THRESHOLD LIMITS FOR MODERATE APERTURE TELESCOPES

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Abstract

Observations have been made of limiting magnitudes indicating that for small to moderate apertures the expression $M_L = 2.8 \times \log D = 1.9 \times \log P + M_O$, where M_O is a constant dependent on the observer, is more useful in determining working limits than is the widely used expression of Dimitroff and Baker.

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I. INTRODUCTION

The limiting magnitude, M_L, of a telescope of aperture D inches is usually found from the Dimitroff and Baker (1945) expression

$$M_L = 8.8 + 5 \log D.$$

Conversion of D to millimeters yields the corresponding metric formula

$$M_L = 1.8 + 5 \log D.$$

Although this formula predicts that stars of magnitude six are the visible limit for the unaided eye, experienced observers can generally see stars substantially fainter than this limit. In fact, Russell (1917) showed that under optimum conditions, stars of magnitude 8.5 could just be seen with the unaided eye.

After extensive laboratory experiments using artificial stars, Langmuir and Westendorp (1931) reported that the minimum intensity required for a point source to be visible on a luminous background is proportional to the square-root of the background brightness. This indicates that the magnitude limit should be dependent on the magnification of the system since the sky brightness varies as the area of the exit pupil when the exit pupil is smaller than the pupil of the eye.

The Langmuir-Westendorp (1931) square-root law predicts point source threshold intensities I and I' for the unaided eye and telescope respectively:

$$I \propto B^{1/2} d^{-2} \quad (1a)$$

$$I' \propto B'^{1/2} D^{-2} T^{-1} \quad (1b)$$

where B and B' are the sky background brightnesses as seen by the unaided eye and the telescope respectively, d is the pupil diameter, D is the telescope aperture in millimeters, and T is the transmission factor of the telescope. Following the argument of Jenkins and White (1957), the background minimum of the eye and telescope are related by

$$B' = BP^{-2} (D/d)^2 T. \quad (2)$$

Thus

$$I'/I = dP^{-1} D^{-1} T^{-1/2}. \quad (3)$$

The magnitude is related to the illumination by

$$E_M = E_1 (1/2.5)^{M-1} \quad (4)$$

with $E_1 = 8.3 \times 10^{-7}$ lumen/m^2. E_1 is arbitrarily chosen as the brightness of a first magnitude star. The magnitude difference is related to the intensities by

$$\Delta M = -2.5 \log(I'/I). \qquad (5)$$

Using Equation (3), one obtains

$$\Delta M = 2.5 \log D + 2.5 \log P - 2.5 \log d + 1.25 \log T. \qquad (6)$$

The magnitude limit is some number, m_o, plus ΔM where m_o is dependent on non-telescopic factors such as spectral response of the observer and sky conditions.

Typical transmissions are in the range of 60% to 80%; although newer, highly efficient coatings described by Cox (1976) may improve these values. Such a variation will cause the last term in (6) to vary by about ± 0.1 magnitude.

The typical pupil diameter of a dark adapted observer ranges from 6 mm to 8 mm [Johnson (1960), Born and Wolf (1970)]. This variation causes the third term in (6) to vary by about ± 0.1 magnitude.

In this investigation the dependence of M_L is studied as a function of aperture, D, and magnification, P. In accordance with the above estimates, the last two terms in (6) may safely be grouped with m_o, bearing in mind that for an individual observer the variations of the contributions due to these two terms will be even smaller than those cited here. Equation (6) then is simplified to

$$M_L = M_o + 2.5 \log D + 2.5 \log P. \qquad (7)$$

where

$$M_o = m_o - 2.5 \log d + 1.25 \log T. \qquad (8)$$

Subsequent observations by Bowen (1947) tend to support this approach. Bowen's results parallel those of Baum (1955). He found that the magnitude limit should increase with aperture until a seeing disk is resolved. For large telescopes, the seeing determines the size of the stellar image. In small telescopes, where the resolving power is rather limited, the stellar diameter depends on the focal length--hence, the magnification and aperture terms combine to give the 5 log D dependence so often quoted.

Baum's interpretation of these results is that the 2.5 log D dependence should commence when the focal length becomes long enough for the image diameter to be determined by the seeing disk and not by the resolution of the instrument.

The investigation by the authors, however, indicates a departure from equation (8).

II. OBSERVATIONS

Observations of limiting magnitudes were originally obtained from eleven reliable and experienced amateur and professional astronomers using various apertures and magnifications of telescopes. The sky regions used were those variable star fields currently under investigation by the AAVSO.

All of the observers reported that their eyes were well dark-adapted and that their telescope objectives were highly reflective or transmissive. The faintest photometered star on a given chart that could be seen was chosen as the magnitude limit. This means that it would be possible that an observer could see slightly fainter stars than were reported.

The authors found, however, that more consistency and less scatter could be realized if isolated observations were not used, and if points resulting from the average of many observations were used. This limited the number of observers to two: Kolman and Locher, but did not significantly reduce the number of observations. A variety of instruments was used by these observers to determine the threshold limit. They included 4, 6, 8, 10, 12, 12.5, 16, and 18 inch reflecting telescopes; 2.4, 3, 6, and 12 inch refracting telescopes; a 3.5 inch Questar, 20 and 50 mm finder telescopes and 50 mm binoculars. The data are listed in Tables II and III.

The problem of atmospheric extinction of starlight was eliminated by observing only those fields which were near the zenith. The problem of variations in AAVSO charts undoubtedly contributed to the scatter of the data; nevertheless the variations are random, so there was little if any systematic error.

III. RESULTS

The experimental expression for the limiting magnitude was assumed to have the same form as equation (8) but with adjusted coefficients

$$M_L = a \log D + b \log P + c. \qquad (9)$$

For each set of values D and P, average observed limiting magnitudes were obtained independently for Kolman and Locher. Values for the parameters a, b, and c in Equation (9) are shown in Table I. We see that the results are quite consistent for the observers and lead to the expression

$$M_L = 2.8 \log D + 1.9 \log P + 3.7. \qquad (10)$$

In this case, D is given in millimeters.

Equation (10) bridges the gap between the extremely large and small apertures encountered by Baum, and as such could be of use to observers using moderate apertures in determining their threshold limits. It should be pointed out, however, that while M_O was consistent for the observers in this study, in general it must be determined individually by each observer.

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REFERENCES

Baum, W. A. 1955, Trans. Int. Ast. Union, 9. 681.
Born, M., and Wolf, E. 1970, Principles of Optics, (London: Pergamon Press), p. 234.
Bowen, I. S. 1947, P.A.S.P., 59, 253.
Cox, R. E. 1976, Sky and Telescope, 51, 283.
Dimitroff, G. Z., and Baker, J. C. 1945, Telescopes and Accessories, (Philadelphia: Blakiston Press), p. 44.
Jenkins, F. A. and White, H. E. 1957, Fundamentals of Optics, (New York: McGraw-Hill), Chapters 7 and 15.
Johnson, B. K. 1960, Optics and Optical Instruments, (New York: Dover) p. 38.
Langmuir, I. and Westendorp, W. F. 1931, Physics, 1, 273.
Russell, H. N. 1917, Ap. J., 45, 60.

TABLE I

EXPERIMENTAL THRESHOLD LIMIT EQUATION COEFFICIENTS

Observer	a	b	c
Kolman	2.6	2.1	3.7
Locher	3.0	1.8	3.6
AVERAGE	2.8	1.9	3.7

TABLE II - KOLMAN'S DATA

D	P	M_L	NUMBER	TYPE
7	1	7.5	10	Unaided
50	7	9.8	5	Refr.
50	85	12.2	1	Refr.
89	160	13.5	1	Catad.
150	160	14.3	10	Refl.
200	160	13.5	1	Refl.
254	200	14.6	5	Refl.
308	240	15.1	4	Refl.
308	310	15.3	6	Refr.
407	160	15.3	2	Refl.
407	320	15.5	4	Refl.

TABLE III - LOCHER'S DATA

D	P	M_L	NUMBER	TYPE
7	1	6.6	5	Unaided
20	4.5	8.4	7	Refr.
50	10	10.3	26	Refr.
90	68	12.7	2	Refr.
110	33	12.8	38	Refr.
110	66	13.2	21	Refr.
110	138	13.7	7	Refr.

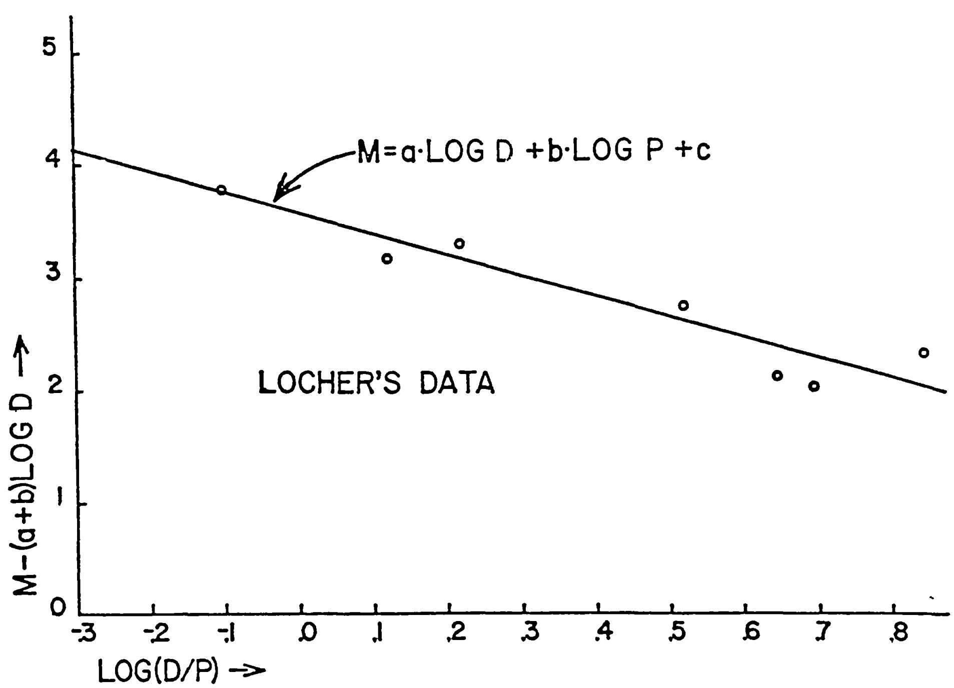

Figure 1. Limiting magnitude, M, as a function of exit pupil diameter, (D/P). Locher's data.

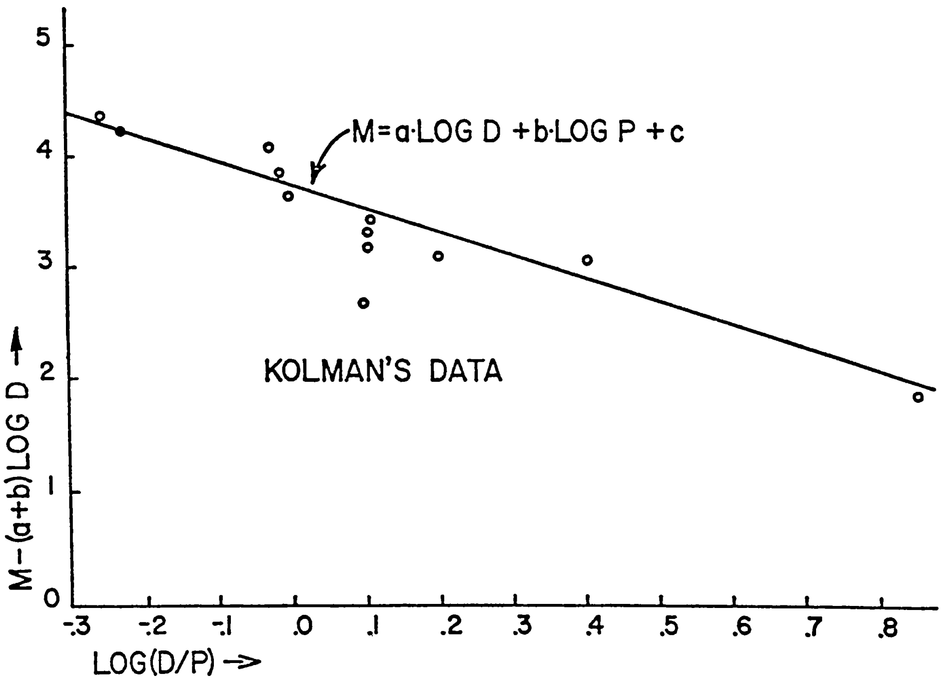

Figure 2. Limiting magnitude, M, as a function of exit pupil diameter, (D/P). Kolman's data.