Self-correlation Studies of RV Tauri Variables and Related Objects

John R. Percy
Farisa Mohammed
Department of Astronomy and Astrophysics, University of Toronto, Toronto, Ontario, Canada

Received August 2003; revised August 2004

Abstract RV Tauri (RVT) variables are old, low-mass, yellow supergiant pulsating variable stars whose light curves show alternating deep and shallow minima. They are related to Population II Cepheid (CW) variables and to yellow semiregular (SRd) variables. We describe the results of self-correlation analysis of a large body of visual photometry of 9 bright RVT and SRd variables, namely AG Aur, AV Cyg, SU Gem, AC Her, SX Her, TT Oph, UZ Oph, TX Per, and V Vul. Self-correlation analysis, which probes the cycle-to-cycle behavior of a variable, averaged over a dataset, has proven to be a useful tool for investigating these stars, because their classification is actually based on their cycle-to-cycle behavior. Our results are consistent with those of our analysis of RVT and CW variables in the Large Magellanic Cloud (Percy, Hosick, and Leigh 2003), and support the view that the RV Tauri phenomenon has two dimensions: (i) the relative depths of the primary and secondary minima, and (ii) the number of cycles over which the alternating minima correlate. If the RVT phenomenon is due to the presence of two pulsation modes, then these dimensions are equivalent to: (i) the relative amplitudes of the two modes, and (ii) the closeness of the ratio of the pulsation periods to 2:1. There is therefore a "spectrum" of behavior from CW to RVT to SRd, depending on the values of these two parameters.

1. Introduction

Population II Cepheid (CW) variables are old, low-mass, yellow supergiants which are pulsating approximately periodically. RV Tauri (RVT) variables are old, low-mass, yellow supergiants whose light curves are characterized by alternating deep and shallow minima; some RVT variables show long secondary periods; those that do are classified as RVb; those that do not are classified as RVa. Yellow semiregular (SRd) variables are old, low-mass, yellow supergiants whose pulsation is semiregular at best. It has long been suspected that CW-RVT-SRd represents a spectrum of behavior. Some CW variables show incipient RVT behavior. In most RVT variables, the behavior is semiregular in the sense that it deviates from the "alternating deep and shallow minima" to a greater or lesser extent (Percy 1993). The distinction between CW, RVT, and SRd is often based on observation of a limited number of cycles; the classification might be different if a larger number of cycles had been observed. In terms of evolutionary status, these groups are often lumped together. They are either undergoing "blue loops" from the asymptotic-giant branch

(AGB) in the Hertzsprung-Russell Diagram, due to "flashes" in their hydrogen- and helium-burning shells or, in the case of the most luminous stars, in transition from the AGB to the white dwarf region. The RVT stars are usually surrounded by a shell of warm dust, presumably ejected from the star during the AGB stage (Jura 1986). For a good, simple, on-line review of RVT variables, see Malatesta (2003).

Percy, Hosick, and Leigh (2003) recently used self-correlation analysis—a simple form of variogram analysis—to study 33 RVT and CW variables in the Large Magellanic Cloud (LMC) using data from the MACHO project—a photometric search for gravitational microlensing by MAssive Compact Halo Objects such as white dwarfs. Self-correlation analysis proved to be useful in confirming the period and classification of the stars; the latter is based on their cycle-to-cycle behavior. Percy, Hosick, and Leigh (2003) also carried out simulations which supported the hypothesis that the RV Tauri phenomenon is due to the presence of two periods whose ratio is close to 2:1. Their simulations also emphasized that the RV Tauri phenomenon has two dimensions: the relative depths of adjacent minima (which depends on the relative amplitudes of the two periods which are present) and the number of cycles over which the alternating minima correlate (which depends on the closeness of the period ratio to 2:1). This would be consistent with theoretical work (Takeuti and Petersen 1983; Tuchman *et al.* 1993; Fokin 1994; Takeuti 1998; Fokin 2001; Ishida 2003), that all concludes that two periods, with a ratio near 2:1, may be present and excited in these stars.

In the present paper, we use long-term visual observations of several bright RVT and SRd variables to study the RVT phenomenon in further detail—especially the relationship between RVT and SRd variables. Visual observations, though less accurate than photoelectric observations, may have the advantages of being more numerous and more sustained, so they may provide a better overall view of the average behavior of the star.

2. Method of analysis—self-correlation

The visual inspection of light curves, and Fourier analysis (power spectra) are commonly used for analyzing variable star data. Self-correlation analysis—a form of variogram analysis (Eyer and Genton 1999)—has proven to be useful, in conjunction with the other two techniques, for some kinds of stars, especially if the stars are somewhat irregular, and if there are "aliases" in the power spectra due to regular gaps in the data. It can detect characteristic time scales, τ, in the data. It determines the cycle-to-cycle behavior of the star, averaged over all the data. The measurements do not have to be equally-spaced.

The algorithm works as follows (Percy, Ralli, and Sen 1993): for all pairs of measurements, the difference in magnitude (Δmag) and the difference in time (Δt) are calculated. Then Δmag is plotted against Δt, from zero up to some appropriate upper limit (which if possible should be a few times greater than the expected time scales, but less than the total time span of the data). The Δmags are binned in Δt so that, if possible, there are at least a few values in each bin; the Δmags in each bin are then averaged. The average Δmag is plotted against the average Δt in a "self-correlation diagram" (hereinafter SCD). The method is so simple that there is no equation involved—just the procedure described above.

The average Δmag will be a minimum at multiples of τ. Each minimum can be used to estimate τ. In the SCD of AG Aur in Figure l, for instance, there are ten minima. The Δt of the Nth minimum, divided by N, gives a measure of τ, so the periods derived from the several minima can be averaged to give a better estimate of the period. The refined period of AG Aur, determined in this way, is 96.70 days.

If the variability were perfectly periodic, and the magnitudes had no error, then the minima should be well-defined, and fall to zero, because measurements which were an integral number of cycles apart should always be exactly the same. In fact, the height of the minima above the zero line is determined by the average error of the magnitudes, and by the degree of irregularity, if any. In Figure 1, the average error, as determined from the first minimum, is about 2.5 "steps."

Measurements which are a half-integral number of cycles apart may have a Δmag ranging from zero to the full amplitude of the variations. As long as there are a sufficient number of Δmags in each bin, the *height* of maximum will average out to about half the total amplitude. The difference between the maxima and the minima is therefore a measure of the average amplitude of the variability, and is approximately 0.5 times the average full range of the light curve. The persistence of the minima to large Δt's is also determined by the degree of irregularity.

For reasons already mentioned, our method requires on the order of ten or more Δmags in each bin. Although our method is not subject to "alias" periods due to the periodicity of the seasonal gaps in the data, there may be gaps in the SCD if there are no pairs of measurements with certain values of Δt—due to regular seasonal gaps in the data, for instance.

The SCD (Figure 1 for instance) is constructed from measurements which are up to a few cycles apart, and is not the same as a light curve or a phase diagram. A phase diagram combines all measurements with a single period. Because RV Tauri stars are defined on the basis of their cycle-to-cycle behavior, our method is well suited for analyzing them. In Figure 3, note that the even-numbered minima are deeper than the odd-numbered ones. This means that minima which are an even number of cycles apart are more similar than minima which are an odd number of cycles apart. Self-correlation analysis can also investigate the correlation between minima which are more than two cycles apart, and can therefore provide information about whether the alternating minima continue to correlate over longer time intervals; see the discussions of individual stars, below.

3. Data

We have used long-term visual observations of bright RVT and SRd variables from several sources: the American Association of Variable Star Observers (AAVSO), whose data were kindly provided by the Director, Dr. Janet A. Mattei (2003); the Association Française des Observateurs d'Étoiles Variables (AFOEV), whose data are available on-line from the Centre de Données astronomiques de Strasbourg (CDS); and the Hungarian Astronomical Association (HAA), whose data were kindly provided by Dr. Laszlo L. Kiss. The method of analysis was as described by Percy, Hosick, and Leigh (2003), using a program which is freely available at: www.astro.utoronto.ca/~percy/index.html. See Table 1 for a list of the stars investigated.

4. Results

The results are best shown through the individual SCDs, which are discussed individually below; the classifications RVa, RVb, or SRd are taken from the *General Catalogue of Variable Stars* (GCVS 4.1; Kholopov *et al.* 1998), via the SIMBAD database:

4.1. AG Aur

The self-correlation amplitude has decreased significantly by Δt = 1000 days (about 10 cycles), indicating some irregularity, as the SRd classification would suggest (Figure 1).

4.2. AV Cyg

The self-correlation amplitude remains substantially constant, at least to Δt = 1000 days, suggesting a high degree of regularity, despite the SRd classification (Figure 2).

4.3. SU Gem

The minima in the SCD rapidly become weaker after the first two or three, and there is little tendency for the odd-numbered minima to be shallower. This suggests that the RVT behavior is weak. The long secondary period in this RVb star complicates the appearance of the SCD, which we have therefore not shown.

4.4. AC Her

In the SCD for this well-known RVT star, the even-numbered minima are deeper than the odd-numbered minima. This means that measurements which are an even number of cycles apart are more similar than measurements which are an odd number of cycles apart. This is what would be expected in RVT variables. (The term "cycle" refers to the separation between adjacent minima; it is sometimes called the "single" period when used in reference to RVT variables.) The SCD is more complex, however: the alternating-minima disappear at 200, 550, and 950 days, but reappear at 350 and 700 days (Figure 3). This may be explainable in terms of two periods whose ratio is not exactly 2:1, and which "beat" against each other every few cycles.

4.5. SX Her

The SCD changes very little out to Δt = 1500 days, though the amplitude decreases slightly, suggesting a fairly high degree of regularity—almost as high as in AV Cyg—despite the SRd classification.

4.6. TT Oph

The amplitude of the SCD remains quite constant out to $\Delta t = 500$ days, suggesting a high degree of regularity (Figure 4). Except in the first two cycles, however, there is little or no evidence of alternating deep and shallow minima, which would cause the odd-numbered minima in the self-correlation diagram to be shallower. The RVT nature of this star therefore seems to be minimal.

4.7. UZ Oph

The amplitude of the SCD remains quite constant out to $\Delta t = 500$ days, suggesting a high degree of regularity. There is no evidence, even in the first two cycles, of alternating deep and shallow minima, which would cause the odd-numbered minima in the SCD to be shallower than the even-numbered ones. The RVT nature of this star therefore seems to be marginal, or perhaps even non-existent, despite its RVa classification.

4.8. TX Per

The amplitude of the SCD has decreased almost to zero by $\Delta t = 1500$ days, indicating some degree of irregularity (Figure 5). Except in the first two cycles, there is no evidence of alternating deep and shallow minima, which would cause the odd-numbered minima in the SCD to be shallower than the even-numbered ones. The RVT nature of this star therefore seems to be marginal, or perhaps even non-existent, despite its RVa classification.

4.9. V Vul

In the SCD, the alternating deep and shallow minima disappear by $\Delta t = 1000$ days, and the amplitude decreases to almost zero (Figure 6). This indicates a fair degree of irregularity in this well-known RVT variable. Furthermore, the shallow secondary minima in the SCD have disappeared by $\Delta t = 1000$ days, having been dominated by the irregularity.

We used the Fourier/CLEAN program TS, available on the website of the AAVSO (www.aavso.org), to look for evidence of periods which were 0.5 or 2.0 times the primary period, in the SRd variables AG Aur, AV Cyg, and SX Her. In SX Her, there was a very weak signal at 0.5 times the primary period. This might indicate incipient RVT behavior, but there is no evidence for RVT behavior in the SCD.

5. Discussion

Unlike the results of Percy, Hosick, and Leigh (2003), the results of the present paper are based on the use of visual measurements of brightness. The lesser accuracy of these measurements is balanced by the larger numbers, and the relatively complete time coverage. They are completely adequate for the purpose at hand.

Several of the SRd variables have proven to be relatively regular, on the basis of their SCDs. The minima in the self-correlation diagram persist to large Δt. This enables us to refine the periods of these stars: 96.70 days for AG Aur; 88.04 days for AV Cyg; 61.24 days for TT Oph; and 76.33 days for TX Per. The regularity of these stars should also make it possible to investigate the period changes in these stars, using the (O–C) method. This, in turn, may shed some light on the evolutionary state of these stars. They are believed to be contracting from the asymptotic giant phase of evolution to the white dwarf phase.

In three RVT stars—TT Oph, UZ Oph, and TX Per—the SCD provides little or no evidence for the RV Tauri nature of these stars. Percy (1993) noted that many RVT stars show as little as 60 per cent incidence of the alternating deep and shallow minima which define this class. The RVT classification is therefore marginal, at best.

The RVT stars AC Her and V Vul show complex SCDs which may indicate the presence of two periods with a ratio close to, but not equal to 2:1. We plan to model the SCDs of these and other RVT variables, using the same method described by Percy, Hosick, and Leigh (2003).

These results support the general conclusion that the divisions between CW, RVT, and SRd variables are not distinct. There are CW variables with mild, almost imperceptable RVT characteristics. As seen in this paper, there are stars which have been classified as RVT which show little or no alternating deep and shallow minima in the self-correlation diagram, and there are SRd variables with strong periodicity.

6. Conclusions

We have successfully used self-correlation analysis to investigate and refine the periods and classifications of several bright RVT and SRd variables. Self-correlation is a useful tool for studying the time behavior of variable stars, especially groups such as these, whose classification is based on their cycle-to-cycle behavior. Visual observations from organizations such as the AAVSO are well-suited to this method of analysis.

7. Acknowledgements

We thank the AAVSO, the AFOEV, and the HAA and their observers, for making and providing the observations on which this study is based. Thanks to Laszlo L. Kiss and the late Janet A. Mattei for providing access to data. This research made use of the SIMBAD database, operated at CDS, Strasbourg, France. FM was a participant in the Ontario Work-Study Program at the University of Toronto. JRP expresses his deepest gratitude to the late Janet A. Mattei for encouraging and supporting numerous undergraduate research projects, like Farisa's, during her 30 years as Director of the AAVSO. We thank the Natural Sciences and Engineering Research Council of Canada for research support.

References

Eyer, L., and Genton M. G. 1999, *Astron. Astrophys. Suppl. Ser.*, **136**, 421.
Fokin, A. B. 1994, *Astron. Astrophys.*, **292**, 133.
Fokin, A. B. 2001, in *Stellar Pulsation—Non-Linear Studies*, ed. M. Takeuti and D. D. Sasselov, Dordrecht, Kluwer Academic Publishers, 103.
Ishida, T. 2003, in *Mass-Losing Stars and their Circumstellar Material*, ed. Y. Nakada, M. Honma, and M. Seki, Dordrecht, Kluwer Academic Publishers, 125.
Jura, M. 1986, *Astrophys. J.*, **309**, 732.
Kholopov, P. N., *et al.* 1998, *General Catalogue of Variable Stars*, 4.1, via the SIMBAD database, Centre de Données astronomiques de Strasbourg (CDS).
Malatesta, K. 2003, www.aavso.org/vstar/vsots/winter03.shtml.
Mattei, J. A. 2003, observations from the AAVSO International Database, private communication.
Percy, J. R. 1993, *Astrophys. Space Sci.*, **210**, 123.
Percy, J. R., Ralli, J. A., and Sen, L. V. 1993, *Publ. Astron. Soc. Pacific*, **105**, 287.
Percy, J. R., Hosick, J., and Leigh, N. W. C. 2003, *Publ. Astron. Soc. Pacific*, **115**, 59.
Takeuti, M., and Petersen, J. O. 1983, *Astron. Astrophys.*, **117**, 352.
Takeuti, M. 1998, in *A Half-Century of Stellar Pulsation Interpretations,* ed. P. A. Bradley and J. A. Guzik, San Francisco, ASP Conf. Series, **135**, 236.
Tuchman, Y., Lebre, A., Mennessier, M. O., and Yarri, A. 1993, *Astron. Astrophys.*, **271**, 501.

Table 1. Program Stars.

Name	*GCVS Class*	*P(days)*	*Range*	*Comments*
AG Aur	SRd	96	10.0–13.1	slightly semiregular
AV Cyg	SRd	89.22	11.0–13.7	periodic
SU Gem	RVb	50.0	9.8–14.1	semiregular
AC Her	RVa	75.01	6.9–9.0	complex
SX Her	SRd	102.9	8.6–10.9	slightly semiregular
TT Oph	RVa	61.08	9.5–10.8	not RVT?
UZ Oph	RVa	87.44	9.9–11.5	not RVT?
TX Per	RVa	78	9.8–12.5	not RVT?
V Vul	RVa	75.7	8.1–9.5	semiregular

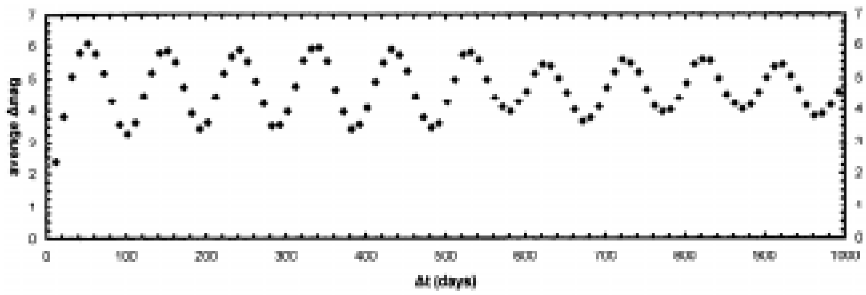

Figure 1. Self-correlation diagram for AG Aur. Note that the amplitude of the diagram decreases for larger Δt, which suggests that the variability is semiregular. This is consistent with the SRd classification of the star. The magnitude is measured in "steps."

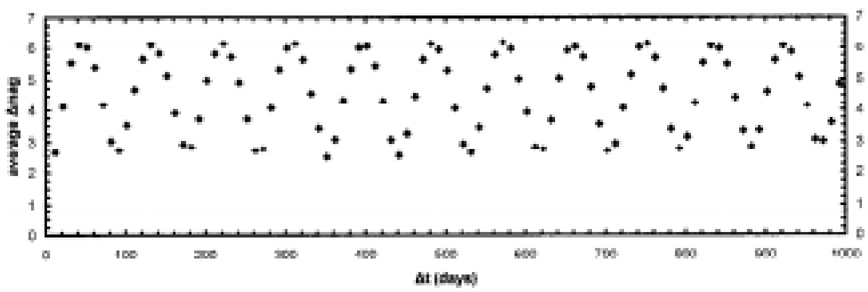

Figure 2. Self-correlation diagram for AV Cyg. Note that the amplitude of the diagram remains constant for larger Δt, which suggests that the variability is regular, despite the SRd classification of the star. The magnitude is measured in "steps."

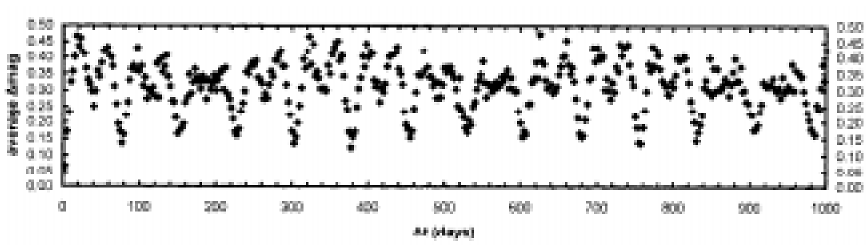

Figure 3. Self-correlation diagram for AC Her. Note the unusual behavior of the diagram; the correlation disappears at 200, 550, and 950 days, and reappears at 350 and 700 days. This might be due to the presence of two periods in a ratio which was close, but not equal to 2:1.

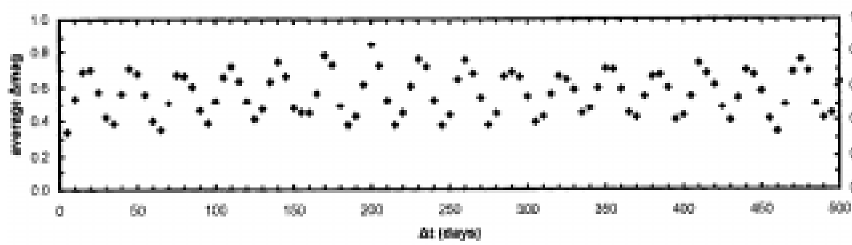

Figure 4. Self-correlation diagram for TT Oph. Note that there is little or no evidence for the odd-numbered minima to be shallower than the even-numbered minima, which suggests that the star may not be an RVT variable. The amplitude of the diagram remains approximately constant for larger Δt, which suggests that the variability is reasonably regular.

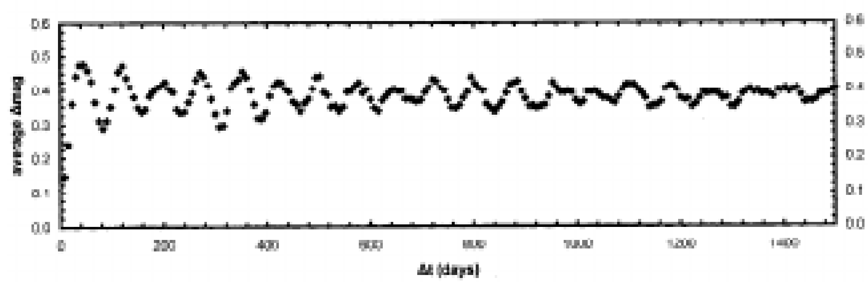

Figure 5. Self-correlation diagram for TX Per. Note that there is little or no evidence for the odd-numbered minima to be shallower than the even-numbered minima, which suggests that the star may not be an RVT variable. The amplitude of the diagram decreases for larger Δt, which suggests that the variability is semiregular.

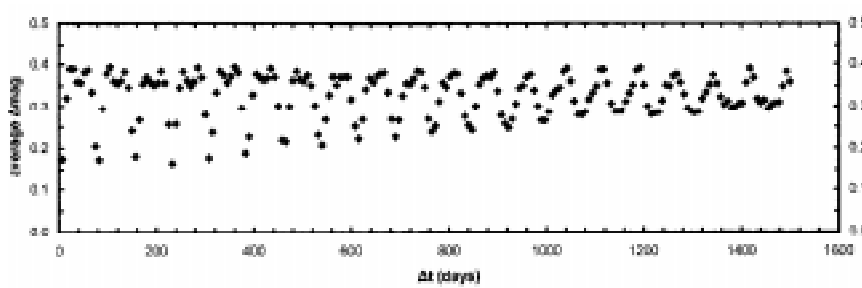

Figure 6. Self-correlation diagram for V Vul, for Δt up to 1500 days. Note that the amplitude of the diagram gradually decreases, which suggests that the variability is semi-regular. Note also that the alternating deep and shallow minima gradually disappear in the diagram.