MONITORING THE EVOLUTION OF CEPHEID VARIABLES

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Abstract

Described here are preliminary results of a pilot project to monitor changes in the ephemerides of northern hemisphere Cepheids using an SBIG camera attached to the 0.4-m telescope of the campus observatory at Saint Mary's University. Epochs of maximum light for fifteen Cepheids have been derived using published light curves for each variable as templates, and the results are being used to update the O-C ephemerides for the program stars. Results for BB Her are presented here. Period changes for Cepheid variables are demonstrated to be an excellent means of pinpointing their evolutionary status, as well as for investigating other peculiarities of the class.

1. Introduction

Classical Cepheid variables are pulsating yellow supergiant stars with masses of about 3–30 $M_\odot$ (Turner 1996). Because they vary regularly in brightness in diagnostic fashion, they are easily recognized even over extragalactic distances. Their periods of pulsation are also directly related to their luminosities via the Cepheid period-luminosity (PL) relation—the more intrinsically luminous a Cepheid is, the longer its pulsation period—which makes them very useful as "standard candles" for establishing distances to nearby galaxies. At the present time the distances of galaxies out to the Virgo cluster can now be measured with the Hubble Space Telescope in almost routine fashion provided only that they contain detectable Cepheids.

Since they are so vital to the reliable determination of distances on extragalactic scales, classical Cepheids have been subject to intense study to learn more about their basic properties. All are post-main-sequence objects that began their lives as hot, hydrogen-burning, B-type stars (e.g., Turner 1996). They have since consumed the original hydrogen fuel in their cores and are now evolving with much larger radii through more advanced stages of nuclear fuel consumption (see Figure 1). They become unstable to radial pulsation perhaps five times during such stages: once during shell hydrogen burning, twice more during core helium burning, and twice again during shell helium burning. A "neat" feature of Cepheids is that the effect of the changes that occur in their overall dimensions as they evolve can be observed. Such changes—either increasing mean radius during evolution towards the cool side of the HR diagram or decreasing mean radius during evolution towards the hot side of the HR diagram—are evident as long-term variations in their pulsation periods, a feature that can be detected by photometric monitoring. The changes are not spectacular by any means, amounting in the most extreme cases to mere seconds or minutes per year in their observed pulsation periods of days to months. The effects are cumulative, however, so the changes are significant and measurable as offsets of several hours or more in observed times for light maximum.

2. Background to the present project

The climate of Nova Scotia is not conducive to a high frequency of photometric nights. The province lies on the tracks of most storm systems travelling northward up the North American Atlantic seaboard or eastward through the heart of the continent. Photometric stability is therefore not a common feature, particularly at sites directly exposed to the Atlantic Ocean.

The Burke-Gaffney Observatory of Saint Mary's University in Halifax was constructed in 1972 one story above the rooftop of the university's 22-story Loyola residence. It is located within a bright light pollution umbrella created by the unshielded lights of downtown Halifax, the nearby Halifax container ship terminal, and adjacent Huskies Stadium. Thus, when formal programs in astronomy were introduced at Saint Mary's in 1974, attempts by astronomy faculty to carry out research projects using the observatory's 0.4-m reflecting telescope were generally unsuccessful. Photographic imaging suffered from a bright sky background, while photoelectric observations faltered because of the infrequency of photometric nights. The situation changed in 1994 when the acquisition of a CCD camera made it feasible to tackle a variety of observational research projects using only campus facilities. One nearly immediate success was the discovery of supernova SN 1995f by amateur astronomer Paul Gray and astronomy technician Dave Lane in February 1995 (Lane and Gray 1995) as a result of a dedicated search. The program described here was initiated as a separate pilot project during the summers of 1996 and 1997.

The photometric monitoring of Cepheids to detect period changes has been performed in the past mainly by means of differential or absolute photoelectric observations. Comparable studies can also be done using CCD imaging, with the degree of sophistication depending upon the experience of the observer and the desired degree of reliability. The availability of CCD imaging with a Santa Barbara Instruments Group (SBIG) ST-6 camera attached to the 0.4-m telescope of the Burke-Gaffney Observatory made it possible to undertake such research on campus. It was initially uncertain, however, how reliable the results would be, given the problems of sky stability described above. As a means of circumventing such difficulties, it was decided to proceed with the program at a very low level of sophistication. Cepheid fields were therefore imaged at blue wavelengths, which represents a compromise between the decreased sensitivity of most CCD devices and the lower brightness of Cepheids at short wavelengths and the larger light amplitudes of Cepheids in the blue. Brightness variations for program objects were also derived by measuring magnitude differences relative to other stars, assumed light-constant, in the same small fields of view using only the software available with the ST-6 camera. Such restrictions were not particularly onerous; only a few potential objects had to be dropped because of the lack of bright, closely adjacent, reference stars. The restriction to the SBIG software, however, meant that a number of small amplitude variables, like Polaris, had to be left off the program because the magnitude estimates were not expected to be very precise. That restriction was dictated by the fact that the observing was done by undergraduate student specialists inexperienced in the use of sophisticated software routines.

Although the SBIG software gives stellar magnitude differences using a box aperture about each star rather than a circular aperture, that restriction was not found to affect the measurements to a great extent. From measures for the full sample of fifteen Cepheids covering blue magnitudes from about 7 to 12, it was found that the typical measuring uncertainties for each Cepheid ranged from about ± 0.03 to ± 0.06 magnitude. The reliability of the estimates naturally varied according to the brightness of the Cepheid and the number of reference stars. The reference stars ranged in brightness from about 9th to 15th magnitude in the blue, with as many as 1–8 being available for each star; the average was 4–5 stars per field.

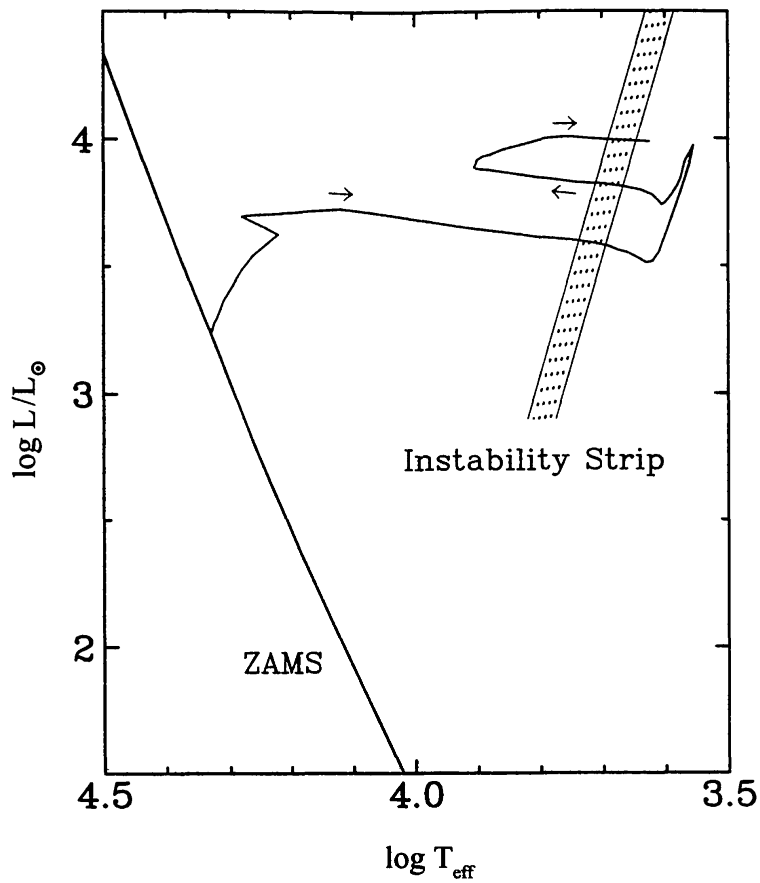

Figure 1. A theoretical HR diagram showing an evolutionary track for a 7 $M_{\odot}$ star. Arrows indicate the direction of evolution for the star once it has left the zero-age main sequence (ZAMS) at left. The location of the Cepheid instability strip is also indicated. As demonstrated by the evolutionary model depicted here, such a star can pass through the region of the instability strip three or more times.

3. Results

Figure 2 shows the light curve derived for BB Her, one of the first Cepheids to be imaged during the project. The variable provides a good example of what limitations can be expected from CCD imaging in a bright sky background. The star was observed on thirteen nights in May and June 1996, with each observation representing the sum of several short exposures on the object. BB Her is not one of the best candidates for imaging at our site since it is 11th magnitude in B. The closest four "reference" stars in the same field also have rather faint blue magnitudes of 12.07 ± 0.05, 13.69 ± 0.07, 14.53 ± 0.15, and 14.59 ± 0.19, as derived by matching the differential light curve for the Cepheid to the photoelectric data for the variable published by Szabados (1980), using least squares. The differential light curve, obtained from magnitude differences Δb measured with respect to the reference stars on dark-subtracted, bias-corrected, and flat-fielded images and normalized to the measures for the brightest visible companion, is still a reasonably good match to those obtained by Szabados (1980) and Berdnikov (1992a) using photoelectric observations. In fact, the nature of the bump on the descending light portion of the light curve suggests a slightly better match of our observations to the photometry of Berdnikov than to that of Szabados.

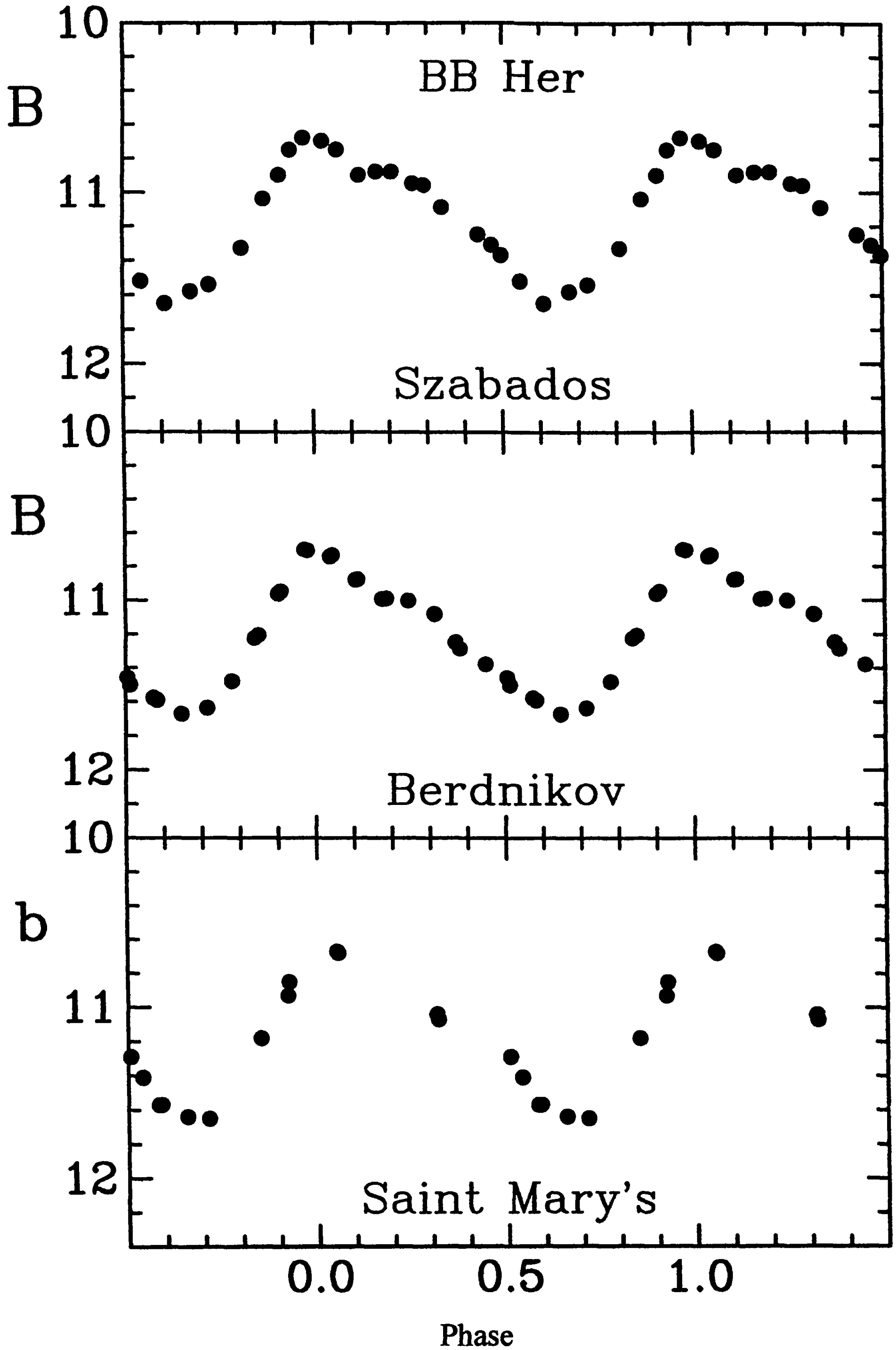

Figure 2. The variations of BB Her in blue light for the photoelectric observations (B) of Szabados (top) and Berdnikov (middle), relative to the CCD observations (b) obtained at Saint Mary's University (bottom) that have been normalized to the photometry of Szabados. The ephemeris used for calculating the phases for the observations is that of Szabados (1980).

In order to plot the light curve for BB Her, the observed magnitude estimates were phased to the ephemeris of Szabados (1980), which is given by:

$$JD_{max} = 2442679.289 + 7.507945\,E. \qquad (1)$$

A comparison of the new observations with the existing light curve allows one to establish if there has been a phase shift in the light curve. For BB Her the phase shift is relatively small. Although Berdnikov (1992b) has developed a rather sophisticated technique for matching new observations to existing light curves to establish phase shifts, we found it practical as a first approximation to match the light curves by eye using identically scaled versions printed on transparencies. In the case of BB Her, the observations of Berdnikov (1992a) are shifted by +0.010 ± 0.005 in phase relative to those of Szabados (1980), while our observations are shifted by +0.015 ± 0.005 in phase relative to those of Szabados. (The uncertainties are visual estimates.) More sophisticated least squares matching of the data sets yields values of +0.006 ± 0.003 and +0.015 ± 0.005, respectively. The corresponding temporal differences for the last two values are calculated from the known period of the Cepheid, from which we obtain a phase shift of +0.001 ± 0.023 day for the observations of Berdnikov relative to the ephemeris of Szabados and +0.069 ± 0.038 day (JD_{max} = 2450217.334 ± 0.038) for our observations relative to the same ephemeris. (The observations of Szabados are shifted by –0.044 day relative to that ephemeris.) The epochs of maximum light for BB Her during our observing period occurred about 2 hours later than expected according to the ephemeris of Szabados.

The O-C diagram for BB Her that is presented by Szabados (1980) can be updated and extended from the new observations made at Saint Mary's, as in Figure 3. Belserene (1986) has provided a detailed description of how one fits a parabola to such data using the method of least squares. A solution of that type is included in Figure 3. The time base

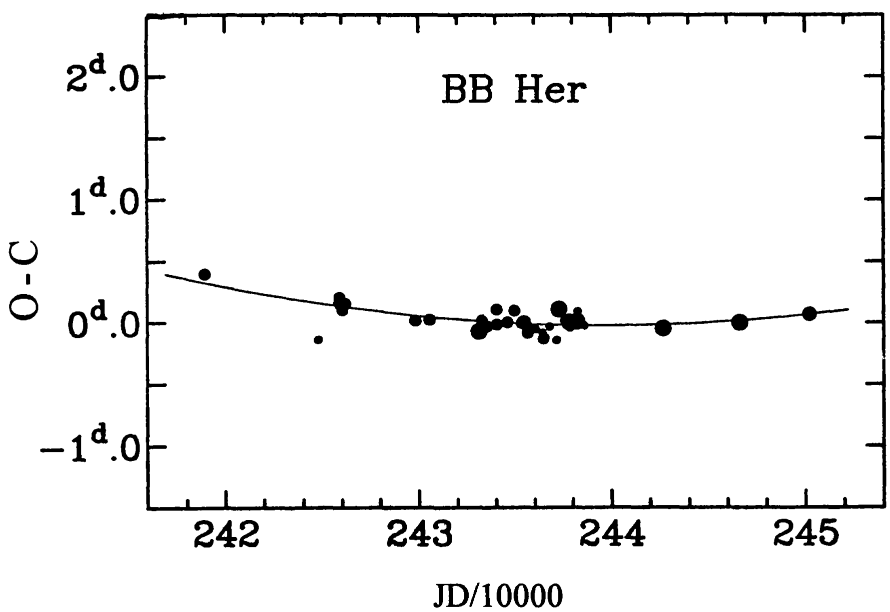

Figure 3. The difference between observed and computed times of light maximum in the Saint Mary's data for BB Her relative to the ephemeris of Szabados (1980). The size of each symbol is proportional to the weight given to it in the least-squares solution. The resulting parabolic fit to the data is shown by the line.

for observations of the Cepheid now covers 86 years, and it can be seen that a simple parabola matches the period variations very well. The observed period change for BB Her amounts to $+0.38 \pm 0.08$ second per year, which turns out to be almost exactly what one predicts for a Cepheid evolving from left to right in the HR diagram in the 3rd crossing of the instability strip.

4. Evolutionary effects on Cepheid periods

The technique of using Cepheid O-C diagrams to detect period changes is well established. Differences between the observed ("O") and computed ("C") times of light maximum for a particular variable can be attributed to various effects, but it is generally recognized that evolution should dominate the detected changes for classical Cepheids (e.g., Fernie 1984). In most cases the observed period changes do agree with theoretical predictions for the evolution of intermediate-mass stars through the Cepheid instability strip (Erleksova and Irkaev 1982; Szabados 1983, 1984; Fernie 1984, 1990; Saitou 1989; Saitou and Takeuti 1990). Described here are the results of another study of the phenomenon.

Extensive collections of O-C data for light maxima of Cepheids have been compiled in the literature, the most comprehensive being that of Szabados (1977, 1980, 1981, 1989, 1991). Results for individual stars tabulated by Szabados were used in 1990 by Stephen Duncan of Saint Francis Xavier University (Antigonish, Nova Scotia) as input for a study of Cepheid period changes carried out under the author's direction (Duncan 1991). The work entailed fitting simple parabolae to Cepheid period changes evident from O-C data using the method of least-squares described by Belserene (1986). Other investigators have completed similar studies, so that with the unpublished study by Duncan (1991), there is a fairly extensive body of such work available. A compilation of published and unpublished period change rates is presented in Tables 1 and 2, where Table 1 presents results for Cepheids with increasing periods and Table 2 presents results for Cepheids with decreasing periods. The values of log P and log dP/dt tabulated there are the derived temporal rates of period change in units of seconds per year. Of course, period changes need not be strictly parabolic (Fernie 1990), and may also be affected by random fluctuations (Eddington and Plakidis 1929; Percy *et al.* 1997). Another possibility that occurs for some stars is the effect of superimposed sinusoidal variations attributable to the cyclical nature of light-travel-time effects in the line of sight for Cepheids undergoing orbital motion about unseen companions (Szabados 1982). Cycle miscounts arising from temporal gaps in the observational record are also important for some stars. However, evolution is likely to be the dominant factor affecting secular period changes for Cepheids, as we demonstrate (once again) here.

Many variables are described as having constant periods of variability based upon the linear behavior of their O-C data. Such objects were examined for secular variations by Duncan (1991), whether or not a parabolic fit to the O-C data was any more significant than a straight line fit, and the results are also included in Tables 1 and 2. The purpose of the present study is not to attribute secular trends to all Cepheids that have O-C data, but rather to establish a data base that makes a comparison of inferred period changes more comprehensive. There are probably a variety of other factors included in the data, and they undoubtedly affect the derived period change rates. In some cases the sense of the deduced period change may even be opposite to the actual secular change. Such problems are generally important only for Cepheids with the smallest rates of period change, however, and can only be investigated properly by means of a longer temporal baseline of observations. Interestingly enough, all of the "questionable" cases tend to exhibit positive rates of period change deduced from the available observations.

The evolutionary models used here for comparison are those of Maeder and Meynet (1988). A variety of improved models could have been used, but most result in rates of

Table 1. Cepheids with increasing periods.

Cepheid	*log* P *(days)*	*log* dP/dt *(sec/yr)*	*Ref.*	*Cepheid*	*log* P *(days)*	*log* dP/dt *(sec/yr)*	*Ref.*
V473 Lyr	0.173	−1.095	1	V600 Aql	0.860	+0.194	2
SU Cas	0.290	−1.839	2	V336 Aql	0.864	−0.214	2
EU Tau	0.323	−0.536	1	R Mus	0.876	+0.021	2
IR Cep	0.325	−0.573	2	BB Her	0.876	−0.352	10
UY Mon	0.380	−0.872	1	VY Cyg	0.895	−1.123	2
BE Mon	0.432	−2.485	2	RX Cam	0.898	−0.546	2
DX Gem	0.497	+0.601	2	U Vul	0.903	−1.413	2
BY Cas	0.508	+0.777	2	DL Cas	0.903	−1.777	2
AD Gem	0.578	−0.758	2	BK Aur	0.903	−0.659	2
DF Cas	0.583	−0.657	2	S Sge	0.923	−0.453	2
SU Cyg	0.585	−1.218	2	IX Cas	0.962	+1.808	2
Y Aur	0.587	−1.238	2	β Dor	0.977	−0.042	2
CM Sct	0.593	−0.858	3	DD Cas	0.992	+0.204	2
α UMi	0.599	+0.505	4	BZ Cyg	1.006	+0.137	2
ST Tau	0.606	−1.210	2	SY Aur	1.006	+0.081	2
SY Cas	0.610	−0.736	2	XX Cen	1.040	+0.650	2
V1726 Cyg	0.627	−0.441	1	TY Sct	1.043	+0.447	5
V912 Aql	0.643	−0.330	5	SV Per	1.046	−0.201	2
GI Car	0.646	+0.104	1	AA Gem	1.053	−0.309	2
FF Aql	0.650	−1.074	2	RY Cas	1.084	+0.459	2
RY CMa	0.670	−0.860	6	SU Cru	1.109	+0.413	6
V1154 Cyg	0.692	−1.167	2	SZ Cas	1.134	+1.691	8
AS Per	0.697	−0.827	2	TT Aql	1.138	−0.484	2
V350 Sgr	0.712	−1.214	2	TX Cyg	1.168	+0.641	8
V386 Cyg	0.721	−0.837	2	VW Cen	1.177	+0.440	6
CV Mon	0.731	−0.377	2	SZ Cyg	1.179	−0.625	2
X Lac	0.736	+0.170	2	SV Mon	1.183	+0.250	2
V924 Cyg	0.746	+0.636	2	X Cyg	1.214	+0.429	2
MY Pup	0.755	+0.695	7	RW Cam	1.215	+0.123	2
FM Cas	0.764	−0.998	2	CD Cyg	1.232	+0.549	2
MW Cyg	0.775	−0.868	2	SZ Aql	1.234	+0.757	8
VW Cas	0.778	−1.282	2	Y Oph	1.234	+1.035	8
FM Aql	0.786	−2.076	2	CP Cep	1.252	+1.295	6
V538 Cyg	0.787	+0.258	2	YZ Aur	1.260	−1.821	2
V733 Aql	0.791	+0.041	2	RU Sct	1.294	+1.004	8
RS Cas	0.799	−0.160	2	VX Cyg	1.304	+0.445	2
X Vul	0.801	−0.352	2	RY Sco	1.308	+1.105	8
RR Lac	0.807	−0.768	2	X Pup	1.321	+1.479	8
AW Per	0.810	−0.391	2	T Mon	1.432	+1.098	2
BB Sgr	0.822	−0.165	2	AQ Pup	1.478	+2.539	6
CS Mon	0.828	+0.338	2	V609 Cyg	1.492	+1.910	9
U Sgr	0.829	−0.486	8	l Car	1.551	+2.074	2
V636 Sco	0.832	+0.042	2	RS Pup	1.617	+2.023	8
V496 Aql	0.833	−0.543	1	GY Sge	1.713	+2.871	9
X Sgr	0.846	+0.172	2	S Vul	1.833	+2.749	8
η Aql	0.856	−0.618	2				

References: 1. Berdnikov *et al.* (1997), 2. Duncan (1991), 3. Olson (1981), 4. Dinshaw *et al.* (1989), 5. Starmer (1989), 6. Vinkó (1991), 7. Szabados (1989), 8. Erleksova and Irkaev (1982), 9. Berdnikov (1994) adjusted (see text), 10. this paper.

Table 2. Cepheids with decreasing periods

Cepheid	*log* P *(days)*	*log* dP/dt *(sec/yr)*	*Ref.*	*Cepheid*	*log* P *(days)*	*log* dP/dt *(sec/yr)*	*Ref.*
V465 Mon	0.434	–0.297	1	TX Del	0.790	+0.450	1
SZ Tau	0.498	–0.311	2	V1954 Sgr	0.791	–0.375	5
AZ Cen	0.507	+0.284	1	VV Cas	0.793	–0.040	1
BD Cas	0.562	–0.040	1	CR Cep	0.795	–0.276	1
RT Aur	0.572	–0.852	3	AO Aur	0.830	–0.120	1
V572 Aql	0.576	+0.589	1	U Aql	0.847	–0.734	1
BF Oph	0.609	–1.122	1	RS Ori	0.879	+0.191	1
V508 Mon	0.616	–1.208	1	GH Cyg	0.893	–0.610	1
SX Per	0.632	–0.580	1	W Gem	0.898	+0.163	1
Y Lac	0.636	–1.401	1	GQ Ori	0.935	+0.472	1
V402 Cyg	0.640	–1.332	1	FN Aql	0.977	–0.434	1
T Vul	0.647	–0.613	1	ζ Gem	1.006	+0.531	1
XY Cas	0.653	–1.164	1	AN Aur	1.012	+0.271	1
S Cru	0.671	+0.371	1	Z Lac	1.037	–0.846	1
VZ Cyg	0.687	–0.692	1	VX Per	1.037	+0.983	1
V Lac	0.698	–0.097	1	XX Cen	1.040	+0.650	1
BG Lac	0.727	–1.690	1	RX Aur	1.065	–0.104	1
δ Cep	0.730	–1.108	1	V1828 Sgr	1.113	+1.538	6
SW Cas	0.736	–0.615	1	RW Cas	1.170	+1.109	1
XX Mon	0.737	+1.084	4	V396 Cyg	1.522	+1.181	7
RZ Gem	0.743	–0.071	1	EV Aql	1.587	+2.492	7
RV Sco	0.783	–0.543	1	SV Vul	1.653	+2.355	1
KL Aql	0.786	–1.440	1	V1467 Cyg	1.686	+2.260	7

References: 1. Duncan (1991), 2. Berdnikov and Pastukhova (1995), 3. Fernie (1993), 4. Vinkó (1991), 5. Wooldridge (1983), 6. Chanover (1989), 7. Berdnikov (1994) adjusted (see text).

period change that are fairly similar (cf. Fernie 1984; Saitou 1989). Slight differences between existing calibrations can usually be tied to the choice of the Cepheid period-luminosity relation used for identifying the location of the Cepheid instability strip. The adopted relation for the present study is an empirical formulation arising from studies by the author of Cepheids in open clusters (see Turner 1992; Turner *et al.* 1994).

The results for galactic Cepheids are shown in Figures 4 and 5. Figure 4 depicts the 91 Cepheids exhibiting period increases (i.e., 1st, 3rd, or 5th crossing of the instability strip), and Figure 5 depicts the 46 Cepheids exhibiting period decreases (i.e., 2nd or 4th crossing of the instability strip). Many short-period Cepheids are suspected from their light curve parameters to be overtone pulsators (e.g., Antonello and Poretti 1986; Antonello *et al.* 1990), but we have not included that extra degree of complication in the diagrams. Theoretical predictions for the various strip crossings are also indicated. Some conclusions evident from the diagrams are listed below:

a. The vast majority of Cepheids with positive period changes are in the 3rd crossing of the instability strip (core helium burning). That is the slowest redward crossing for such objects.

b. At least three Cepheids seem likely to be in the 1st crossing of the instability strip (shell hydrogen burning). The three objects—BY Cas, IX Cas, and DX Gem—have light curves that closely mimic pure sine waves and have the fastest rates of period change for their pulsational periods. It has been argued by Efremov (1968) that all s-Cepheids—the original designation for Cepheids with sinusoidal light curves, but one that is used in a slightly different context at present—may be in the 1st crossing of the

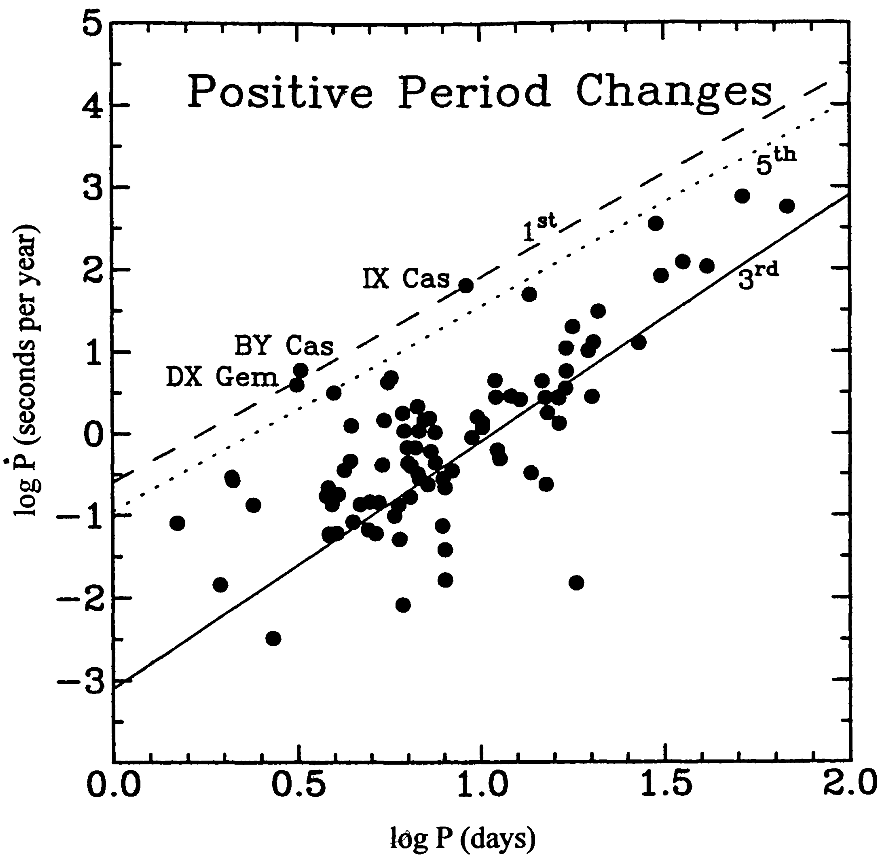

Figure 4. The dependence on period of the rates of period increase for Cepheids in Table 1. The different lines represent the predictions for 1st, 3rd, and 5th crossings of the instability strip. The three s-Cepheids that may be 1st crossers are identified.

instability strip. Yet there are several other s-Cepheids in Figures 4 and 5 that have rates or directions of period change inconsistent with such a hypothesis. A more reasonable conclusion is that all 1st crossers develop sinusoidal light curves. It is interesting to note that Polaris (α UMi) may also fall in this group. Its rate of period increase is only slightly smaller than what would be predicted for a 1st crossing of the instability strip.

c. Most Cepheids with negative period changes are in the 2nd crossing of the instability strip (core helium burning). That is the slowest blueward crossing for such objects.

d. The number of Cepheids likely to be in the 4th and 5th crossings of the instability strip (shell helium burning) is small, but reasonably consistent with the numbers predicted for such rapid evolutionary phases.

e. Rates of period change seem to provide no unambiguous information about Cepheid pulsation modes.

f. The three objects in point b are probably fundamental mode pulsators. Their number is consistent with the expected number of 1st crossers for a sample of this size (137 objects), whereas if they were overtone pulsators, their period change rates would be more consistent with a 5th crossing of the instability strip. Also, there would then be no Cepheids in the sample consistent with a first crossing. Our conclusions do depend somewhat upon the model predictions, but are in general remarkably consistent with the observational data available.

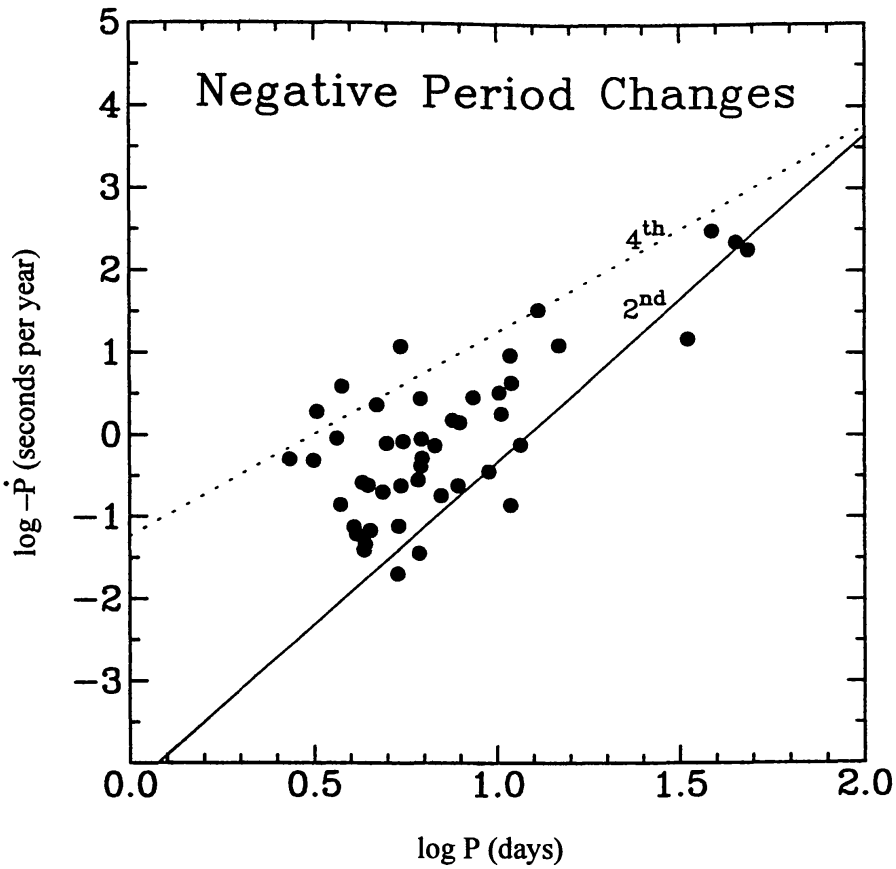

Figure 5. The dependence on period of the rates of period decrease for Cepheids in Table 2. The different lines represent the predictions for 2nd and 4th crossings of the instability strip.

5. Future work

It is not immediately evident from the data in the tables or the figures, but there are a few anomalies among published rates of period change that need to be examined in more detail. For example, there seems to be a misprint in the rates of period change for long-period Cepheids published by Berdnikov (1994), since they require an adjustment by a factor of ~100 to match the results obtained by Duncan (1991) for overlapping stars using essentially the same data. That change was made to the values cited in Tables 1 and 2. Such discrepancies need to be clarified if we are to learn as much as possible about the nature of Cepheids. Berdnikov *et al.* (1997) have also pointed out the need to monitor Cepheid light curves in order to examine the curious sinusoidal variations evident in the O-C data for some Cepheids. Such cyclical variations cannot be explained by light-travel-time effects in binary systems, since the inferred masses for the systems would be unrealistically large. If they are caused by random fluctuations, then it seems imperative to understand why only specific objects are affected. It is for such reasons that the pilot program described here is continuing. The results for other program Cepheids observed during the 1996 and 1997 summer observing periods have also been analyzed. Full details will be presented in a separate paper (Turner *et al.* 1998).

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