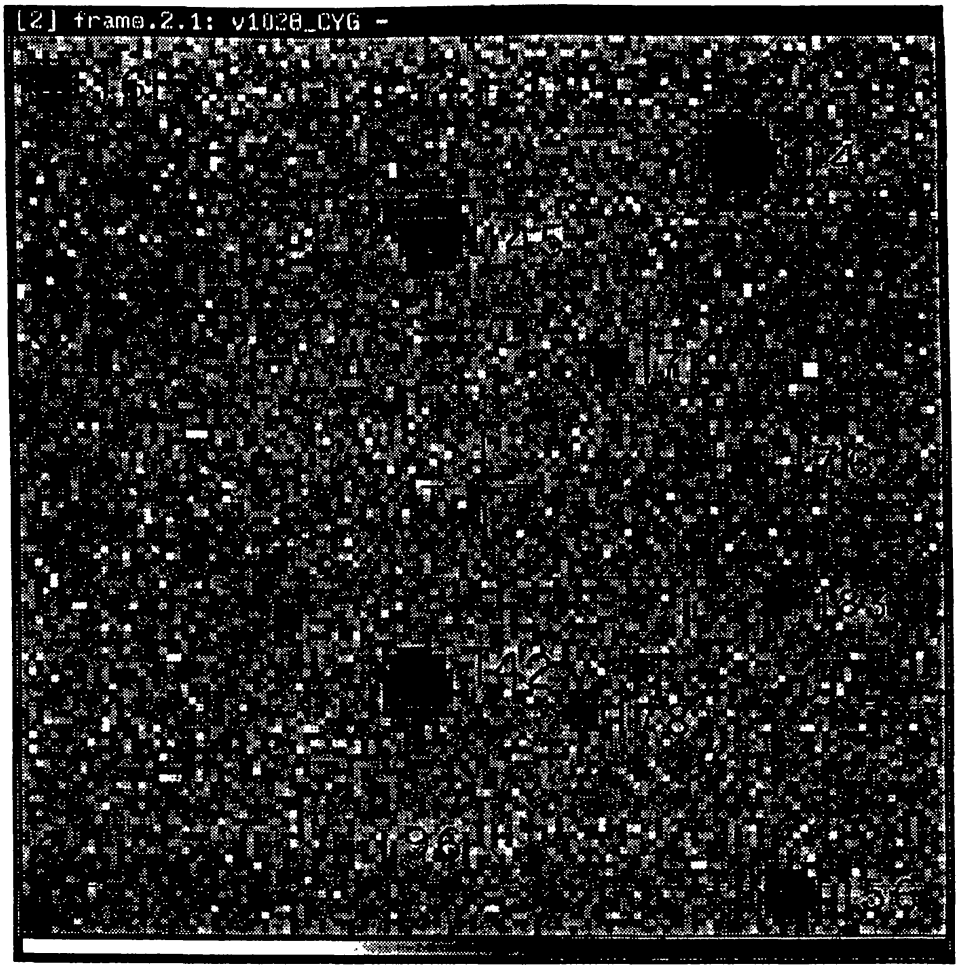

Figure 2. 1800-second CCD(V) exposure of V1028 Cyg at minimum magnitude = 20.3 +/- 1.3. The faint stars of Figure 1 are more prominent in this longer exposure.

$$\frac{S_*}{N_*} = \frac{S_*}{\{N_*^2 + N_s^2 + N_d^2 + N_r^2\}^{1/2}} \qquad (2)$$

The variable C_* in equation (3) is the signal rate in ADU/sec for the star. C_s and C_d are the signal rate in ADU/sec/pixel for the sky and dark signal. The read noise is given by r in units of ADU/pixel. ADU is Analog to Digital Units.

$$\frac{S_*}{N_*} = \frac{C_* t^{1/2}}{p^{1/2}\left(\frac{C_*}{p} + C_s + C_d + \frac{r^2}{t}\right)^{1/2}} \qquad (3)$$

Examination of equation (3) shows that the S/N increases proportionally to the square root of the exposure time, as we observed before. The longer the exposure, the smaller the contribution from read noise since the term r^2/t decreases with time. Therefore, one long exposure will be better than the sum of many short ones. The main disadvantage of one long exposure is that a guiding error can make it useless. It is often better to sum a number of intermediate-length exposures. Any exposures with guide errors can be discarded and the rest used.

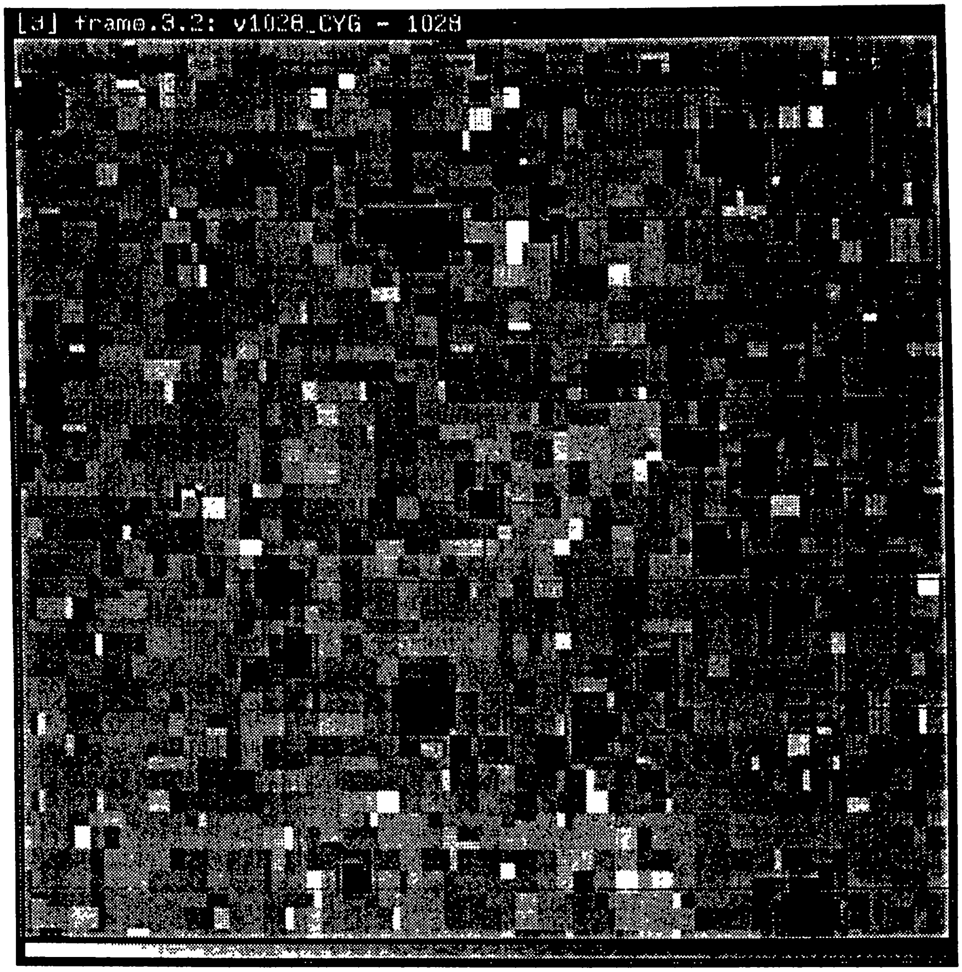

Figure 3. POSS digital scan of the same region as Figure 2. Compare the stars visible on both images. There are some faint star-like spots on the CCD image that are not on the deeper POSS image. These are fluctuations in the background noise and should not be claimed as “detected” stars.

Introducing the number of pixels in a star image, p, accounts for seeing variations. Equation (3) shows that S/N increases as p decreases. As the seeing improves, the star image covers fewer pixels. The value of “seeing” used is the Full Width at Half Maximum (FWHM) of a star image. When measuring an image, an aperture is chosen with a radius which includes ninety percent of the star image. The number of pixels within this aperture is the value p in equation (3).

We could conclude that the best situation would be when all the star light falls within one pixel. That would mean that the sky, dark, and read noise from only one pixel would increase the total noise. This is true in an algebraic sense but in reality it would cause problems.

Imagine the case where the star image falls exactly on the corner of four adjacent pixels. When that image is read out there will be noise added from four sky, dark and read pixels instead of just one. The error of measure for two equal-magnitude stars would be different, depending on where the image fell on the chip. Over-sampling of an image gives a more uniform error calculation. It is also needed for finding the centroid of an image for astrometry, where the centroid can be found to a fraction of a pixel.

3. Calibrations

In order to use equation (3) to predict an exposure time, it is necessary to determine the value of the star signal, sky brightness, dark signal, and read noise of your system. If you are engaged in multicolor photometry, the star and sky values must be determined for each color used. The dark signal and read noise are, of course, independent of the filter used.

My system is composed of a 24-inch reflecting telescope with a Photometrics CH250 camera with a TH7896 chip of 1024 x 1024 pixels. The pixels are 0.019 mm square. The image scale is 0.635 arcsec/pix. Typical seeing is 3–5 arcsec. On rare nights the seeing has become diffraction-limited! Both components of β Del, with a separation of 0.6 arc-sec, would fall within one pixel on such a night.

The test report that came with the camera lists the read noise and the ADU conversion factor. These are shown in Table 1 along with the rest of the needed values. Determination of the dark signal in units of ADU/sec/pixel is straightforward. Five bias frames were taken and summed. Then a 30-minute dark frame was taken, followed by five more bias frames. The mean bias frame was subtracted from the dark frame and the result divided by 1800. A program that performs image statistics was then used to give the mean value of a single pixel. A similar procedure is used each night to calculate the number of dark frames that might be needed in the evening. The biases and a long dark exposure are taken after the chip has reached its operating temperature. The dark exposure, less the mean bias, is divided by the appropriate number to give 10-, 30-, 90-, 120-, 300-, 600-, and 900-second dark exposures, and the mean bias is added back onto each. This gives the short dark frames much less noise than a single short exposure.

The dark signal is strongly dependent on the temperature of the chip. For most silicon chips the dark signal doubles for each 7° C increase in temperature, which is why temperature regulation is very important.

The AAVSO CCD star chart for W Leo was used as a calibration source. B, V, R, and I magnitudes are given for a number of comparison stars in the field. Exposures were made in each color and analyzed in Image Reduction Analysis Facility (IRAF) to give the rest of the values in Table 1.

Table 1. Values of the constants needed to evaluate equation (3) for the telescope-camera system at Williston Observatory.

C is signal count rate with subscripts d, s, and * referring to dark, sky, and star; r is read noise; subscripts B,V,R,I refer to the standard filter colors of Blue, Visual, Red, Infrared (deep red). Star signal is given for that of a magnitude-10 star. Other magnitudes can easily be scaled from these values. Sky signal is also given in terms of magnitude-per-square-arcsec.

1 ADU = 4.07 e^-
r = 1.92 ADU/pix

C_d = 1.62 ADU/sec/pix @ -45.3°C
C_{sB} = 0.102 ADU/sec/pix 19.7 mag/$''^2$
C_{sV} = 1.41 ADU/sec/pix 18.2 mag/$''^2$
C_{sR} = 1.60 ADU/sec/pix 18.1 mag/$''^2$
C_{sI} = 1.17 ADU/sec/pix 17.6 mag/$''^2$

C_{*B} = 1446 ADU/sec
C_{*V} = 4856 ADU/sec
C_{*R} = 1853 ADU/sec
C_{*I} = 1862 ADU/sec

the rest of the values in Table 1.

The sky values were determined by placing the software aperture at various star-free areas of the image. IRAF reduction files recorded the mean value of the sky in ADU/pix. It was only needed to divide by the exposure time to get the sky signal in ADU/sec/pix. Also, the sky signal was converted to an equivalent value of a star magnitude per square arcsec.

The star signal was measured in a similar manner through IRAF. The IRAF output file lists the star and sky signal and the number of pixels within the aperture. After the sky signal is subtracted and the result divided by the exposure time one has the value of the star signal in ADU/sec. However, this value is for the particular magnitude of the star measured. Each star in the field will have a different value. Each particular star signal was converted to the equivalent signal of a star of tenth magnitude for convenience and to allow a mean value of all comparison stars in the field to be calculated.

4. Nomographs

Nomographs were a popular method in the past for finding quick solutions to complex problems. They are still useful in this age of digital computers. One may ask, "Why do I need a nomograph to find a number that my computer can determine more quickly and to many more decimal places?" The advantage of a nomograph is that one can scan the area around a solution to see a number of equivalent choices, rather than have a computer print out a single exposure time of 325.876341 seconds. How much can one deviate from this exposure and still get a reasonable S/N ratio?

Equation (3) was evaluated for each of four colors, B, V, R, and I , and within each color, for three values of seeing, 5, 3, and 1 arcsec. The Log_10 of S/N values ranging from 0 to 4 were plotted against the Log_10 of the exposure time, ranging from 0 to 4. The curves for magnitudes 10 to 20 were then plotted. These nomographs are shown in Figures 4, 5, 6, and 7, for B, V, R, and I, respectively.

To use the nomograph, first select the needed S/N. A Log S/N of 1.0 is equivalent to +/- 0.1 magnitude, 2.0 is equivalent to +/- 0.01, 3.0 is equivalent to +/-0.001 magnitude, etc. Next, select the plot for the color being used and the appropriate seeing. Lay a straight edge horizontally at the selected Log S/N value. Locate the proper magnitude line for the star being measured. Find the intersection of the straight edge with the magnitude line. Drop straight down and the read the Log Exposure Time. Look around the area of solution to see if you have a dark exposure saved that is the close to the needed exposure. Will a slightly shorter exposure give too small a S/N? What if the star is a little fainter than predicted? Will a longer exposure be better? All these questions can be answered quickly with a nomograph.

For those unaccustomed to thinking in logarithms, it may be more convenient to label the tic marks on the nomographs in their equivalent values of 1, 3,10, ... seconds and the algebraic values of S/N.

How well do the nomographs predict the correct exposure? The IRAF photometry output file contains much information about the image being measured. It gives an instrumental magnitude value along with an uncertainty of the value calculated from the actual scatter in the pixel intensities of the image and background. These calculated uncertainties match the nomograph values to within ten percent or, often, better.

References

Association of Universities for Research in Astronomy (AURA) 1993, *The Digitized Sky Survey*, disk 70.

National Optical Astronomy Observatories (NOAO) 1987, *Kitt Peak National Observatory Facilities Book.*

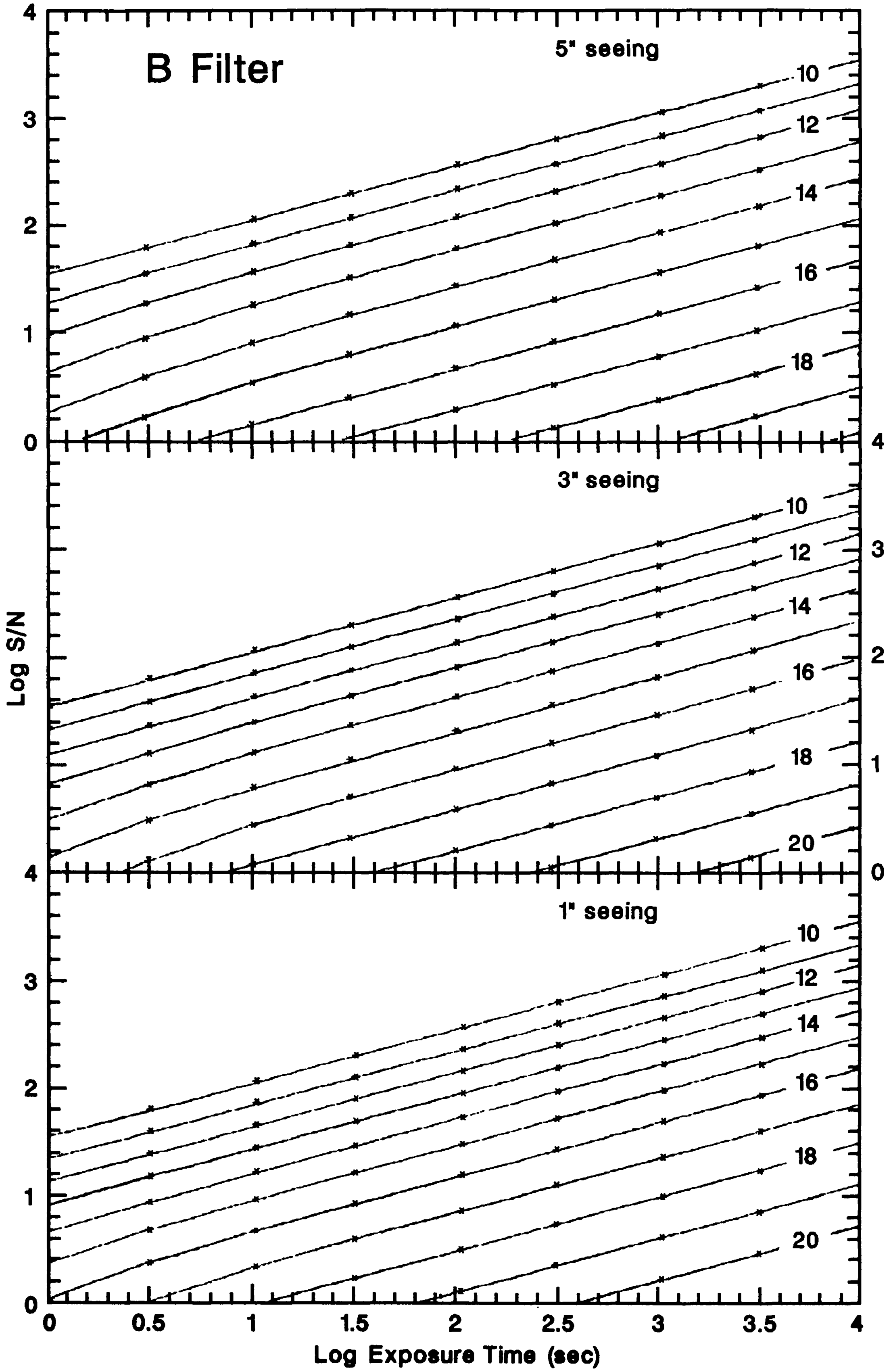

Figure 4. Nomograph for B filter. Seeing values of 5, 3, and 1 arcsec.

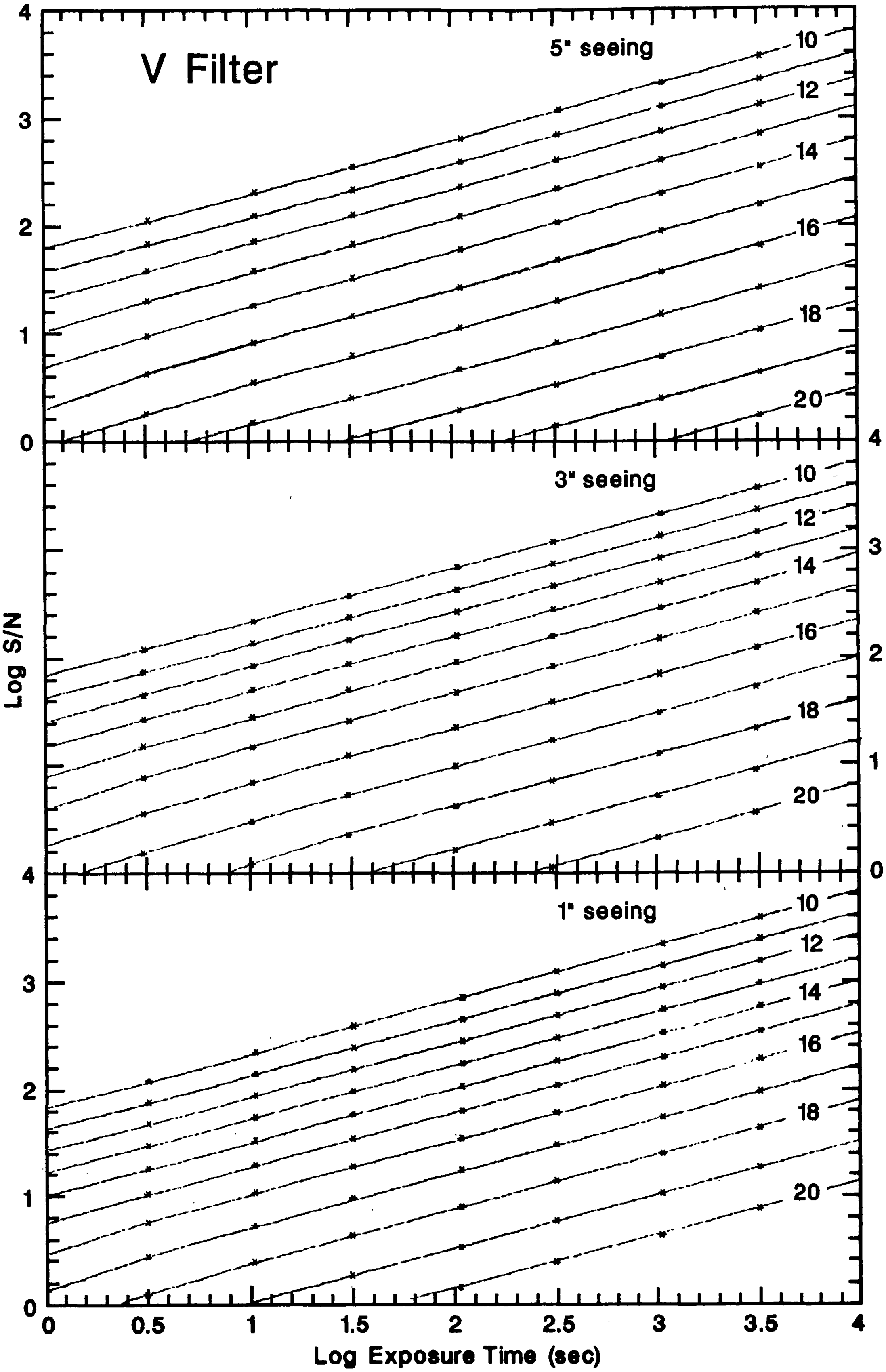

Figure 5. Nomograph for V filter. Seeing values of 5, 3, and 1 arcsec.

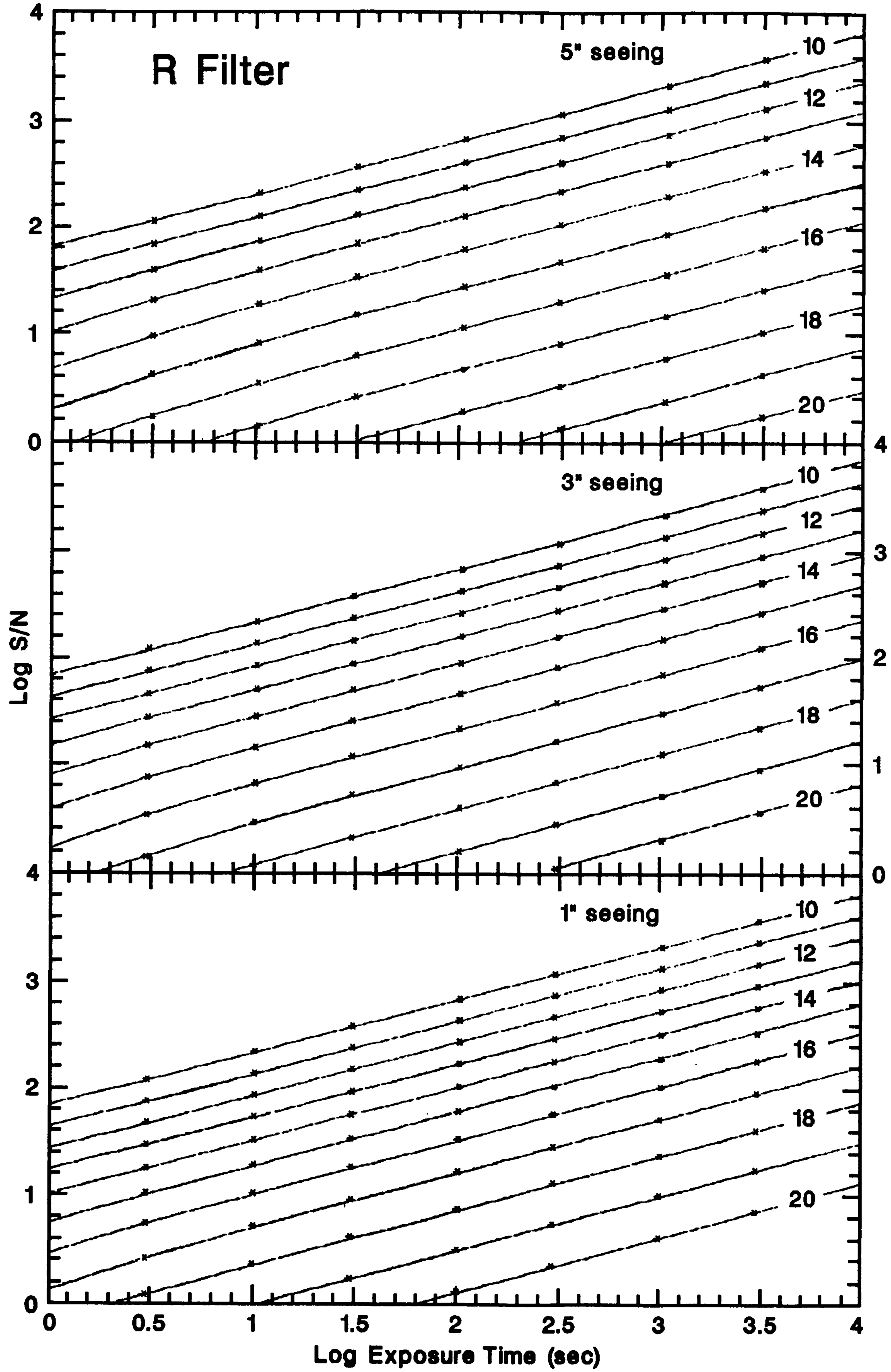

Figure 6. Nomograph for R filter. Seeing values of 5, 3, and 1 arcsec.

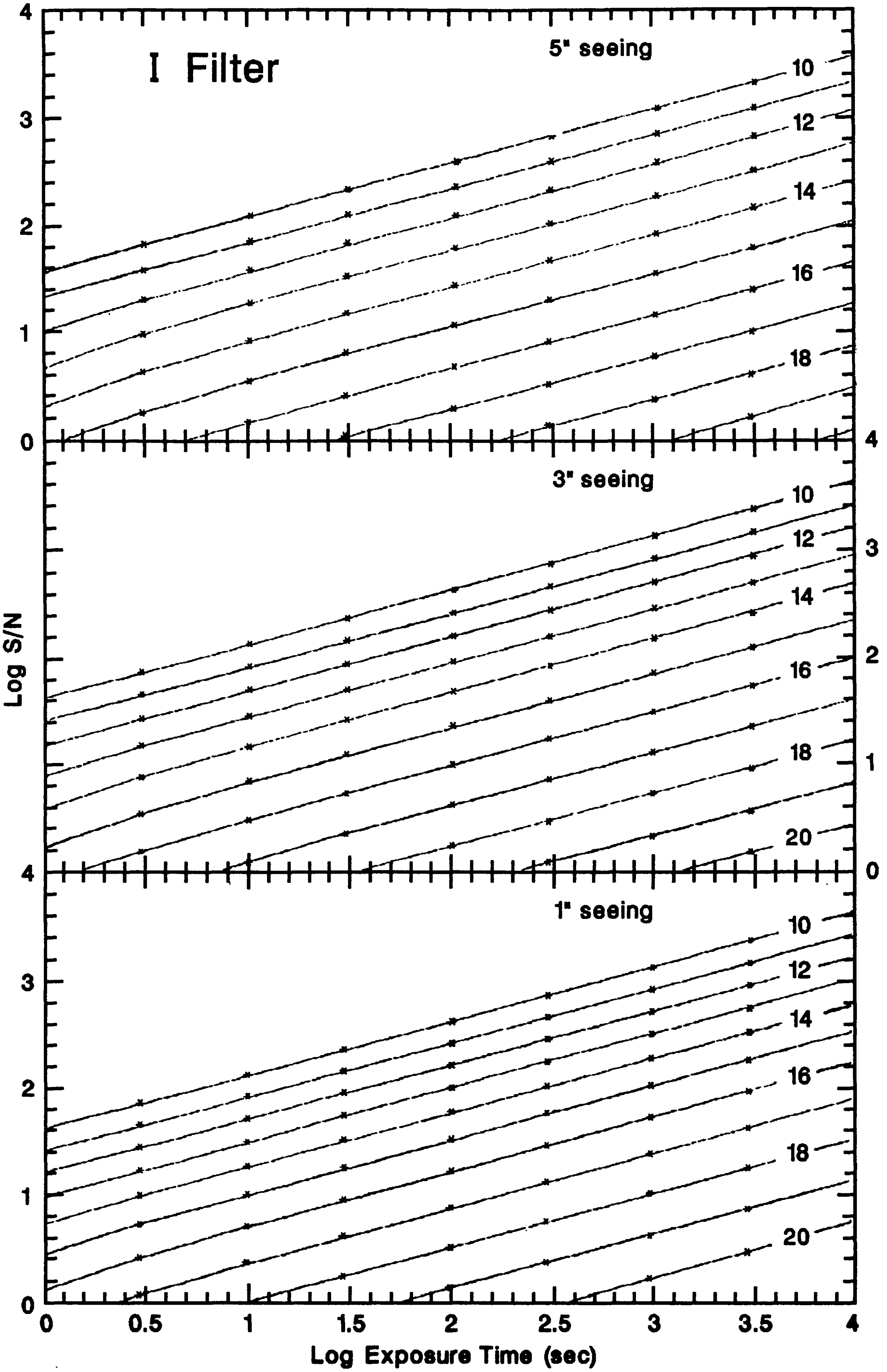

Figure 7. Nomograph for I filter. Seeing values of 5, 3, and 1 arcsec.