A METHOD OF DETERMINING THE PERIOD OF SHORT-PERIOD VARIABLE STARS

With Application to EL Comae and Three New Variables

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The determination of the period of a short-period variable star is, mathematically, a simple procedure. In practice, however, the analysis of one star may require weeks or even months of effort. The main difficulties arise from a lack of data and from a periodicity which is often inherent in the observations themselves. This paper outlines the method used at the Maria Mitchell Observatory for determining periods from photographic observations. To give the reader an understanding of the practical problems encountered, each step is illustrated with data on Variable 13,* an RR Lyrae star in Coma Berenices discovered by Dr. Dorrit Hoffleit in 1969.

The basis for all estimates and calculations of the period of a variable star is data on the fluctuation of the star's brightness. Observations of magnitude may be made in terms of arbitrary units corresponding to the magnitudes of adjacent comparison stars. For example, b4 denotes that the variable is fainter than comparison star b by 4/10 of the interval between b and c. Estimates may be left in this form or may be converted to their numerical equivalents for the purpose of calculating the period. If the observations are not converted, estimates of the magnitude intervals between comparison stars should be made in order to assess scattering more correctly. The accuracy of the data and the number of cycles represented by the time span of the observations determine the number of significant figures to which the period can be calculated.

If observations of a star could be made continuously over, say, a 48 hour interval, most difficulties in analyzing a short-period variable would be eliminated. The period would simply be equal to the average time between similar phases of variation, for example, between successive maxima or minima. Unfortunately, actual observational runs are frequently interrupted by poor weather, sunlight, and moonlight. For most stars whose periods are over 1/3 of a day, the entire cycle cannot be observed in sequence. Often only one observation of a variable can be made per night, the photograph representing a small fraction of the period. The exact shape of the light curve and length of the period are rarely obvious from the raw data.

A graph of magnitude versus Julian date, on a scale of approximately ten days to the inch, is helpful in assessing the general shape of the light curve. In the case of a long-period variable star, periodicity is immediately evident from this graph, but on graphs of short-period stars trends may be evident only in occasional night runs. The relative dis-

* Provisional designation for an as yet unpublished star.

tribution of data points between maximum and minimum magnitude may, however, be characteristic of a certain pattern of light variation. If the majority of points are at maximum and very few observations are of minima, the star is probably an eclipsing binary; if most observations are of minima and the star rises quickly to maximum, a cluster variable is suspected; if most points are of intermediate magnitude, the star may be a cluster variable, a cepheid, or a W Ursae Majoris star. A knowledge of the form of the light curve is useful in estimating the probable length of the period and in judging the merits of a trial period.

A lower limit to the period of a particular star can be established by constructing a composite light curve from the data of several night runs. This procedure is illustrated in Figs. 1 and 2 for Variable 13. Series of two or more consecutive observations are plotted on the scale of one Julian day per inch. These fragments are then pieced together in accordance with the proposed pattern of light variation.

Since most observations of Variable 13 are of intermediate or minimum brightness, a light curve of the RR Lyrae prototype is constructed.

Fig. 1 SERIES OF TWO OR MORE OBSERVATIONS OF VARIABLE 13

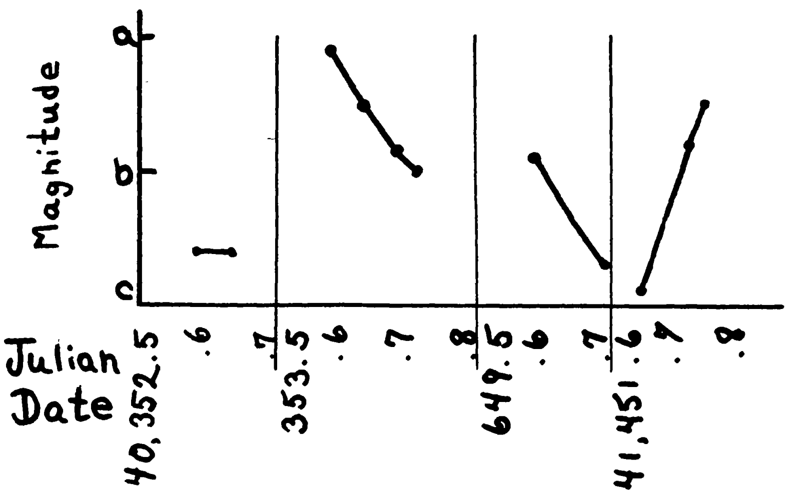
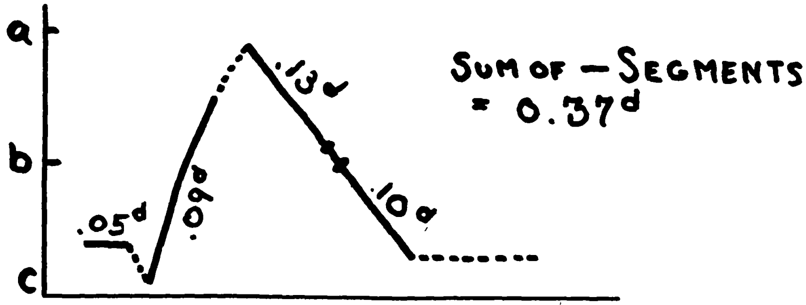


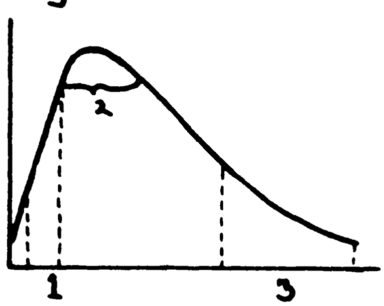
Fig. 2 COMPOSITE LIGHT CURVE



Allowing for overlap in the observations and for minor errors in estimates, the composite curve for Variable 13 establishes a minimum period of approximately 0.4 days. 0.5 or 0.6 days would be educated guesses as to the actual length of the period. A composite light curve does not set a strict upper limit to the period because in all probability it does not account for the entire duration of minimum. A rough estimate of the upper limit may be made, however, based on the rates of ascent and decline, and the percentage of observations of minimum. In the case of Variable 13, it would be reasonable to assume that the period is somewhat less than one day.

A more reliable upper limit to the period may be determined if observations are available of the same phase of variation separated by a small number of days, and thus by a small number of cycles. The time intervals between these observations may be calculated, the smallest first difference defining the upper limit. The reference phase should correspond to the steepest portion of the light curve. When dealing with a star suspected of being a cluster variable, the reference phase should be chosen on the ascending branch of the curve (interval 1 in Fig. 3). Inasmuch as it is often difficult to determine whether a single observation is on the ascending or the descending branch, maximum may be chosen as the reference, although error is increased from interval 1 to interval 2. Clearly, first differences computed using points at minimum would be of no value since interval 3 is so long. Differences between minima are effective, however, in determining the periods of eclipsing binaries.

Fig. 3



Variable 13 was found to vary between a0 and c0; observations between a0 and a5 were considered maxima. Had more data been available, only points closer to a0 would have been judged maximum, and the errors in the first differences would have been reduced. All observations of the star at maximum separated by less than twenty-five days are listed in Fig. 4. The dates from successive plates are averaged. In this example, even the smallest first difference is too large to be considered a plausible upper limit.

Fig. 4

JD of Maximum	First Differences	Greatest Common Factor
39 256.599 – 268.662	12.063	4.021
40 003.627 – 022.531	18.904	3.781
353.585 – 357.582	3.997	3.997
357.582 – 377.654	20.072	4.014
734.598 – 738.632	4.043	4.043
738.632 – 746.683	8.051	4.025

As is so often the case with short period variables, the first differences and their average common factor are very close to multiples of one day. Since a tentative upper limit for the period of Variable 13 was set at one day by the composite light curve, the average greatest common factor 3.980 must be divided by at least 4 (=.995) and more probably by 5 (=.796), 6 (=.663), 7 (=.568), 8 (=.498), 9 (=.442), 10 (=.398), 11 (=.362), or 12 (=.332). Judging from the composite light curve (Fig. 2), .568 and .663 would be the most likely trial periods.

The merits of any trial period are easily judged by plotting magnitude versus phase, where phase is expressed either in days or in fractional parts of the period. Perhaps the most obvious means of determining the phase of an observation is expressed by the formula:

$$\phi_d = JD - JD_o - nP. \qquad (1)$$

The phase in terms of days (ϕ_d) equals the Julian date of the observation, minus the Julian date of a convenient reference epoch, minus an integral number of periods (n). The quantity ($JD - JD_o$) represents the time elapsed since the reference date. The value of <u>n</u> is chosen so that ($JD - JD_o$) - nP is the smallest possible positive number.

Example: If JD_o = 40,348.570 (the phase of that observation is then defined as 0.00) and P = 0.75^d,
Then the ϕ_d of an observation at JD_1 = 40,359.691 is given by

$$\phi_d = 40{,}359.691 - 40{,}348.570 - 14(.75) = 0.621^d.$$

This result indicates that the observation at JD_1 is 0.621 days farther through the cycle than the observation at JD_o.

This method is effective but cumbersome, inasmuch as one multiplication and two subtraction processes are required to calculate the phase of each observation.

A much more efficient procedure, based on the same principle, is evident when equation (1) is divided through by the period:

$$\phi_P = JD\left(\frac{1}{P}\right) - JD_o\left(\frac{1}{P}\right) - n \qquad (2)$$

$$\text{or } JD\left(\frac{1}{P}\right) = JD_o\left(\frac{1}{P}\right) + n + \phi_P \qquad (3)$$

The second form reveals that if the Julian date of any observation is multiplied by the reciprocal of a trial period, the result is equal to a constant $JD_o(1/P)$, plus an integer <u>n</u>, plus a fraction which is the phase expressed in terms of the period. The constant can be ignored. Therefore, JD(1/P) equals a whole number plus a fraction which is the desired phase.

Example: If P = 0.75^d

JD = 40,359.691, then

$JD(\frac{1}{P})$ = 53,812.921, and from equation (3), in which

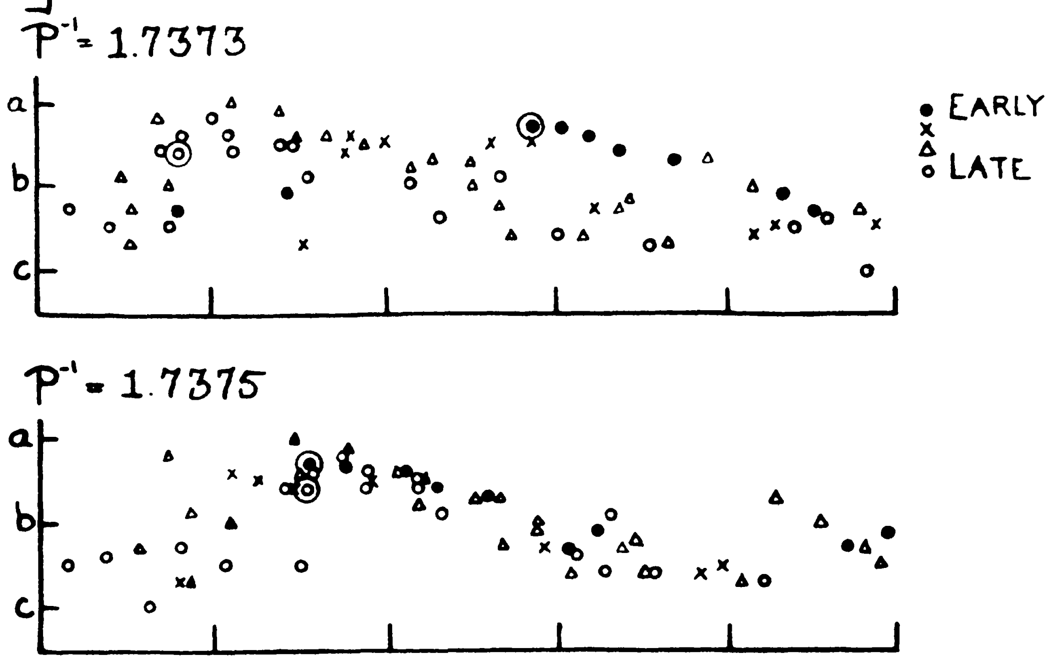
$JD_o \equiv 0_1$, ϕ_P = 0.921.

When the phases of the observations have been calculated for a trial period, a graph of magnitude versus phase will hopefully take the form of a smooth curve to within the accuracy of the data. A convenient scale for phase is 0.2 periods per inch. The extremely high probability that the first trial period will not be the correct one, however, makes it advisable to plot only one or two years' observations at once. The proper correction to the trial period may be found more easily if earlier data points can be distinguished from the later ones; this can be accomplished by either symbol or color coding.

The graph of a trial period often reveals that more recent observations are slightly out of phase with earlier ones (see Fig. 5). The calculation of the necessary correction is based on the positions of representative points from each group of observations. $\Delta\phi_p \div \Delta JD$ equals the absolute value of the correction, where $\Delta\phi_p$ is the desired change in position of one point relative to another, and ΔJD refers to the time interval between the two representative points. If late points are to be moved forward, the correction must be <u>added</u> to the trial period in accordance with the following line of reasoning: an increase in the reciprocal period results in an increase in all ϕ_p's, but the increase will be larger for points with a larger Julian date than for those with smaller dates, therefore, late points will move forward relative to early points. By similar reasoning, a decrease in the reciprocal period will cause late points to move back relative to earlier ones.

This method of correction is illustrated by the graphs in Fig. 5.

Fig. 5



A reciprocal period of 1.7373 was found for Variable 13 which fits the observations of the years corresponding to X's, Δ's, and O's. The graphing of the earliest observations (●'s) indicated that the reciprocal period needed to be increased. The correction was calculated for the circled points as follows:

$$JD_1 \simeq 40{,}738.6$$

and $$JD_2 \simeq \underline{38{,}887.7}$$

Therefore $$\Delta JD = 1{,}850.9$$

From Fig. 5 take $$\Delta \phi_P = 0.41$$

Thus $$\frac{\Delta \phi_P}{\Delta JD} = 0.0002$$

A correction could also be calculated which would move the early •'s forward by a phase of 0.59. Graphs of both corrections indicated, however, that +0.0002 was the more effective correction.

As a result of insufficient data, periods are sometimes found which seem to fit the observations but are actually only related to the true period. The most obvious example of this situation is a period which is n times the true period. This confusion arises when there are no closely spaced observations from which to determine a composite light curve.

Another set of erroneous periods, referred to as "spurious periods," arise because of a periodicity in the observations themselves. The most common observational period is related to the day-night cycle. Photography is limited to the darkest part of the night, and an object is usually photographed when it is on the meridian. Other observational intervals correspond to the month, since photography is interrupted when the moon is up; to the year, for all but circumpolar stars; and to any interval peculiar to a particular observing schedule. Such periodic observations resemble those made with a stroboscope which imposes an observational period on the process being viewed and can even make the sequence appear to run backwards.

If all observations were made at a precise interval, say that of a sidereal day, an infinite number of periods would seem to fit the data equally well. The relationship between the true period (P_0), a spurious period (P_1), and the observational period (q) is derived as follows:

According to equation (2)

$$\phi_0 = \Delta JD \left(\frac{1}{P_0}\right) - n_0 \qquad (4)$$

And $$\phi_1 = \Delta JD \left(\frac{1}{P_1}\right) - n_1 \qquad (5)$$

where ϕ_x refers to the phase in terms of the period P_x, and n is an integer. A spurious period occurs when $\phi_0 = \phi_1$ for all observations when $\Delta JD = q$. Subtracting (5) from (4), therefore, gives:

$$\frac{q}{P_0} - \frac{q}{P_1} = n_0 - n_1 = N,$$ a positive or negative integer, so that

$$\frac{1}{P_1} = \frac{1}{P_0} \pm \frac{N}{q} \qquad (6)$$

The observational interval relevant to a particular star is the one which corresponds most nearly to its period. For short-period variables the sidereal day interval is the most important; the synodic month interval is critical for stars with periods around thirty days. Under most circumstances, the yearly interruption in observations would not affect calculations for a short-period variable. At the Maria Mitchell Observatory, however, this interval must be taken into account because of the small fraction of the year during which Coma Berenices can be observed and the small number of plates taken each year. The reciprocals of the three most common observational intervals are calculated below:

$$\frac{366^{d}.2422}{365.2422} = 1.002738 \text{ for mean sidereal day interval.}$$

$$\frac{1}{29^{d}.53059} = 0.0338632 \text{ for synodic month interval}$$

$$\frac{1}{365^{d}.2422} = 0.0027379 \text{ for tropical year interval}$$

Spurious periods can be avoided whenever observations made outside the regular interval are available. Photographs taken when the star is at a large hour angle accomplish this purpose. Additional data from other observatories at different longitudes are also very helpful. Minimum and maximum periods as derived from composite light curves and first differences eliminate spurious periods corresponding to a large $\underline{N}$ in the formula:

$$\frac{1}{P_1} = \frac{1}{P_0} \pm \frac{N}{q} \,. \qquad (6)$$

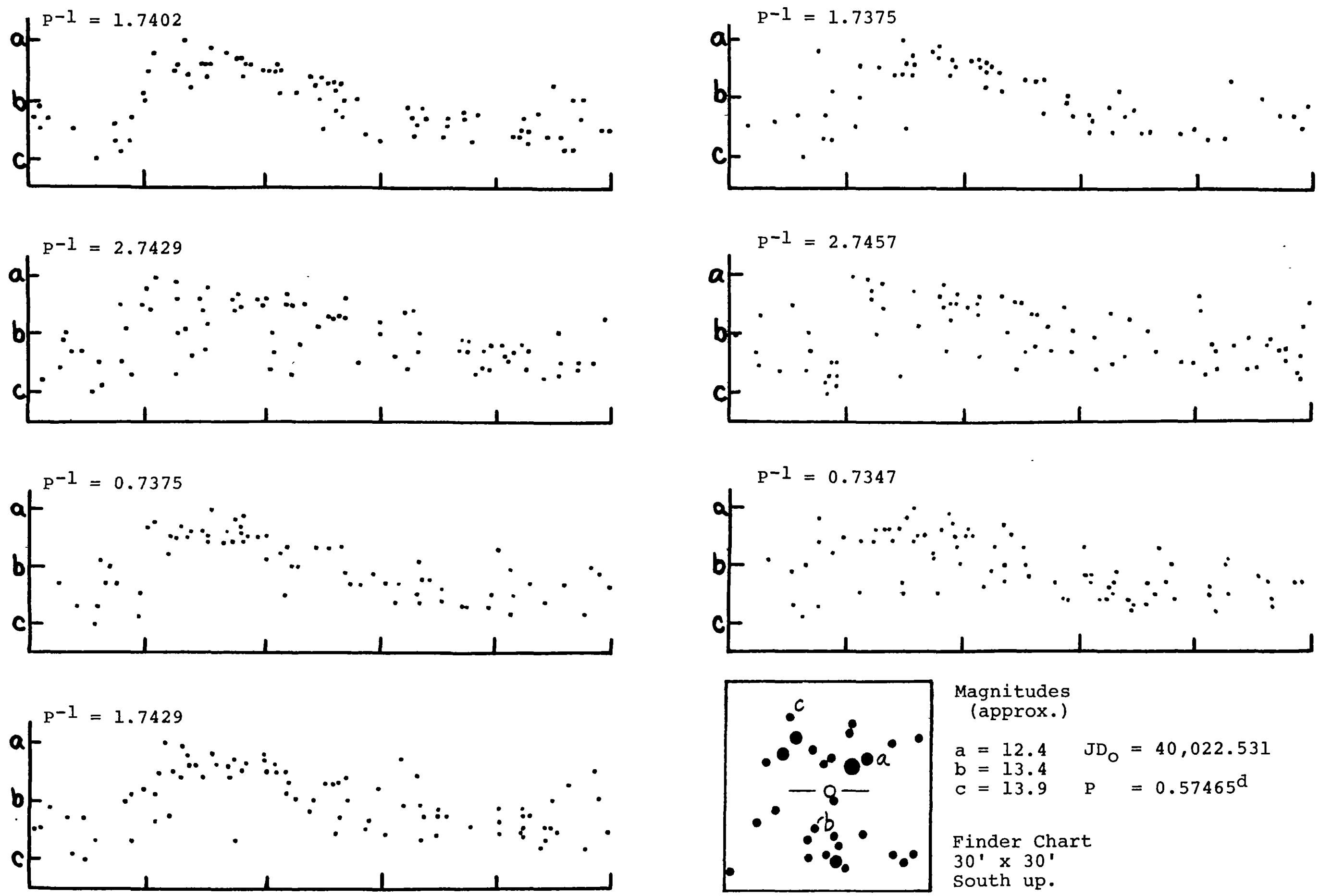
Whenever a trial reciprocal is found which seems to fit the data, all plausible related spurious periods should be investigated. If one of these periods also fits the observations, a choice between these periods must await further data.

The light curves corresponding to the correct period of Variable 13 and related spurious periods are illustrated in Fig. 6:

1.7402 =	true reciprocal period		
2.7429 =	reciprocal period plus day interval		
0.7375 =	"	"	minus day interval
1.7429 =	"	"	plus year interval
1.7375 =	"	"	minus year interval
2.7457 =	"	"	plus day and year interval
0.7347 =	"	"	minus day and year interval

Figure 6. True and Spurious Periods of Variable 13

Position: $12^h\ 46^m\ 21^s$, + 33° 30.0' (1900)



The reason that all the spurious periods do not fit the observations equally well, as previously assumed, is that the observational period at the Maria Mitchell Observatory is not constant. When possible, Coma Berenices is photographed on the meridian, but as the season progresses, the constellation must be photographed at an increasingly large hour angle. Thus the observational interval is neither precisely a solar nor a sidereal day. Although the spurious periods do not fit the data as well as the true period does, periodicity is evident in all the graphs in Fig. 6.

As a National Science Foundation undergraduate research participant at Maria Mitchell Observatory, I spent the past summer studying six variable stars in Coma Berenices under the direction of Dr. Dorrit Hoffleit. The method outlined in this paper for determining short periods was successfully applied to EL Comae and to Variables* 1, 12, and 13. Variables 1 and 12 were discovered with the Rodman blink microscope by Dr. Hoffleit in 1968 and 1969, respectively. Light curves of these stars are shown in Fig. 7.

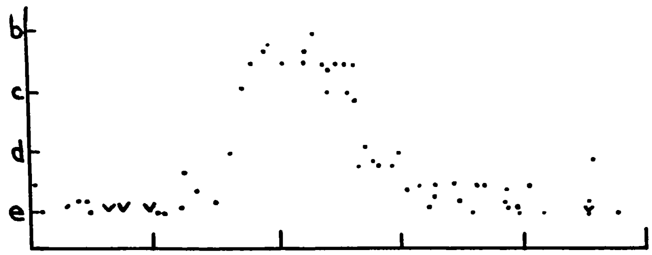
The spurious period relationships were used in the analysis of all four stars. The period first found for Variable 1 was improved by the subtraction of the yearly interval, and the period of Variable 12, by the addition of the yearly interval. EL Comae, rediscovered on the Nantucket plates by Dr. Hoffleit in 1969, was studied in an attempt to refine the period of 0.55^d published by Kukarkin in 1969. While the reciprocal period of 2.91265 is graphed in Fig. 7, an alternative reciprocal of 1.9098 fits the observations nearly as well. These figures differ by 1.00285, approximately the spurious interval related to the observational period of one sidereal day. This illustrates the difficulty in deciding between two related periods on the basis of data distributed over small ranges of hour angle and season, and emphasizes the need for cooperative efforts among variable star observers.

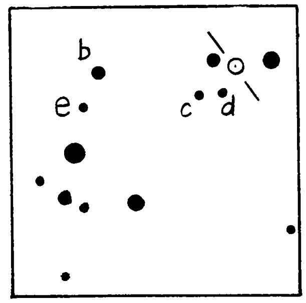
* Provisional designations for as yet unpublished stars.

Figure 7

VARIABLE 1
Position: $11^h\ 58^m$, +28° 30' (1900)

Finder Charts, South Up.
15' x 15' (approx.)





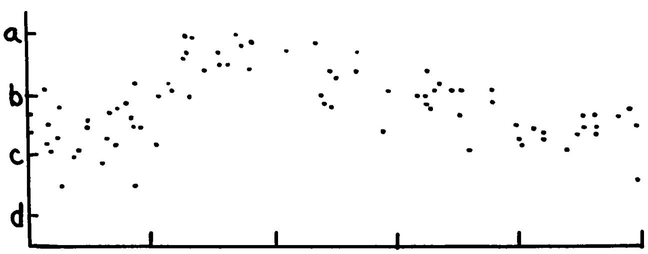
JD_o = 39,286.617

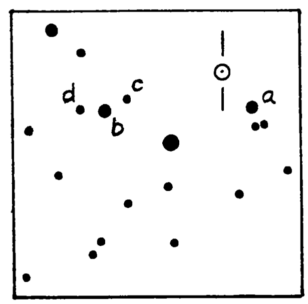
P^{-1} = 2.24837

P = 0.444766^d

Magnitudes (approx.)
b = 13.7
c = 14.3
d = 14.5
e = 15.8

VARIABLE 12
Position: $11^h\ 56^m\ 28^s$, +32° 27.4' (1900)





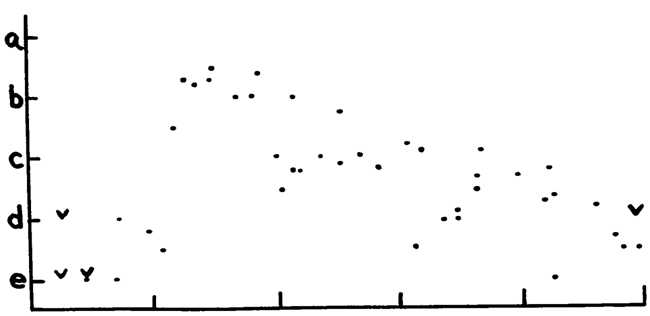
JD_o = 40,382.627

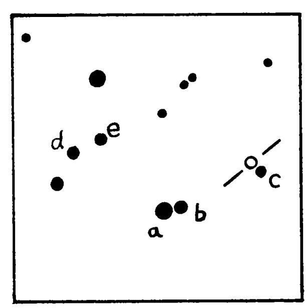
P^{-1} = 1.6385

P = 0.61031^d

Magnitudes (approx.)
a = 13.3
b = 13.7
c = 14.0
d = 14.5

VARIABLE EL COMAE
Position: $12^h\ 46^m\ 37^s$, +24° 40.2' (1900)





JD_o = 38,532.557

P^{-1} = 2.91265

P = 0.343329^d

Magnitudes (approx.)
a = 13.4
b = 13.9
c = 14.1
d = 14.7
e = (15