"THEORETICAL GLUE": UNDERSTANDING THE OBSERVED PROPERTIES OF MIRAS WITH THE HELP OF THEORETICAL MODELS

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Abstract

Measurement of observable quantities of stars, variable or otherwise, is only part of the process of understanding their nature. First of all, quite a lot of theory-based analysis goes into a measurement of a desired basic property for the star (such as mass, radius, or luminosity). Second, theoretical models are used to check relationships among the observed parameters. Interpreted using models that also give consistent relations among the resulting properties of the star, can we say that we know what the properties of the star really are? We are not there yet for the Miras, although great progress has been made in recent years, both on the theoretical side and on the observational side.

1. Introduction

The previous speakers have talked about a variety of observational studies of the Mira variables. Each observing program has as its goal to determine some quantities describing these stars. However, observations alone do not give us an understanding of what we are seeing. Theoretical models are needed both to connect what is observed to the qualities of the star, and to link the various measurements into one coherent picture of its nature. It is in this sense that a good model is a kind of "glue" holding the picture together.

In Figure 1, I illustrate the concept of "theoretical glue" by showing how the luminosity L, the radius R, and the effective temperature T_{eff} are related by the theoretical (and lab-tested) model of a blackbody "perfect radiator." Such a perfectly radiating surface emits power per unit surface area that increases as the fourth power of T (in Kelvins*), so a doubling of the temperature gives a 16-fold increase in the total amount of radiation (light, infrared, ultraviolet, X-rays and so on) coming from each patch of the surface. If stars were, in fact, ideal blackbodies, then their radiation would be completely known and the problem of relating L, R, and the temperature of the surface would be trivial. However, real stars are gas spheres; we can only see into the atmosphere on the average down to the apparent surface, the photosphere. We define the "effective" temperature T_{eff} as the temperature of a blackbody of the same size as the star that radiates the same total power, L. This gives the equation $L = 4\pi r^2 \sigma T_{eff}^4$, with $\sigma = 5.670 \times 10^{-8}$ watts per square meter per second per $(\text{Kelvin})^4$. The stellar atmosphere is not all at one single temperature, and we see to different depths at different wavelengths; as a result the spectrum we see has high and low spots compared with an ideal blackbody spectrum. Typically, the effective temperature is close to the gas temperature at the photosphere, but is not identical to it. We have to use a theoretical model atmosphere to relate effective

* Historical/cultural note: Degrees from absolute zero are properly called Kelvins, not degrees Kelvin; this decision was made some years back when small (for astronomy) corrections to the absolute scale were introduced by condensed matter physicists working on experiments very close to 0K.

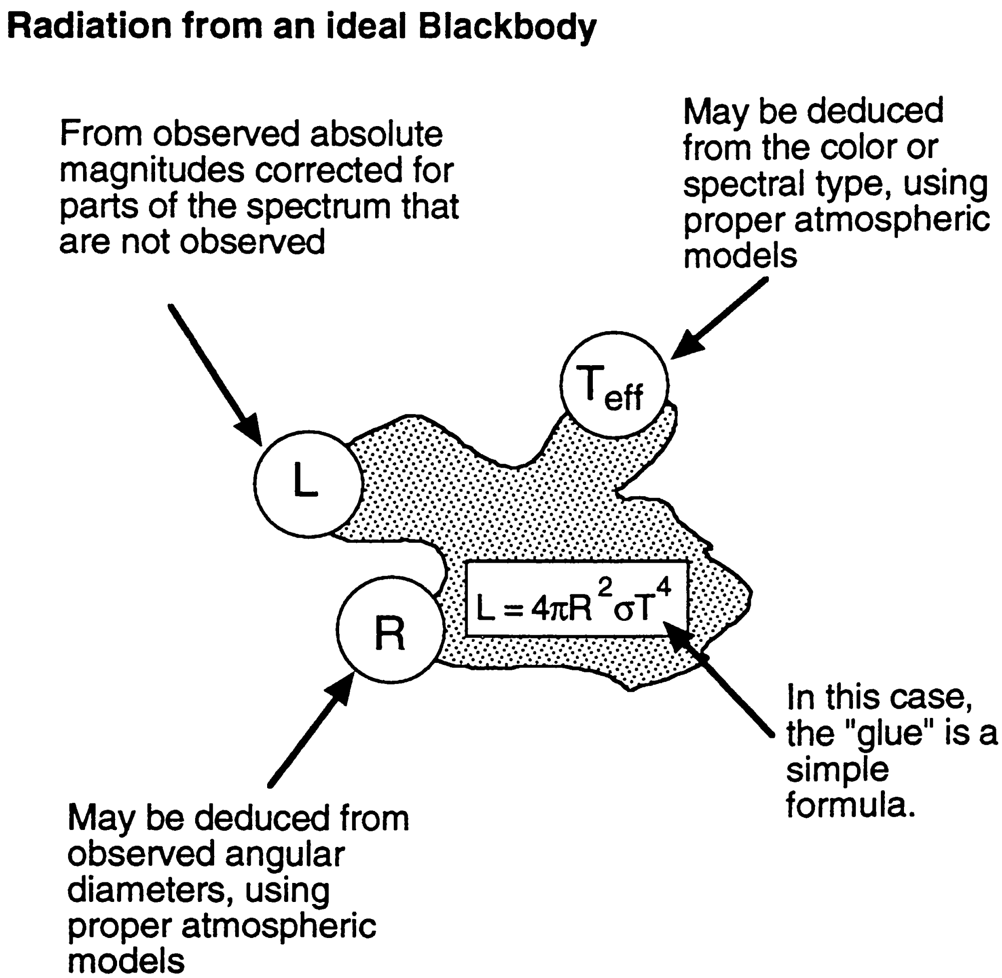

Figure 1. The definition of effective temperature (T_{eff}) illustrating the concept of theory as "glue."

temperature to the spectrum we see and to the temperature at the photosphere.

Many observations of Miras are made with the goal of determining their fundamental properties. These include quantities that are also determined for other kinds of stars: luminosity, mass, radius, effective temperature, and surface composition. But for Miras, even more than for most stars, the process of translating "what is actually observed" into "what the star is really like" can lead the incautious investigator astray. For example: To get the luminosity—the total power output, energy per second—we observe the visible part of the spectrum and, if we are lucky, also the near-infrared and perhaps the ultraviolet, in detail or using broad-band photometry. We then use a model of some sort to estimate how much light we are missing in the parts of the spectrum that we can't see, and finally we correct for distance. How good our final result is will depend on how good the model is that we use to fill in the "missing bits," as well as on how much of the spectrum we could actually observe and how accurately we know the distance. The expected (and sometimes observed) spectra of Miras are very far from the simplest case—a blackbody spectrum—so detailed models are essential. Worse, the molecules that produce some of the deepest spectral features in Miras are also found in Earth's atmosphere, and so we are selectively less likely to observe the depressed parts of the spectrum.

2. Classical model atmospheres

The calculation of a classical stellar atmosphere begins with a choice of stellar parameters—for example, composition, effective temperature, and surface gravity. The propagation of energy from the interior of the star through the atmosphere and into space is then calculated, taking into account the effects of the different atoms, ions and perhaps molecules that can absorb and emit light. The result of a classical atmosphere calculation may include any or all of a predicted spectrum, a model for the pattern of brightness (limb-darkening) that you would see if you could get close to the star, and predicted values for the photometric colors. Such models play a key role in the determination of the luminosities of stars—as I have noted above—and also in the derivation of their radii.

To get an estimate for the radius or diameter of a star, we may use an interferometer or a lunar occultation to get a pattern of fringes that is the interpreted using a model for the brightness pattern on the star (uniform brightness or limb-darkened, for example). Or, we may try to relate the appearance of the spectrum to the effective temperature T_{eff}, using a detailed model atmosphere, and then deduce R from L and T_{eff}. If these methods give the same answer, it increases our faith that the model is close to describing what happens on the star.

To find the composition of the atmosphere, the line spectrum is analyzed using a stellar model atmosphere. Thirty years ago, most such calculations were made using some reference model atmospheres and looking for differences using methods such as the "curve of growth." Today, it is possible to carry out most analyses by making model atmospheres with a range of compositions and selecting the composition pattern that produces a spectrum that best matches the observations.

Figure 2 illustrates how a classical stellar model atmosphere glues together observable and non-observable quantities for stars. In a classical model atmosphere there is no net outflow of matter—no stellar wind—and there are no systematic motions, such as one might get from pulsation. Obviously, this is not going to work perfectly for modeling Miras! Also, in classical atmospheres, each part of the atmosphere is in radiative equilibrium—meaning that the radiant energy flowing into a sample volume of the gas per second exactly equals the radiant energy flowing out of the same sample volume per second. In more modern model atmospheres, other forms of energy are also considered, and energy is allowed to shift from one form to another—for example, from soundwaves to radiation. There are still relatively few models, however, that include dynamical effects and outflows.

One of the important ingredients in a stellar atmosphere model—whether classical or modern—is the surface gravity $g = GM/R^2$, where G is the gravitational constant. This combination of M and R turns out to be important for the spectrum, because higher gravity compresses the atmosphere more. One can deduce a gravity by computing synthetic spectra for models with a range of surface gravities, and then picking the model whose spectrum best matches the star, as long as the model does produce a good match. This works reasonably well for most non-variable stars, but does not do a good job with the variable ones. Other methods are needed for these, at least for now.

3. Glue from pulsation and evolution studies

In principle it should be easier to derive a value for the surface gravity for a pulsating star, because the material in the atmosphere is moving in response to gravity during much of the cycle. Thus, one might observe the change in velocity over some interval

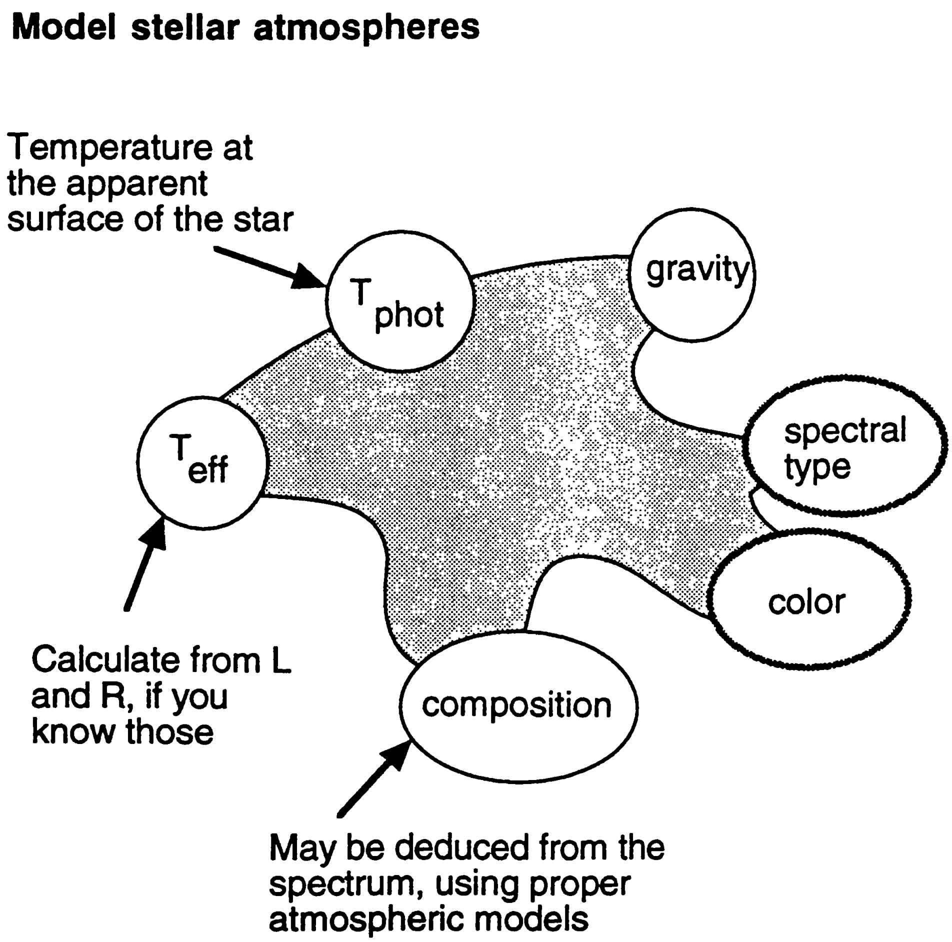

Figure 2. Classical model atmospheres as "glue" for the colors and spectra.

of time and estimate $\Delta v/\Delta t = g$. Because the excursion in radius is large (so g is not the same at all parts of the path) and because pressure forces are also important, the above method is pretty bad; it typically underestimates g by a factor of five or so. Instead, a dynamical atmosphere model needs to be used to interpret the result. Also, you need a good radiative transfer model to be able to interpret the observed Doppler shift in terms of the motions of parts of the atmosphere, because only part of what you see is moving towards or away from you. To get a meaningful Δv from the Doppler shift requires a fairly detailed model for the atmosphere, and this correction is still rather rough for most variable stars.

For pulsating stars there is another way to get a combination of M and R. A given star is usually able to pulsate only in one or a small number of modes, each with a distinct period associated with it. Detailed models for the interior of a pulsating star can be analyzed to reveal the period(s) that are possible, and these can be related through formulae such as

$$P = a\,R^b\,M^c, \qquad (1)$$

as is illustrated in Figure 3. (Usually b is between 1.5 and 2, and c is between -0.5 and -1.) A special case of this is given by the formula for the so-called "pulsation

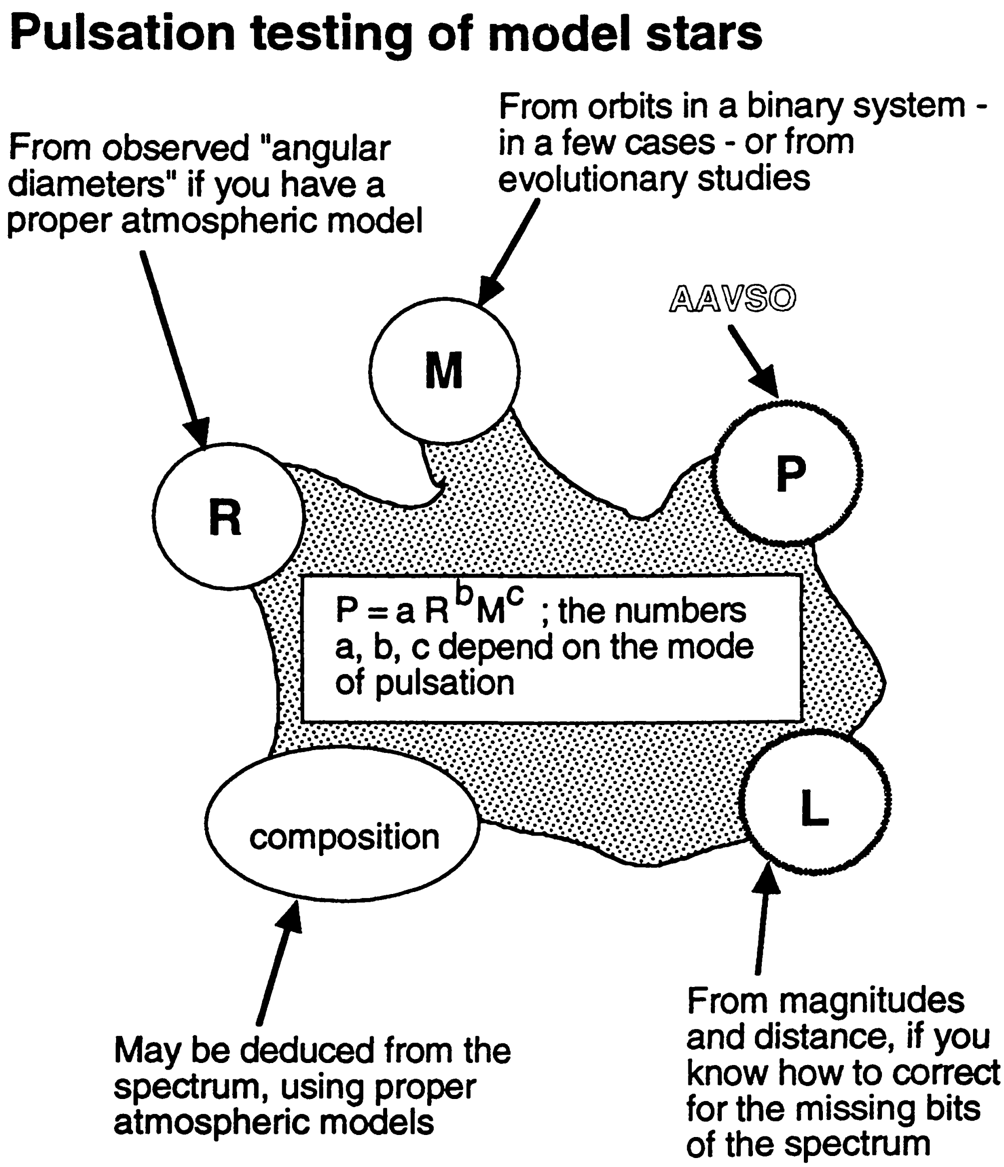

Figure 3. Pulsation periods from model stellar interiors connect evolutionary models to stellar parameters.

constant," which gives a very rough estimate for the likely pulsation period:

$$P = Q\,(R/R_{\odot})^{3/2}(M/M_{\odot})^{-1/2}, \qquad (2)$$

where P and Q are both given in the same units (usually days). For radial pulsation for a wide range of types of stars the models give Q = 0.01 to 0.15 day, with most models falling near the middle of this range—0.03 to 0.06. The fundamental mode pulsation of Miras is given by another "PMR relation":

$$P_{\mathrm{Mira,\,F\,mode}} = 0.012\,(R/R_{\odot})^{1.92}\,(M/M_{\odot})^{-0.73}, \qquad (3)$$

according to Ostlie and Cox (1986). It is perhaps worth mentioning that there are assumptions that go into this kind of modeling that need to be tested more thoroughly than has been possible so far: for example, the period of pulsation of a star pulsating at full amplitude may not be the same as the period derived looking at very small pulsations in a model for a static star.

A PMR relation is often used to estimate the mass of a pulsating star, M, given its radius, R (which may have been derived from L and T_{eff} or from angular diameter measurements) assuming that one knows the mode of pulsation. It can otherwise be used to determine the pulsation mode(s), if one is confident of M and R from other measures. For most classes of stars this is relatively easy to do, and the results are consistent with whatever else one knows about the stars. However, for Miras the radii and masses are still sufficiently uncertain that this method does not even yield an incontrovertible result about the pulsation mode, much less useful estimates for their masses.

The process for calculating PMR relations starts with a detailed stellar model describing the internal structure of the star—how the temperature, density, and composition vary from the center to the surface. This model is usually taken from evolutionary models that follow the evolution of the internal composition as nuclear reactions modify it.

4. Models for stellar evolution

The most important "theoretical glue" in stellar astronomy is the study of how stars evolve. Starting with some composition (assumed to apply throughout the star) and a mass, M, a model is found that obeys relevant physical equations and is in hydrostatic equilibrium. For all but the lowest-mass stars the model will include energy generation by nuclear reactions in the core. These reactions modify the composition at the center, so some time later the star's structure will be a little different and its L and R may also be a little different. By building a sequence of static models that are related by the condition that the change of composition comes from the nuclear reactions, one can model the evolution of the star.

In most evolutionary calculations the mass M is not allowed to change with time, although there are times in the life of a star when the mass decreases as the result of mass loss from the surface. The change in mass that comes from the conversion of mass to energy in the nuclear reactions is almost always small enough to ignore. One time when the mass loss is particularly important is the Mira stage, and this fact is a major reason why Mira models are not yet in a settled state.

Since much of stellar evolution proceeds at constant mass, and since L and T_{eff} are the easiest quantities to estimate directly from observations, we traditionally plot tracks for constant mass stars in a diagram of L versus T_{eff}, one variant of the Hertzsprung-Russell diagram. One may then think of evolutionary tracks as linking L, M, T_{eff} or R, initial composition, and age for the star (Figure 4).

In addition to the problem of how to include mass loss in a realistic way, evolutionary models also suffer from our lack of detailed understanding of convection in stars; of rotation inside stars; of the effects of magnetic fields in stars; and of the events associated with fast changes such as the "helium core flash."* Most astronomers assume that these effects will turn out to be small, but I would not be surprised to learn that some of them affect the "big picture."

* The helium core flash occurs at the end of the first red giant stage of evolution, when hydrogen has been converted to helium in a thin shell around an inert helium core. When the He core reaches about 1/2 $M_{\odot}$, in low mass stars (including the progenitors of most Miras) the reactions that convert helium to carbon and oxygen start with an abrupt event that, for reasons we don't understand fully, does not disrupt the star.

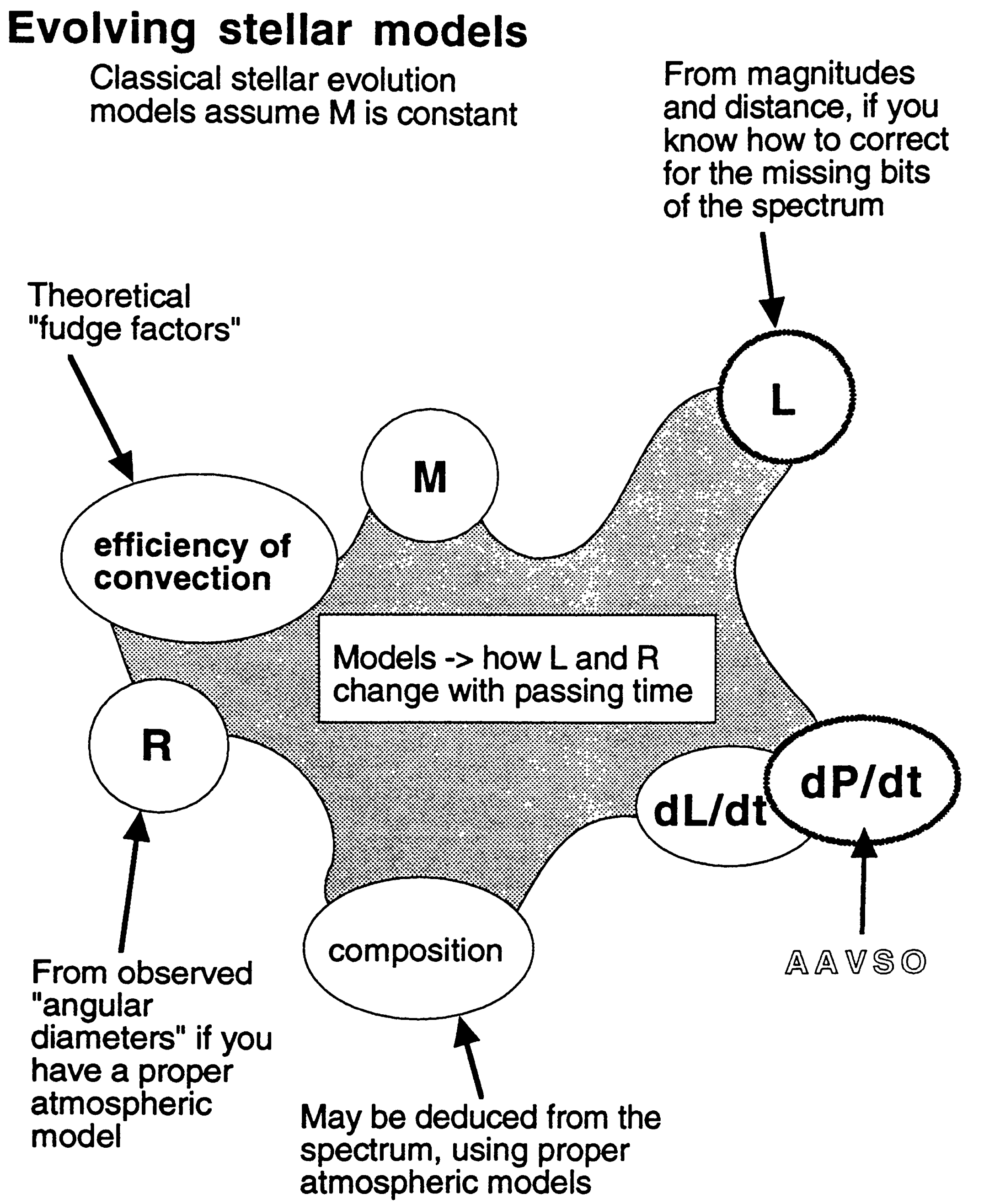

Figure 4. Evolutionary models link stars in different evolutionary states as well as relating stellar properties and (occasionally) rates of change of those properties.

5. Dynamical models for the atmospheres of pulsating stars

If we know how a star is pulsating, then we can model the response of the outer parts of the star (the atmosphere) to this pulsation. Figure 5 shows the connections that can be made this way. In practice, L, M, and T_{eff} or R are assumed; also a pulsation period P is assigned (using a PMR relation) and the bottom of the atmosphere is made to move in and out with period P.

Dynamical models require some understanding of the interaction between the gas and the radiation. The pulsation generates waves that compress the gas, heating it. It then cools by radiating away energy, and also by expanding. Depending on the density of the gas, the conversion of internal energy into radiation may be fast (compared with the pulsation time) or slow. Where it is fast, the material cools to roughly the equilibrium temperature that it would have in a static model, and then as it expands it may be refrigerated below the temperature it would have in the static case.* Where the density is lower, the cooling is less efficient; there, the temperature may never fall as low as the equilibrium temperature. Some dynamical model results are very sensitive to the treatment of these processes; mass loss is one example. Since the details of how the gas emits or absorbs radiation at low density involve many non-equilibrium chemical processes, this is definitely one of the frontier areas in dynamical atmosphere modeling.

A detailed treatment of the interaction between gas and radiation—the radiative transfer problem—is also required in order to synthesize the spectrum and colors that would be observed, as well as the light curve. So far, there is no model for Miras or other pulsating stars that includes enough detail to do this effectively. However, dynamical models that are now available provide important insight into the motions of the atmospheres and the mass loss rates that result. For example, Bowen's models (Bowen 1988, 1990) have atmospheric motions and conditions that match what we deduce from observations—shocks with velocity amplitudes of 20 to 30 km/s, warm regions in some, dust formation in others, and so on. In fact, the success of dynamical models in matching velocity variations observed in the infrared CO lines is the best evidence we have about the mode of pulsation of these stars—fundamental mode models match well and overtone models (with larger radius at a given P) are quite far from matching, as was first noted nearly 20 years ago (Hill and Willson 1979).

6. Some results of recent "glue" production

Bowen's latest grid of dynamical atmosphere models is a collection of models that are constrained by stellar evolution calculations: once L, M, and initial composition are chosen the evolutionary calculations are used to derive R, and then a theoretical PMR relation gives P. This single step of requiring the stars to fall on a single set of evolutionary tracks turns out to make quite a big difference in the way that the mass loss is understood to develop. The choice of which tracks to use is not so important as is the fact that using tracks forces certain relationships between models: For a given mass, as a star increases in luminosity it also increases in radius with (slightly) decreasing T_{eff}. Two stars with the same L and different masses will be separated in T_{eff} or R. Two stars of the same L and M but different composition will also be separated in T_{eff} or R: lower metallicity stars are hotter and smaller at a given L.

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* Since the density is highest just after the gas is compressed, there is a region in the atmosphere where it can lose energy to radiation immediately after compression but has a harder time regaining energy near the end of its expansion. We can describe this approximately by saying that the shock is nearly isothermal—it returns to the radiative equilibrium temperature quickly—but the expansion between shocks becomes nearly adiabatic—without gain or loss of energy.

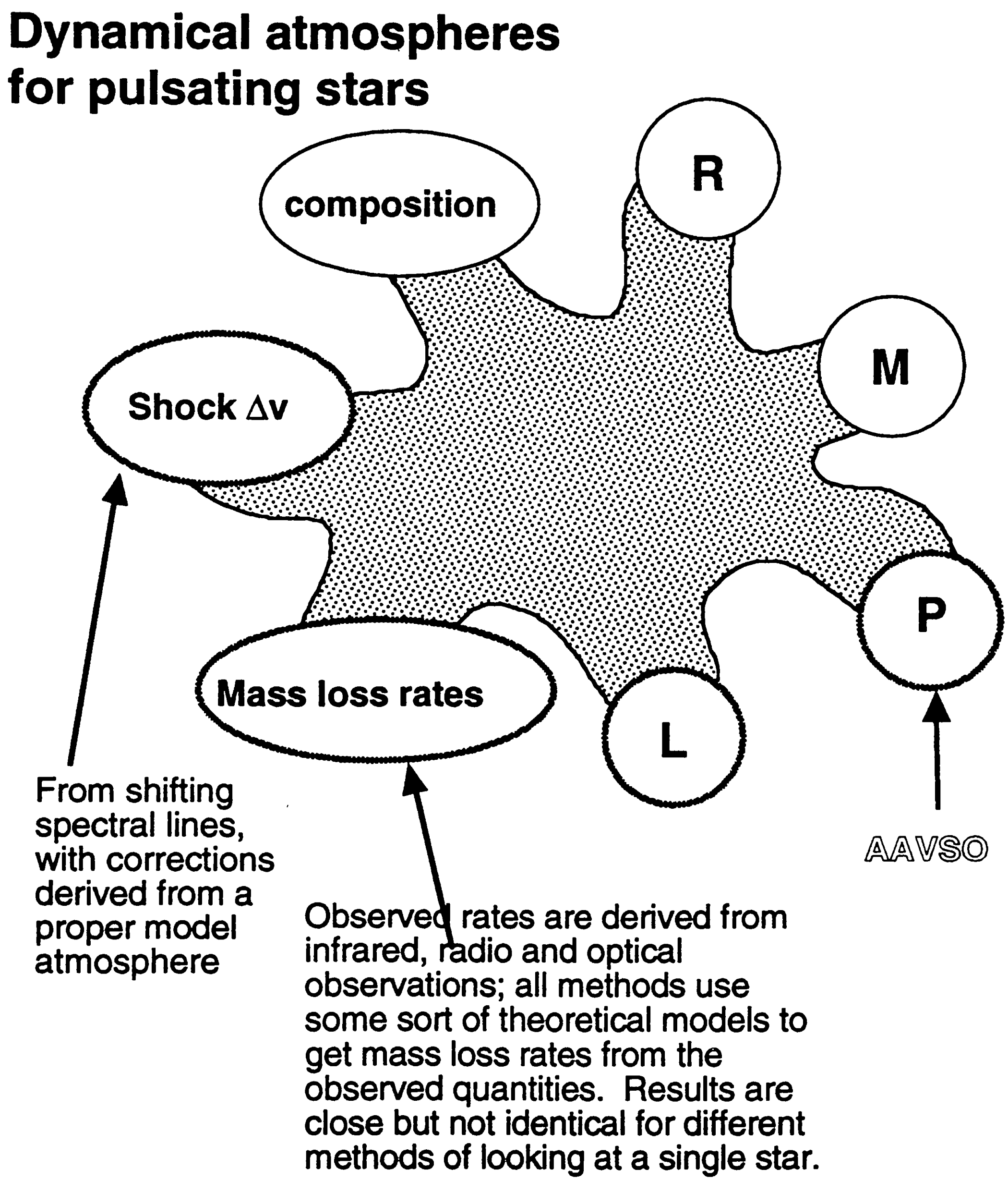

Figure 5. Dynamical model atmospheres are required for pulsating stars, such as Mira variables.

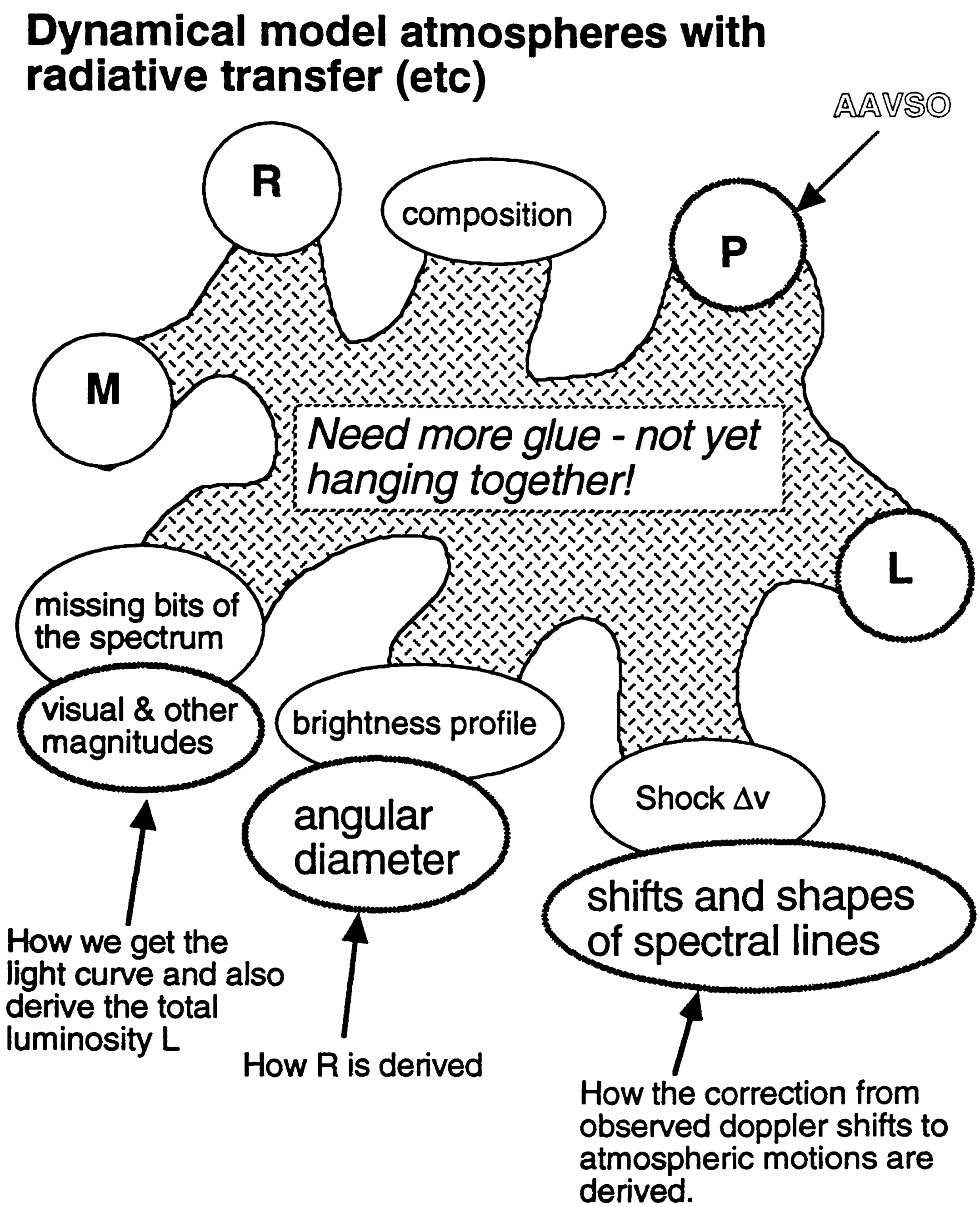

Figure 6. Ideally, radiative transfer, non-equilibrium chemistry, and detailed hydrodynamics are included in the models. In practice, no one model yet includes all the details that are needed to reproduce all the observations.

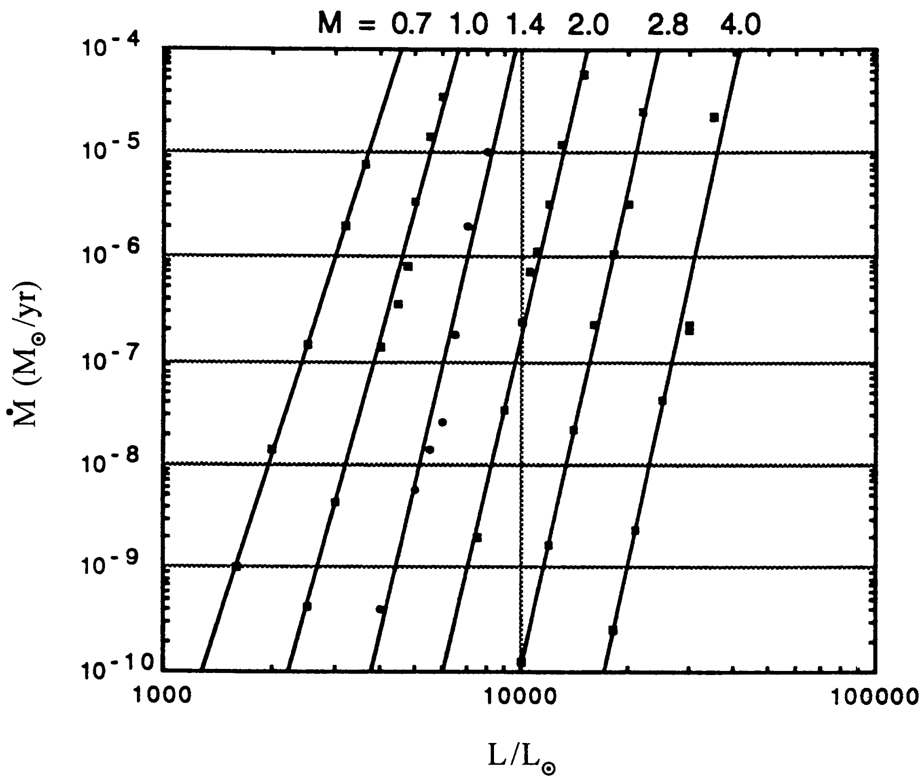

Figure 7. The dependence of mass loss rates for Miras on the luminosity, on a doubly logarithmic scale, where L is understood to include those changes in R and T_{eff} that accompany changes in L as a star evolves up the AGB. Straight-line fits to the model results are shown; these are thus power law fits to $\dot{M}$ versus L.

Bowen's models, constrained in this way, predict mass loss rates that are very sensitive to stellar L, R, and M. Since all of these parameters vary in a predictable way along a given evolutionary track, we can display the results as mass loss rate $\dot{M}$ versus L for a given mass, where for a given metallicity L and M together also determine R, T_{eff}, and P as well. Figure 6 shows the result of these calculations for stars whose composition matches that of the Sun.

Exponents (dlog$\dot{M}$/dlogL) in the fits shown in Figure 7 range from 11.2 to 15.5. Comparing pairs of models with the same L and different M we also find very steep dependence of $\dot{M}$ on M: exponents (Δlog$\dot{M}$/ΔlogM) range from 16 to more than to 22.4. A mass loss law with such large exponents is surprising to many observers, because empirical relations have tended to suggest much gentler variation. The models are okay, and the observations are also okay, but the problem has been the interpretation of the observations. These are, in fact, dominated by selection effects, as I shall now demonstrate.

Low mass loss rates (< 10^{-7} $M_\odot$/year) are hard to detect, and early surveys found mostly stars with rates higher than this. At the same time, mass loss rates above 10^{-5} $M_\odot$/year remove most of the mass of the star in 10^5 years or less—an astronomical instant—

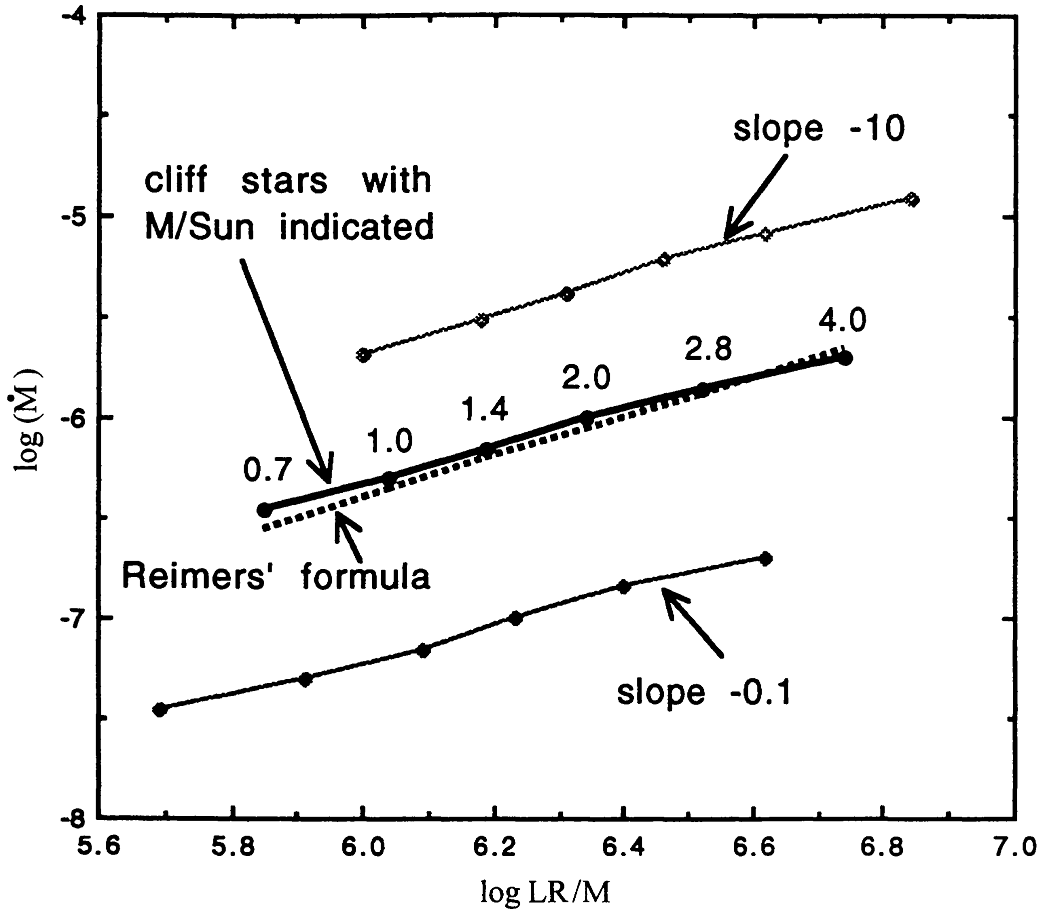

Figure 8. Given mass loss rates that increase rapidly with increasing luminosity, the stars that are observed to be losing mass will be those in a relatively narrow range between the lowest detectable mass loss rate and the mass loss rate that destroys the star quickly. Applying simply this selection criterion to the Bowen models reproduces the reults of observational surveys, such as those used to support the prescription suggested by Reimers (1975).

so relatively few stars are seen at such high mass loss rates. Also, early surveys checked stars that could be detected in visible light, and above about $10^{-5}M_{\odot}$/year the wind becomes an opaque shroud visible only in the infrared and radio parts of the electromagnetic spectrum. Thus we expect surveys to select just those stars that fall in the relatively narrow range of mass loss rates around 10^{-6} $M_{\odot}$/year.

One can define a "critical mass loss rate" where the rate of evolution of the star in mass is equal to its rate of evolution due to nuclear processes, or in a mathematically convenient form:

$$(1/L)\ dL/dt = -(1/M)\ dM/dt, \qquad (4)$$

which can also be written as

$$d\log L/dt = -d\log M/dt \qquad (5)$$

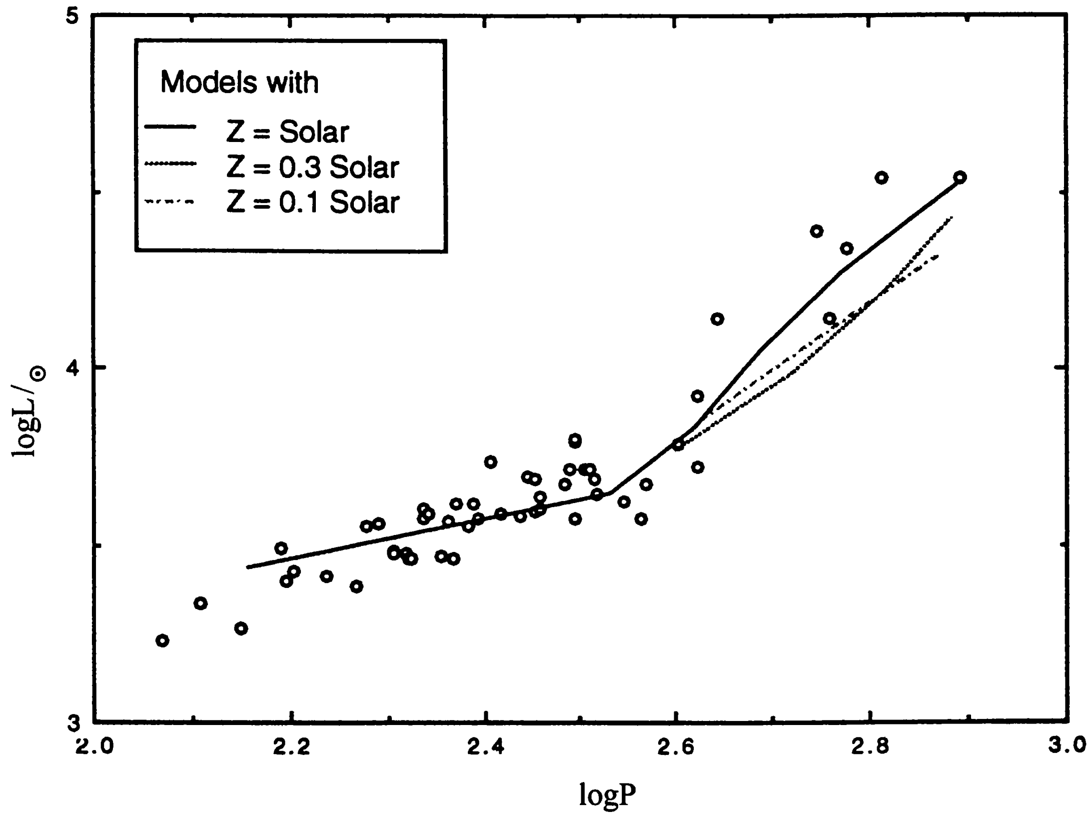

Figure 9. Identifying Miras as those stars that are in the final mass loss stage on the AGB leads to a predicted period-luminosity diagram that matches the observed one.

(The minus sign comes in because L is increasing and M is decreasing, so technically what we call the "mass loss rate" is $\dot{M} = |dM/dt| = -dM/dt$.)

Taking those stars for which the mass loss rate computed by Bowen equals the critical mass loss rate, we find that the result matches well what was found in mass loss surveys of red giants and related types of stars. The most widely used of these empirical relations is "Reimers' Law" (1975) which takes the form

$$\dot{M} = 4 \times 10^{-13}\, LR/M. \qquad (6)$$

This matches exactly the models with the critical mass loss rate (Figure 8).

We conclude that Reimers' formula results from selection effects and a steep dependence of $\dot{M}$ on L and M—it is widely *misused* as a formula describing how much mass loss to expect for a given star.

For reasons that probably have to do with the effects of pulsation and mass loss on the atmospheric structure, we also identify stars as Miras when they are near this critical mass loss rate. We can compare observed properties of Miras with expected properties of such stars and we find another good fit, as shown in Figure 9.

This good agreement with the observed period-luminosity relation supports the hypothesis that Miras are exactly those stars that are entering a final stage of precipitous mass loss. In a way that none of us fully anticipated, the dynamical atmosphere calculations now both confirm the trends of standard evolutionary tracks and give

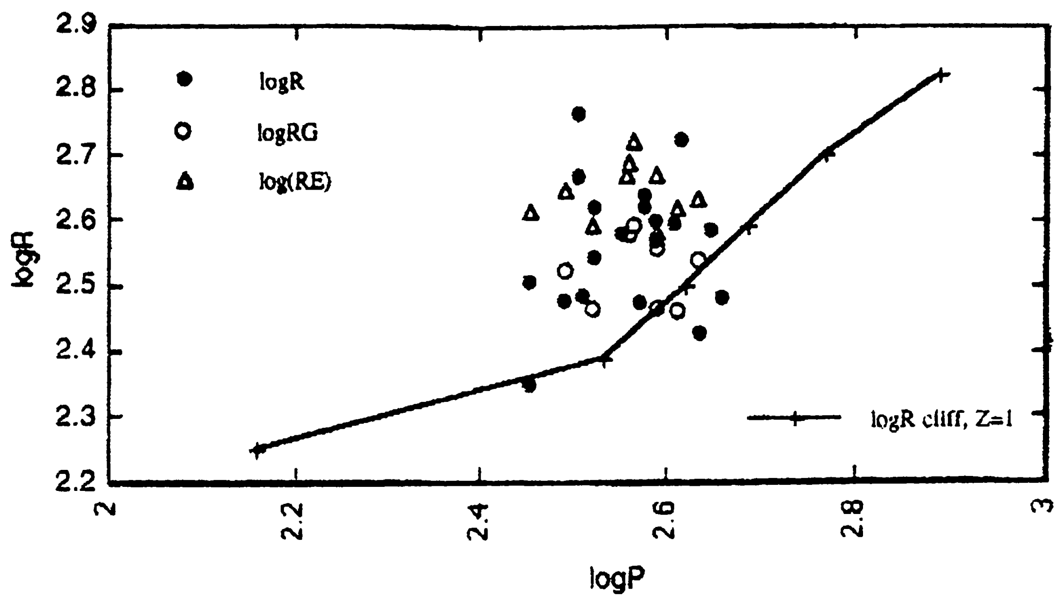

Figure 10. The black dots show data from van Belle *et al.* (1996; Wyoming Interferometer); the open symbols show two different fits to data by Haniff *et al.* (1996)—a gaussian intensity distribution (RG) and a model atmosphere (RE).

information to those calculating stellar evolution models about how to end the "asymptotic giant branch" stage of evolution.

7. Where we need some new glue

Bowen's calculations, which were so successful in reproducing mass loss and luminosity relations, also predict a relation between the stellar radius and the pulsation period. This is a rather strong prediction, because the behavior of the models is very sensitive to R. When observed angular diameters are interpreted using simpler atmospheric models, the derived radii do not agree at all with the predicted trend shown in Figure 10.

There is a likely source of error for the angular diameters. These are derived assuming a brightness distribution for the star. Some such distributions are illustrated in Figure 11. The models that have been most often used for fitting interferometric observations are the uniform disk, the limb-darkened case, and an extreme limb-darkened case. However, there is reason to believe that the star will be surrounded by an apparent "halo" that would cause the measurements to be systematically too large. Based on the Bowen models, we expect that more detailed models, when fitted to the interferometric observations, will yield results in much better agreement with the other parameters. This requires a closer blending of dynamical models with non-equilibrium chemistry with radiative transfer calculations in full spherical geometry than has been done so far. Other factors may also play a role; Karovska (these proceedings) has reached similar conclusions from a careful analysis of the observational data, and also notes that there are asymmetries in the measured stellar "disks" that affect the results and need to be understood.

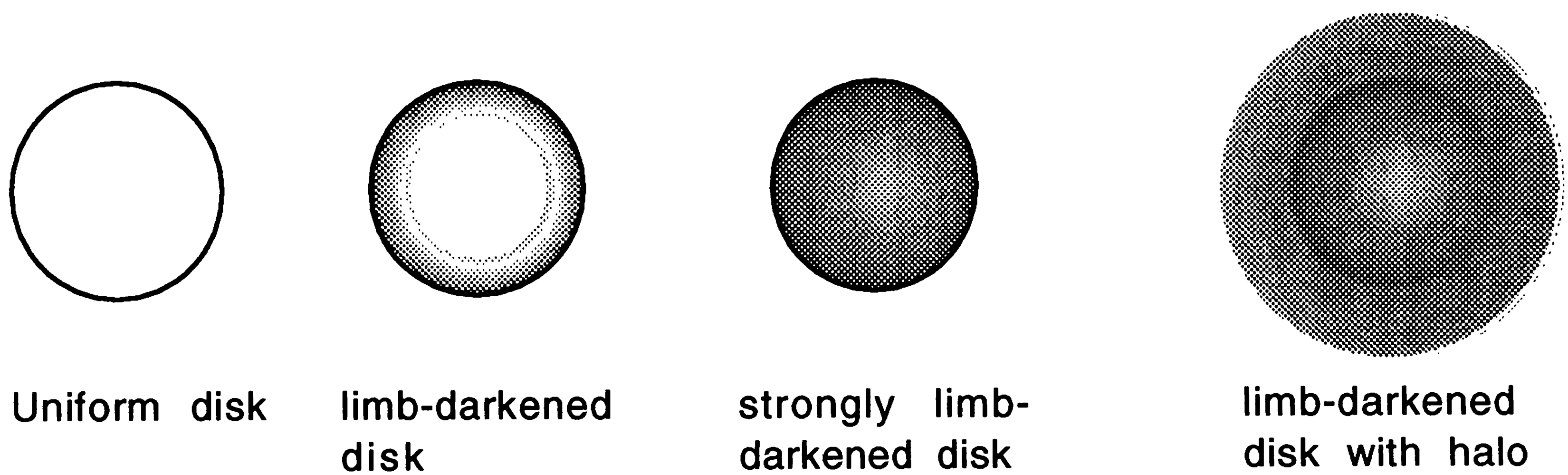

Figure 11. An illustration of various assumptions that are made in interpreting interferometric observations to obtain stellar radii. The greater the limb-darkening, the larger the estimated radius. If the star has a halo, but this is not taken into account in interpreting the observations, then the derived radius may be even more of an estimate.

8. Concluding points

I have discussed some of the better-quality astrophysical "glue" available to us for use in understanding Mira variables and other stars: (1) stellar evolution calculations; (2) classical and modern models for static stellar atmospheres, with detailed radiative transfer; and (3) dynamical model atmospheres.

Many pieces of the puzzle are well-glued together by these theoretical calculations, but some very basic properties of Miras remain "unglued." The outstanding problem in the case of the Miras remains the determination of their absolute sizes; here, uncertainties of factors of two or more are still a problem. Clearly we need more and better "glue"—new models that incorporate more of the physics that we already know is important in producing what is observed.

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