Frequency Analysis of Long-term AAVSO Visual Observations of TU Cas

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Abstract Forty years of visual data in the AAVSO International Database for the double-mode Cepheid variable TU Cas were analyzed for possible changes in pulsation frequency and amplitude. The data were divided into four epochs for frequency analysis to search for any changes in the pulsation periods. The fundamental and first-overtone periods showed no significant variation during the forty-year span. The values determined for these frequencies agree well with other studies based on PMT and CCD data.

1. Introduction

TU Cas is a member of the "beat" or "double-mode" δ Cephei class of variable stars. These stars pulsate in two frequencies, usually the fundamental and first overtone (1O) radial modes, rather than the single frequency seen in normal Cepheid variables. Several studies of TU Cas' pulsation modes have been published, the most recent being Pardo and Poretti (1997), based on several decades of CCD and photomultipier tube (PMT) data. An analysis by Faulkner (1977) detected the presence of a second overtone frequency, although this has not been confirmed by later studies.

Studies of beat Cepheids have suggested that the fundamental and first overtone pulsations may change in frequency and/or amplitude ratios on time scales of decades. Hodson *et al.* (1979) proposed that the *V* amplitude of the first overtone had decreased by 40%, based on nearly seventy years of data. The AAVSO International Database contains almost 5,000 visual observations of TU Cas spanning sixty years. An analysis of this large data set was undertaken to search for any changes in the pulsation periods or amplitudes.

2. Analysis

The AAVSO visual data (AAVSO 2008) exhibited four distinct epochs of increased observation activity, separated by periods of relatively low activity. These four epochs were analyzed separately to search for long-term variations in amplitude ratio or pulsation frequencies. Table 1 shows the statistics for each of the four epochs. A single data point from JD 2432480 (October 1947) was omitted from data set 1 because it was more than 7,000 days from the nearest data point and introduced a number of aliases in the frequency analysis. After omitting this data point the complete set of data used for analysis covered 41.6 years and contained 4,948 data points.

The AAVSO database includes the AAVSO Observer Initials (unique observer identification) for each data point. The data in sets 1, 3, and 4 were normalized by adjusting the measurements for observers with large numbers of observations (generally more than eighty points) such that the average for the observer matched the average of the entire set. Generally this adjustment was on the order of 0.1 to 0.2 magnitude. Data set 1 was also adjusted by +0.20 magnitude to shift its average to match the other three data sets; this offset is most likely due to changes in comparison star magnitude values or due to the use of different comparison star sequences. Observer-specific adjustments were not applied to data set 2 because the bulk of the observations (over 90%) were contributed by a single observer. The adjusted data in sets 1, 3, and 4 and the unadjusted data from set 2 were also merged into a single forty-two-year collection (referred hereafter as the "combined" data set) and analyzed in the same manner as the individual data sets.

Frequency analysis of each data set was conducted using PERIOD04 (Lenz and Breger 2004). After zero-point-subtraction, Fourier analysis in the range of $0 < f < 2d$ was performed and the strongest signal was selected. PERIOD04's least squares fit was run using the selected frequency to find the best amplitude, phase, and frequency. The Fourier analysis and least squares fit was repeatedly performed using the residuals from the previous iteration, with the strongest signal found in each cycle being added to the list of fitted frequencies. The Fourier analysis and fitting cycle was terminated when the residuals value determined by PERIOD04 was no longer decreasing. The signal-to-noise ratio (SNR) was then determined using a box size of 5 for each frequency in each fitted set. Frequencies with SNR < 4 were eliminated from each data set. The residuals in each data set were on the order of 0.2 magnitude after fitting these significant frequencies. Sample power spectra for the combined data set are shown in Figures 1 through 3.

3. Results and Discussion

Tables 2 through 6 list all frequencies with SNR > 4 found in data sets 1 through 4, and the combined set, respectively. Each table lists the frequency, the calculated visual magnitude amplitude and the error estimates (in parentheses) from the least squares fit, along with the SNR calculated using PERIOD04's Calculate Noise module.

The fundamental radial mode (*f0*) was the most significant frequency in all data sets. The first overtone (*f1*) was also detected in all data sets, though in Data Sets 1 and 2 it was detected as a 1-day alias (1–*f1*). Most of the data after JD 2451667 (May 2000) used the standard AAVSO chart of the field dated "12/96." The AAVSO data before this date also do not include comparison star information, making it impossible to adjust the estimates based on the comparison sequence on the 12/1996 chart. Data prior to the chart issuance in December 1996 presumably used a variety of comparison star sequences, resulting in significantly noisier data that obscures the overtone and coupled frequencies.

The data in sets 3 and 4 were all collected after the standard chart was issued in 1996, and the analysis of these sets revealed additional frequencies above the SNR = 4 threshold. Significant frequencies were detected with amplitudes down to 0.04 magnitude in both data sets. This result validates the value of visual estimates, and is impressive, given the large number of observers, the uncalibrated nature of the data, and the fact that visual estimates were only reported to 0.1 magnitude precision.

The fundamental radial mode and first overtone were detected in all four individual data sets. The primary mode (*f0*) had values of 0.46744(1), 0.46748(3), 0.46745(1), and 0.46746(1) cd^{-1} in sets 1 through 4, respectively. The first overtone frequency, *f1*, had values of 0.65862(2), 0.65864(6), 0.65863(2), and 0.65865(2) cd^{-1} in data sets 1 through 4, respectively. These values all agree within 1σ and do not indicate any statistically significant variation between the epochs, in agreement with the findings of Pardo and Poretti (1997). The values are also consistent with the values of 0.467442 and 0.658635 reported in that paper.

The combined data set Fourier analysis yielded values of 0.467448(2) cd^{-1} and 0.658655(9) cd^{-1} for *f0* and *f1*, respectively. The value for *f1* is consistent with Pardo and Poretti (1997), but *f0* is only marginally consistent with the value of 0.467442 reported in that paper. No error estimate for the frequencies are given in the Pardo and Poretti paper so the significance of this discrepancy cannot be definitively evaluated. After prewhitening these two frequencies a weaker peak was detected for each of the two modes. These residuals had frequencies of 0.467482(6) and 0.65868(2) cd^{-1} for *f0* and *f1*, respectively; Figure 3 shows the residual *f0* signal. These signals indicate that the frequencies of the two pulsation modes may have changed during the forty-two-year span, possibly switching between the two frequencies detected for each mode. The low sampling rate in the data set makes it difficult to do a finer analysis since further subdividing of the individual data sets will result in larger error values, overwhelming any frequency changes.

The second overtone frequency that Faulkner (1977) detected at *f* = 0.79843 cd^{-1} is not detected in any of the data sets. Faulkner found an amplitude of 0.05 magnitude for this pulsation, somewhat greater than the 0.035-magnitude amplitude found for the weakest signal above the SNR = 4 cutoff in this study. This non-detection is in agreement with Matthews *et al.* (1992), who did not detect any additional overtone frequencies at the 0.004-magnitude level.

In addition to frequency changes, some studies have suggested that the relative amplitudes of the primary and first overtone pulsations have changed on decade time scales, with Hodson *et al.* (1979) proposing a 40% change over seventy years. The ratio of brightness amplitudes between *f*0 and *f*1 in data sets 1 though 4 have values of 2.5(5), 2.3(6), 2.7(2), and 2.4(2), respectively. The error estimates for these ratios are rather high, especially in data sets 1 and 2, due to the uncertainties in the amplitudes from the Fourier analysis. Assuming that the 40% change over seventy years detected by Hodson *et al.* is reasonably linear, we would expect to see roughly a 25% change in the AAVSO data set, but the amplitude ratios do not vary at a statistically significant level, and a 25% change over the period can be ruled out.

4. Conclusions

Frequency analysis of the forty-two years of visual data for TU Cas in the AAVSO International Database yielded good measurements of the star's primary and first overtone pulsation frequencies. The values are in good agreement with studies performed using PMT and CCD measurements with much higher precision, showing the value of the AAVSO visual data. The frequencies determined from the AAVSO data are in good agreement with previous studies, and also confirm that they have remained stable over at least the last four decades.

References

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Table 1. AAVSO data sets for TU Cas.

Data Set	*Date Range (JD)*	*Number of Observations*	*Average Observations per Day*	*Average Magnitude (unadjusted)*
1	2439340–2444217	1245	0.26	7.65
2	2444577–2446787	334	0.15	7.88
3	2448673–2452730	1462	0.36	7.84
4	2452752–2454519	1907	1.08	7.86
Combined	2439340–2454519	4948	0.33	n/a

Table 2. Frequency analysis results for data set 1.

Frequency ID	*Frequency (c/d)*	*Amplitude (V mag)*	*SNR*
f0	0.46744(1)	0.16(1)	11.7
1-day artifact	1.00143(2)	0.07(1)	5.3
2*f0*	0.93542(3)	0.06(1)	4.5
1–*f1*	0.34138(2)	0.06(1)	4.7
1-day artifact	1.00512(3)	0.06(1)	4.6

Table 3. Frequency analysis results for data set 2.

Frequency ID	*Frequency (c/d)*	*Amplitude (V mag)*	*SNR*
f0	0.46748(3)	0.27(2)	13.9
1–*f1*	0.34136(6)	0.12(2)	6.0
f0+*f1*	1.12607(9)	0.10(2)	4.9
2*f0*	0.93494(9)	0.09(2)	4.5

Table 4. Frequency analysis results for data set 3.

Frequency ID	*Frequency (c/d)*	*Amplitude (V mag)*	*SNR*
f0	0.46745(1)	0.286(7)	30.5
f1	0.65863(2)	0.105(7)	11.2
2*f0*	0.93487(2)	0.097(7)	10.4
f0+*f1*	1.12609(2)	0.069(7)	7.4
2*f0*+*f1*	1.59362(3)	0.061(7)	6.5
f0–*f1*	0.19125(4)	0.038(7)	4.0

Table 5. Frequency analysis results for data set 4.

Frequency ID	*Frequency (c/d)*	*Amplitude (V mag)*	*SNR*
f0	0.46746(1)	0.292(7)	33.2
f1	0.65865(2)	0.122(7)	13.8
2*f0*	0.93492(3)	0.103(7)	11.7
f0+*f1*	1.12612(4)	0.078(7)	8.9
2*f0*+*f1*	1.59354(5)	0.053(7)	6.0
1-day artifact	1.00191(5)	0.055(7)	6.2
3*f0*	1.40236(7)	0.040(7)	4.6
f0–*f1*	0.19106(7)	0.042(7)	4.7
1-day artifact	0.99761(6)	0.043(7)	4.9
f0 alias	1.46980(7)	0.039(7)	4.4

Table 6. Frequency analysis results for the combined data set.

Frequency ID	*Frequency (c/d)*	*Amplitude (V mag)*	*SNR*
f0	0.467448(2)	0.229(9)	40.5
f1	0.658655(2)	0.10 (5)	18.1
2*f0*	0.934890(2)	0.088(4)	15.6
f0 lobe	0.467482(6)	0.081(4)	14.3
f0+*f1*	1.126101(2)	0.064(4)	11.4
f1 lobe	0.65868 (2)	0.05 (5)	8.8
?	0.000170(3)	0.048(6)	8.6
2*f0*+*f1*	1.593534(3)	0.047(4)	8.3
1-day artifact	0.998283(5)	0.033(5)	5.9
1-day artifact	1.001848(5)	0.030(5)	5.2
3*f0*	1.402312(5)	0.030(4)	5.4
f0–*f1*	0.191200(5)	0.030(4)	5.2
1-day artifact	2.005046(5)	0.028(5)	4.9
1-year artifact?	0.001403(5)	0.028(5)	5.0
1-day artifact	0.994755(5)	0.027(4)	4.8
?	2.397052(6)	0.025(4)	4.5
4*f0*	2.525188(6)	0.024(4)	4.2
1-month artifact	0.032680(6)	0.022(4)	4.0
4*f0*	2.525188(6)	0.024(4)	4.2

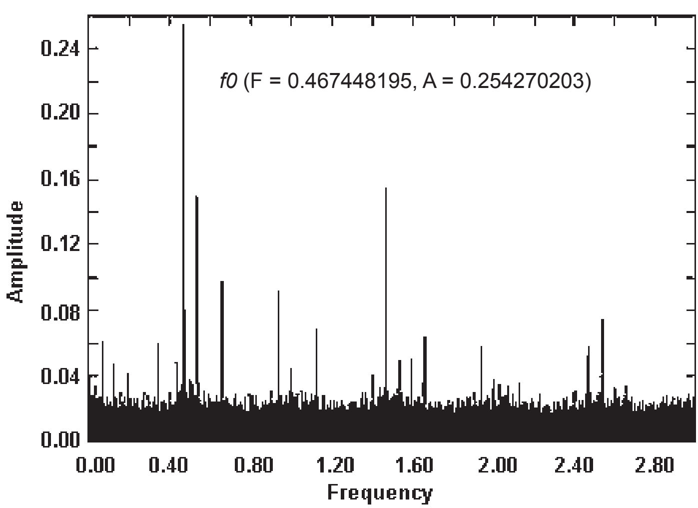

Figure 1. The initial Fourier spectrum for the combined data set, prior to prewhitening steps. Frequencies *f0*, *f1*, and their multiples and cross-terms are readily apparent, as well as several aliases and sampling artifacts.

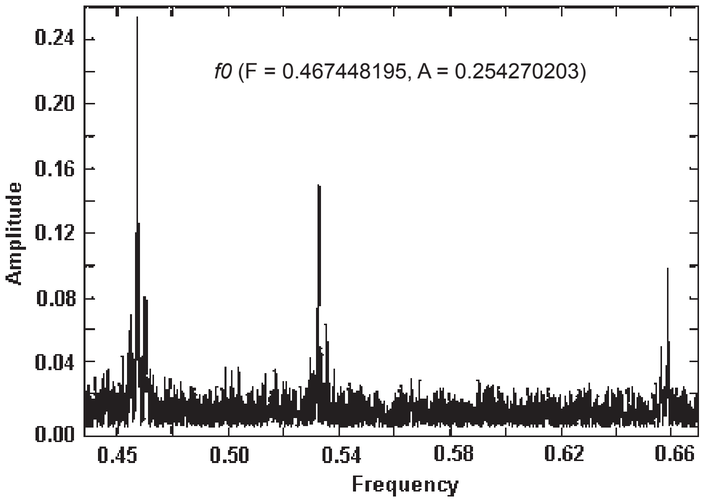

Figure 2. An enlarged view of the initial Fourier spectrum for the combined data set showing the region around *f0* and *f1*.

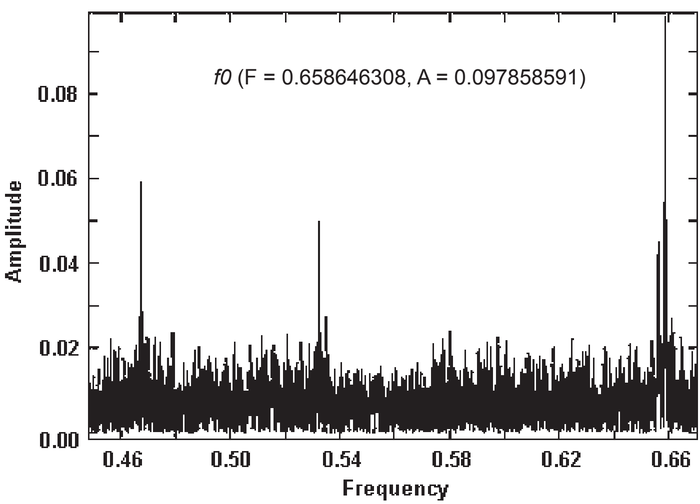

Figure 3. The same enlarged view as Figure 2, after prewhitening with the primary frequency found for *f0*. Note the residual signal from *f0* that remains, near 0.46 cd^{-1}.