STUDIES OF YELLOW SEMIREGULAR (SRd) VARIABLES

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Abstract

We have used the Hipparcos database of epoch photometry to study the variability of several yellow semiregular (SRd) variables. For some of the stars (V487 Cas, RW Cep, BM Sco, CE Vir), the results were inconclusive. For SX Lac and TY Vir, the periods found were consistent with the *General Catalogue of Variable Stars* (190 and 50 days, respectively). For UU Her, the known periods of about 45 and 72 days were recovered. For WY And, RU Cep, and SX Her, reliable periods were found; we used archival data to construct (O-C) diagrams to study both random and systematic period changes in these three stars.

1. Introduction

Almost all yellow (spectral type F, G, and K) supergiants are variable in brightness. Those which are periodic are usually classified as Cepheids—Population I or II, depending on whether they are young and massive, or old and less massive. Those which show alternating deep and shallow minima are classified as RV Tauri stars—RVB if there are long-term variations in mean brightness, and RVA if there are not. The most luminous yellow supergiants—ρ Cas, for instance—are often classified as "hypergiant" variables. The rest are included in the general SRd classification, "semi-regular variable giants and supergiants of spectral types F, G, K sometimes with emission lines in their spectra. Amplitudes of light variation are in the range from 0.1 to 4 magnitudes; the range of periods is from 30 to 1100 days," according to the *General Catalogue of Variable Stars* (GCVS) (Kholopov *et al.* 1985).

Because they are a "mixed bag," the SRd variables are rather poorly-studied, though they have been included in some important studies and reviews of RV Tauri stars (Preston *et al.* 1963; Dawson 1979; Wahlgren 1993). In recent years, Zsoldos (e.g., 1993, 1995) has been a leader in photometric monitoring of RV Tauri and SRd stars, and in studying their long-term behavior.

Most RV Tauri stars (and probably most of the SRd stars) are believed to be related to low-mass AGB (asymptotic giant branch) stars, either executing "blue loops" which carry them into the yellow giant or supergiant region of the Hertzsprung-Russell diagram as a result of nuclear shell-burning "flashes" (Vassiliadis and Wood 1993), or contracting to the white dwarf stage. The shell-burning flashes typically occur on a time scale of a few tens of thousands of years; the contraction to the white dwarf stage is faster. The hypergiant variables are much more massive stars in an advanced stage of evolution.

In the present study, we make use of Hipparcos epoch photometry, and data from the AAVSO and other sources, to investigate the light curves, periodicity, and period changes of several SRd variables. The stars in our sample are listed in Table 1, and described individually in Section 3. They are those SRd stars which are bright enough to be contained in the Hipparcos catalogue. Spectral types are taken from the GCVS. In a sense, this is a "pilot study" for a future analysis of a larger sample of SRd variables, using AAVSO and other data.

Table 1. SRd stars

Star	*GCVS Amplitude (mag.)*	*GCVS Period (d)*	*Type of Data**	*Calculated Period (d)*	*O-C Diagram*
WY And	1.4	108	H, A	110.8	Yes
V487 Cas	0.3	134	H	98.7	No
RU Cep	1.6	109	H, A, V	109	Yes
RW Cep	2.1	346	H	approx. 600	No
SX Her	2.3	102.9	H	102.2	Yes
UU Her	2.1	80.1	H	45 and 75	No
SX Lac	1	190	H	197	No
BM Sco	1.9	815	H		No
CE Vir	2.3	67:	H		No
TY Vir	0.53	50	H		No

* H = Hipparcos, A = Archival, V = AAVSO

2. Data and analysis

The primary source of data for this study is the Hipparcos database of epoch photometry (ESA 1997). The challenges of analyzing variable stars using this database have recently been discussed in this Journal by Marinova and Percy (1999). Many measurements are made within a few hours. For the purpose of analyzing variable stars with periods of weeks to months or more, these “clusters” of measurements are effectively single measurements. The intervals between these clusters are often non-randomly distributed; they tend to be in the range of 20–30 days.

We have analyzed the data using light curves and phase diagrams, and autocorrelation analysis and Fourier analysis (Percy *et al.* 1993; Marinova and Percy 1999). We have used the ASTROLAB program written by Matt Szczesny (Percy *et al.* 1996) for autocorrelation analysis. We used the program DCDFT, kindly made available by Dr. E.P. Belserene, and the AAVSO time series analysis program TS11 (available from the AAVSO website http://www.aavso.org/adata/software.stm) for Fourier analysis. To construct phase diagrams, we have used the on-line software on the Hipparcos website astro.estec.esa.nl/Hipparcos/hipparcos.html, which we recommend heartily for both research and educational purposes.

In order to test for possible random, cycle-to-cycle fluctuations in period, we used the formalism of Eddington and Plakidis (1929): if $< u(x) >$ is the mean absolute difference between (O-C)’s which are x cycles apart, then $< u(x) >^2 = 2a^2 + \varepsilon^2 x$, where a is the mean observational error in measuring the time of maximum or minimum, and ε is the mean fluctuation in period in days, per cycle.

To test for possible evolutionary changes in period, we fit the (O-C) diagram with a parabola, using the method of least-squares. If the change in period is linear, then the (O-C) diagram will be parabolic.

3. Results for individual stars

3.1. WY And

WY And (HIP 116883, Sp G2e-K2(M3)) has an amplitude of 1.4 magnitudes (photographic), and a period of 108 days, according to the GCVS. There were 29 “clusters” of Hipparcos measurements which could be used for analysis. The intervals between the clusters were relatively uniformly distributed between 1 and 140 days. The autocorrelation diagram showed minima at Δt’s of 120, 240, and 360 days, which suggested a period of about 120 days. Fourier analysis showed one main period of 110.8 days. The phase curve, using this period, is shown in Figure 1. The scatter is quite satisfactory for a semiregular variable.

Zsoldos (1990) has already carried out a comprehensive study of the long-term period changes in this star. His (O-C) diagram extends from 1900 to 1988, but there are two significant gaps, at 1950–1965 and 1975–1984. The number of cycles in each gap is uncertain. Zsoldos favors an interpretation in which the period is constant in the long term (at 107.4 days), but with random variations.

We decided to concentrate on the interval JD 2427137 (1933) to 2429504 (1939), when the coverage was densest. Times of maximum brightness (Beyer 1948) were kindly provided by Dr. Endre Zsoldos, Konkoly Observatory, Hungary. The data cover less than 25 cycles. Nevertheless, these maxima lead to an (O-C) diagram which appears to be parabolic (Figure 2). The equation of the best-fitting parabola is:

$$(\text{O-C}) = 0.0507\text{N}^2 - 1.455\text{N} - 6.47 \text{ days} \qquad (1)$$

where N is the number of cycles elapsed since the epoch given (here, JD 2447882.83).

The corresponding rate of period change ($\dot{\text{P}}$) is rather large: 0.343 day/year; the characteristic time required for the period (P) to change would be $(\text{P}/\dot{\text{P}})$ = 108/0.343 = ~315 years. This large rate of period change is not inconsistent with Zsoldos' (1990) (O-C) diagram if an extra cycle is added in each of the gaps mentioned above. On the other hand, Zsoldos' more conservative interpretation of the (O-C) diagram would imply a much slower evolution rate.

The $< u(x) >^2$ diagram is linear to $x = 9$, with a slope which corresponds to an average period fluctuation of 3.8 days (0.03 cycle) per cycle, and an intercept which corresponds to an average error of 2.8 days in the times of maxima. These random fluctuations can explain the wave-like features seen in the (O-C) diagram.

3.2. V487 Cas

V487 Cas (HIP 5239, Sp G0-G4Ia) has an amplitude of 0.3 magnitude (visual), and a period of 134 days, according to the GCVS. There were 32 "clusters" of Hipparcos measurements which could be used for analysis. The intervals between these clusters ranged mostly from 1 to 50 days. The autocorrelation diagram was almost flat from 20 to 100 days (the lowest points were at 80 and 100 days, but the highest point was at 90 days). Fourier analysis gave periods of 98.7 and 5.7 days, with comparable power, and comparable scatter in the phase diagram; there are a variety of other periods with slightly lower power. The results are therefore inconclusive.

3.3. RU Cep

RU Cep (HIP 6325, Sp G6-M3.5III) has an amplitude of 1.6 magnitudes (visual), and a period of 109 days, according to the GCVS. There are 34 "clusters" of Hipparcos measurements which could be used for analysis. The intervals between these were almost all less than 50 days. The autocorrelation diagram showed minima at 110, 230, 340, and 440 days, which suggests a period of 110 days. The highest peak in the power spectrum was at a period of 110.25 days. This was used to produce the phase curve (Figure 3), and for subsequent (O-C) analysis.

Times of maximum brightness were taken from Beyer (1948). These times cover the interval of Julian Date 2427130 (1933) to 2429370 (1939)—an interval of only 20 cycles. These produced an (O-C) diagram which was reasonably straight, but had a slope of 0.1 cycle per cycle, suggesting that the period was 10 per cent longer at that time. We have combined these times of maximum with more recent times of maximum, derived from visual observations in the AAVSO International Database (Mattei 1998) covering the interval JD 2438224 to 2450960 (1963–1998). These produce the (O-C) diagram shown in Figure 4. There is a gap of about 80 cycles; the exact number of cycles cannot be known with certainty. Figure 4 is reasonably straight; there is no significant curvature. The slope suggests that the average period has been about 1 per cent longer than the one used (109 days), and has been approximately constant. The wave-like structures in the (O-C) diagram may be due to cycle-to-cycle fluctuations in period.

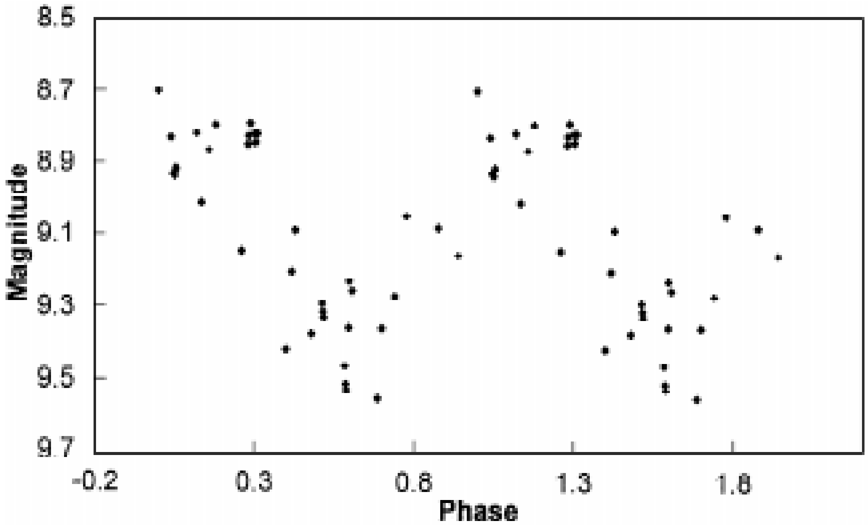

Figure l. The phase curve for WY And, using Hipparcos photometry, a period of 110.8 days, and an epoch of JD 2447882.83.

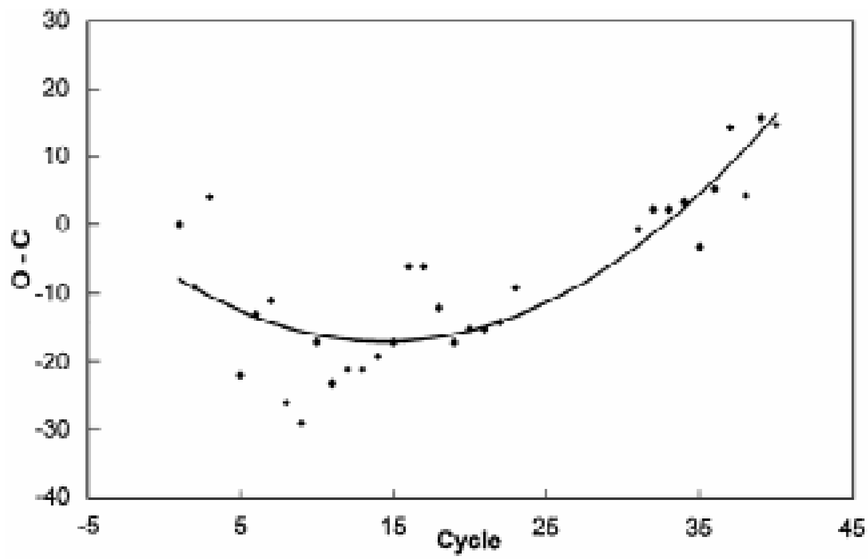

Figure 2. The (O-C) diagram for WY And, using a period of 108 days and an epoch of JD 2447882.83, and data from Beyer (1948).

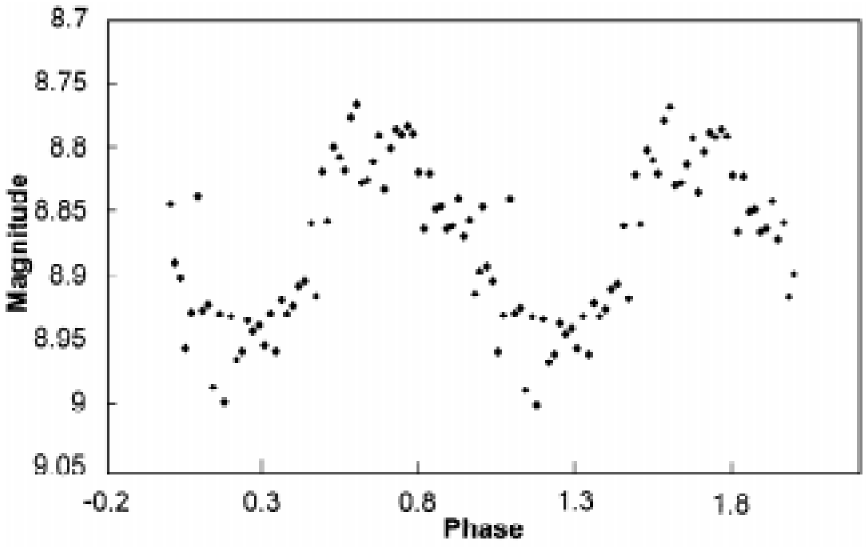

Figure 3. The phase curve for RU Cep, using Hipparcos photometry, a period of 110.25 days, and an epoch of JD 2437601.

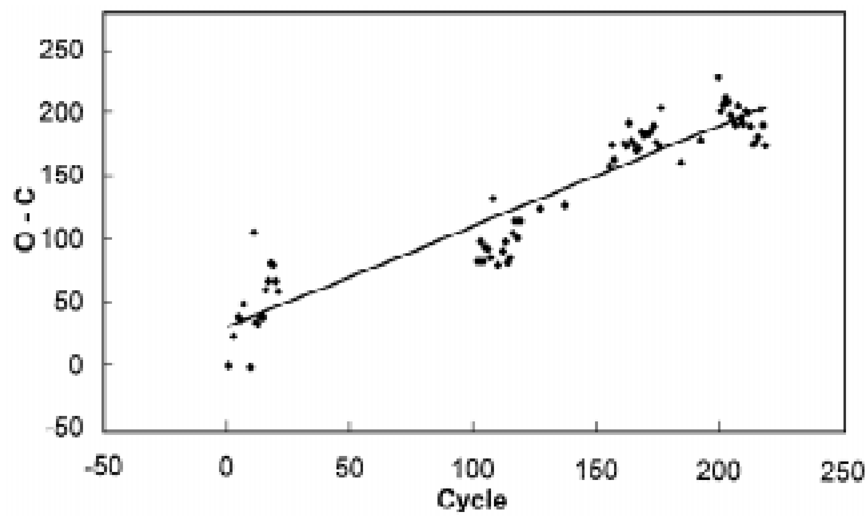

Figure 4. The (O-C) diagram for RU Cep, using a period of 109 days and an epoch of JD 2437601, and data from Beyer (1948) and AAVSO (Mattei 1998).

However, the $< u(x) >^2$ diagram is linear only to about $x = 5$, which suggests that the hypothesis of random, cycle-to-cycle period fluctuations in this star is weak.

3.4. RW Cep

RW Cep (HIP 110504, Sp K0Ia-0) has an amplitude of 2.1 magnitudes (photographic), and a period of 346 days, according to the GCVS. There were 36 "clusters" of Hipparcos measurements which could be used for analysis. The Hipparcos light curve showed three irregular cycles over 1200 days, for an average cycle-count period of 400 days. Understandably, in view of the short dataset, the autocorrelation diagram and Fourier analysis provided very little information, other than that the period was several hundred days.

3.5. SX Her

SX Her (HIP 78994, Sp G3ep-K0(M3)) has an amplitude of 2.3 magnitudes (photographic), and a period of 102.9 days, according to the GCVS. There are 33 "clusters" of Hipparcos measurements which could be used for analysis. The intervals between these were mostly less than 50 days. The autocorrelation diagram showed a maximum at a Δt of 65 days, and a minimum from 80 to 110 days. The highest peak in the power spectrum was at a period of 102.9 days. This period produced an acceptable phase diagram (Figure 5), and was used for subsequent (O-C) analysis.

Times of maximum brightness were taken from Gerasimovic (1929). They cover the interval JD 2415091 to 2425369 (1900–1928), which corresponds to about 100 cycles. The (O-C) diagram is shown in Figure 6. There is some suggestion of curvature. The best-fitting parabola is:

$$(O\text{-}C) = 0.002467N^2 - 0.2205N - 7.77 \text{ days} \qquad (2)$$

The curvature corresponds to a rate of period change of 0.0175 day/year. The characteristic time required for the period to change would be $(P/\dot{P}) = 102.9/0.0175 =$ ~5800 years.

The $< u(x) >^2$ diagram is linear to about $x = 9$, but with some scatter; the slope corresponds to an average cycle-to-cycle period fluctuation of 2.0 days (0.02 cycle) per cycle, and the intercept corresponds to an average error of 3.7 days in the times of maximum.

3.6. UU Her

UU Her (HIP 81272, Sp F2Ib-G0) has an amplitude of 2.1 magnitudes (photographic), and a period of 80.1 days, according to the GCVS; it is the prototype of a class of high-latitude yellow supergiant variable stars with unstable periods (Sasselov 1981). For a general review of high-latitude supergiants, see Sasselov (1993). UU Her has shown periods of 72 and 45 days (Fernie 1993).

There were 27 "clusters" of Hipparcos measurements which could be used for analysis. The intervals between these were relatively uniformly distributed between 1 and 60 days. The autocorrelation diagram showed a maximum at a Δt of 30 days, and a minimum at 50 days (but based on a small number of points). The two highest peaks in the power spectrum were at 75.2 and 45.6 days, in good agreement with the previously-known periods (especially considering the short dataset which was analyzed). Each of these periods produced a phase diagram with reasonably small scatter. The GCVS period of 80.1 days produced a phase diagram with significantly more scatter.

3.7. SX Lac

SX Lac (HIP 113252, Sp K2) has an amplitude of 1 magnitude (photographic), and a period of 190 days, according to the GCVS. There are 30 "clusters" of Hipparcos measurements which could be used for analysis. The intervals between these were mostly less than 50 days, with a few that were higher. The autocorrelation diagram

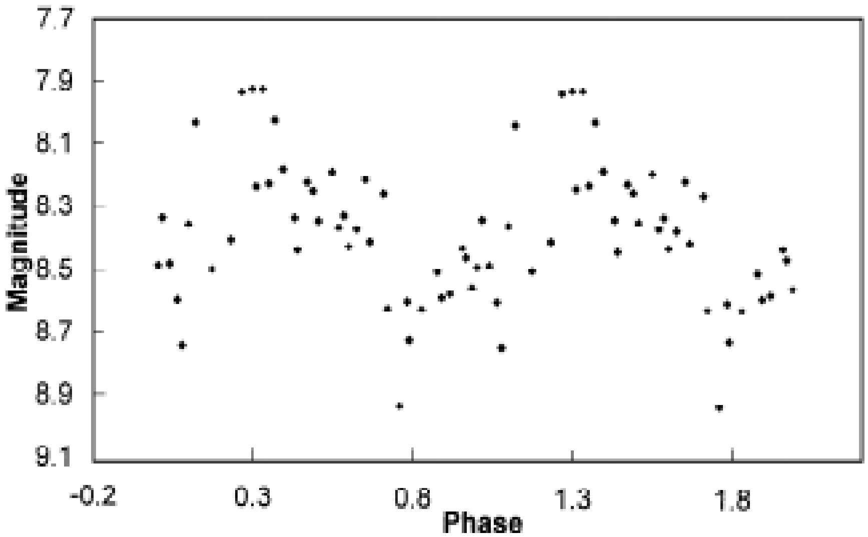

Figure 5. The phase curve for SX Her, using Hipparcos photometry, a period of 102.9 days, and an epoch of JD 2433059.9.

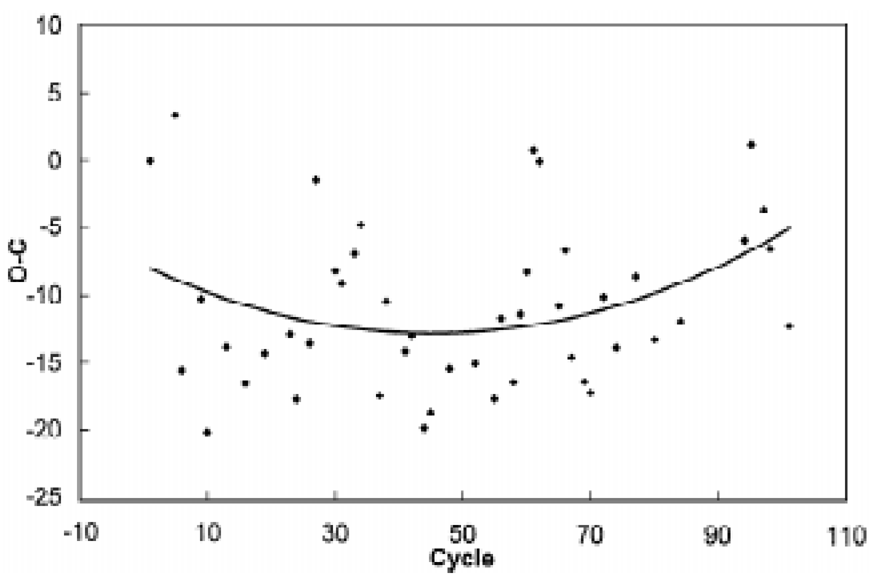

Figure 6. The (O-C) diagram for SX Her, using a period of 102.9 days and an epoch of JD 2433059.9, and data from Gerasimovic (1929).

showed a maximum at a Δt of 90 days, and a minimum from 150 to 200 days. The highest period in the power spectrum is at 197 days; all other peaks are significantly lower. This period produced a phase diagram with low scatter. The GCVS period of about 190 days is confirmed.

3.8. BM Sco

BM Sco (HIP 86527, Sp K2.5Ib) has an amplitude of 1.9 magnitudes (photographic), and a period of 815 days, according to the GCVS. There were only 15 "clusters" of Hipparcos measurements which could be used for analysis. The light curve showed variations on time scales of 500 to 1000 days, with an amplitude of about 0.3 magnitude in the Hipparcos system. A phase diagram, using the GCVS period, was not convincing. The results are therefore inconclusive. The period is unusually long for a class Ib supergiant.

3.9. CE Vir

CE Vir (HIP 67439, Sp G-K) has an amplitude of 2.3 magnitudes (visual), and a period of 67: days, according to the GCVS. There were only 20 "clusters" of Hipparcos measurements which could be used for analysis. The intervals between these clusters were either 1–40 days or greater than 120 days. This made it almost impossible to produce a meaningful autocorrelation diagram. The highest peak in the power spectrum was at 13.2 days, but there were peaks at 145.5, 79.7, and several shorter periods, which were almost as high. All of these periods (including 67 days) produce equivalent phase curves with some scatter. The results are therefore inconclusive.

3.10. TY Vir

TY Vir (HIP 57850, Sp G3pIb) has an amplitude of 0.53 magnitude (visual), and a period of 50 days, according to the GCVS. There were 40 "clusters" of Hipparcos measurements which could be used for analysis. The intervals between these were mostly less than 50 days or greater than 120 days. The autocorrelation diagram showed a maximum at a Δt of 20 days, and a flat minimum from 30 to 50 days. The power spectrum showed many comparable peaks, mostly for periods less than 4 days. The highest peaks for longer periods were at 60.6 and 46.2 days, each of which produced an acceptable phase curve. These results are not inconsistent with the GCVS period of 50 days.

4. Discussion

The ten stars studied are obviously a "mixed bag," as can be seen from their spectral types, periods, and degrees of regularity. The time scales of variation of these stars range from 50 to over 800 days. These are not obviously correlated with the luminosity classes of the stars.

The behavior of the stars ranges from nearly periodic to rather irregular. One hypothesis about the behavior of the yellow supergiant variables is that they represent a sequence of non-linear behavior from periodic (the Cepheids), through period-doubling (the RV Tauri stars), to chaotic (the SRd stars), with effective temperature as the control parameter (Buchler and Kovacs 1987). We should point out that the small-amplitude and medium-amplitude red variables also show irregularity of unknown origin.

We have determined (O-C) diagrams for three of the more periodic variables. The (O-C) diagrams can be affected by several processes, including random error in determining the time of maximum light, and random cycle-to-cycle fluctuations in period (Eddington and Plakidis 1929; Percy and Colivas 1999), and by evolutionary effects. Of the three stars (with similar periods!) examined, two showed random fluctuations of 0.02–0.03 cycle/cycle, and one did not.

A linear change in period, due to evolution, produces an (O-C) diagram which is a parabola; the curvature of the parabola is a function of the rate of change in period (or the "characteristic time" of evolution). For WY And, the characteristic time may be as short as a few hundred years, but could be longer. For SX Her, the characteristic time is a few thousand years. For RU Cep, the characteristic time may be longer still. If the SRd stars are low-mass stars undergoing nuclear shell flashes in their interiors (Vassiliadis and Wood 1993), then the evolution times are tens of thousands of years.

5. Conclusions

Despite their irregularity, and their heterogeneous nature, the SRd variables can potentially tell us about the pulsation processes, and evolution in yellow giants and supergiants. We urge visual and photoelectric observers to continue to monitor a selection of these interesting stars.

6. Acknowledgements

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