Do Eclipsing Variable Stars Show Random Cycle-to-cycle Period Fluctuations?

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Abstract AAVSO observers and others have measured the times of minima of hundreds of eclipsing binaries over many decades. These times can be used to construct (O–C) diagrams that can be used to refine the periods of the stars, and to look for changes or fluctuations in the periods. We have applied the Eddington-Plakidis (1929) model to the (O–C) data on 100 stars in the AAVSO Eclipsing Binary Program, to determine whether the (O–C) diagrams can be explained by the cumulative effect of random, cycle-to-cycle fluctuations in period. The stars can be divided into three groups: 25–35% showing (O–C) fluctuations due only to measurement errors; 40–50% showing small, random cycle-to-cycle period fluctuations (typically a few times 10^{-4} of a cycle), and 20–30% showing (O–C) variations which do not fit the Eddington-Plakidis model and therefore cannot be explained by the accumulation of random fluctuations. We discuss possible explanations for these three groups.

1. Introduction

For several decades, AAVSO observers and others have systematically recorded the times of minima of eclipsing binaries. These observations have been used to plot (O–C) diagrams (Sterken 2005), which can be used to refine the periods and epochs of the stars, and to look for changes in the periods (e.g. Mallama 1987, who used AAVSO and other observations from amateurs to study period changes in several eclipsing variables). If the period is constant, but not equal to the assumed value, the (O–C) diagram is a straight line with non-zero slope. If an incorrect epoch has been assumed, the intercept on the (O–C) axis is not zero. If the period is changing, the (O–C) diagram is not linear. It is parabolic if the rate of period change is constant.

In many pulsating variables, however, some of the apparent period changes are due to the cumulative effect of random cycle-to-cycle period fluctuations. The formalism for studying these was developed by Eddington and Plakidis (1929), and this formalism, and others, have subsequently been applied to a wide variety of pulsating variables; see Turner *et al.* (2009) for a recent short review. The average fluctuation in period ε is found to be proportional to period

P: ε = 0.0136P (±0.0069 s.d.), i.e. ε / P = 0.0136. One possible cause of these fluctuations is the effect of large, random convection cells in the outer layers of the stars.

The signature of these fluctuations in the (O–C) diagram is a random wave-like pattern. Since such patterns are seen in the (O–C) diagrams of many eclipsing binaries (Nelson, undated), it seemed worthwhile to apply the Eddington and Plakidis (1929) algorithm to the data for some of the stars in the AAVSO database. In this way, using this new method of analysis, we can hope to extract some meaningful science from the observations that have been so diligently made.

2. Data and analysis

Times of minima of the stars listed in Tables 1 and 2 were taken from the AAVSO website:

http://www.aavso.org/observed-minima-timings-eclipsing-binaries (groups 1 and 2).

The timings were originally published, in eleven groups of fifty stars, in the *AAVSO Times of Minima of Eclipsing Binaries* series. It is not clear whether there was any bias in selecting the stars in each group, especially the first two groups. We note that group 2 seems to contain more stars with longer periods. In our sample, there is a good distribution of stars with type, range, and period.

Author Mohajerani wrote a computer program in JAVA to implement the algorithm of Eddington and Plakidis (1929). This program was used to generate values of $<u(x)>^2$ as a function of x, the number of elapsed cycles between times of minimum. If the star shows random cycle-to-cycle period fluctuations, the graph of $<u(x)>^2$ as a function of x should be a straight line with slope ε^2, where ε is the average value of the fluctuation, in days, and intercept $2\alpha^2$ where α is the average random observational error in determining the time of minimum, in days. Note that α is the accidental error of observation, and ε is the actual fluctuation in cycle length. Linear least-squares fits were performed on the graphs to determine the slopes and intercepts and their standard errors.

In some stars, the graph is curved upward, and the intercept is negative, which is inconsistent with the model since the intercept is proportional to the square of a real quantity. In this case, it is appropriate to use the value of $<u(1)>^2$ in place of the intercept, to determine α. If no value of $<u(1)>^2$ was available, $<u(2)>^2$ or $<u(3)>^2$ was used. Intercepts determined in this way are marked with an asterisk (*) in the tables.

The referee has correctly pointed out that, in the Eddington-Plakidis algorithm, "a correcting factor is required when x is a considerable fraction of the whole number of periods employed in determining the mean period" (Eddington and Plakidis 1929). In our case, we assume that the periods have been derived from a much larger and longer dataset than we are using. For a few stars, we repeated the analysis with a smaller maximum value of x. The intercept and slope changed somewhat (perhaps due to curvature in the diagram), but the changes were much smaller than the range of values of the intercept and slope in Table 1.

As we shall discuss in Section 4, it is important to note that the minimum-to-minimum intervals are not necessarily a measure of the orbital period of the binary system at that epoch. For instance, the minima may be distorted by spots or pulsations on the stars, or by light from circumstellar disks or streams.

3. Results

Tables 1 and 2 list the names of the stars, their type according to SIMBAD, the period in days, the approximate magnitudes of maximum and minimum, the slope and its error, the intercept, and the values of α, ε, and ε / P. The types are those used in the *General Catalogue of Variable Stars* (GCVS, Kholopov, *et al.* 1985), and are based on the light curve shape (EA, EB, EW), the state of detachment of the components (DM, SD), the ellipsoidal shape of the components (KW), or the degree of activity (RS: RS CVn). The value of α is the average error in measuring the time of minimum. To assume that this is a fixed value is rather simplistic, since the minima are measured in various ways, by various observers (though the AAVSO Eclipsing Binary program uses a standard method for determining the times of minima). If the slope is negative, then the value of ε is undefined (N/A in Table 1).

Figures 1 and 2 show $<u(x)>^2$ diagrams for two representative stars: ZZ Cep, whose diagram can reasonably be fitted with a straight line, and U Cep, whose diagram cannot.

Most of the $<u(x)>^2$ diagrams can be fitted with a straight line, with a (positive) intercept which is consistent with the expected observational error. Specifically: a histogram of the intercepts shows that slightly more than half are clustered between 2 and 9×10^{-5} with a mean value of about 4×10^{-5}. This corresponds to a value of α of a few times 0.001, which is reasonable: the times of minima are given to the nearest thousandth of a day and, where there are multiple measurements of the same minimum, they tend to differ by a few times 0.001. This value is also consistent with the estimate by Batten (1973), namely, "of order of 10^{-3} to 10^{-4} in the most favorable cases."

Some diagrams cannot be fitted with a straight line; the curve is initially flat, then curves upward (Figure 2). This results in an intercept that is negative, which is unphysical because it is proportional to α^2, where α is a real quantity. Predictably, the (positive) magnitude of the slope is then correlated with the (negative) magnitude of the intercept.

A histogram of the slopes (Figure 3) shows that about a third are clustered between -5 and $+5 \times 10^{-9}$. Although their formal errors generally exclude a slope of zero, we are inclined to assume that these stars have little or no random fluctuations. The rest of the stars have slopes that exceed 10×10^{-9}, and are likely significant. Almost half fit the Eddington-Plakidis model, with a small but significant slope. About a quarter have significant slopes but do not fit the Eddington-Plakidis model. The distributions of both the intercepts and the slopes were similar for all three types of binaries—EA, EB, and EW.

We looked for correlations between various quantities in Tables 1 and 2. As mentioned, stars with large slopes had large negative intercepts, but this is an artifact of fitting a linear function to non-linear data. The stars with the largest slopes—greater than $\times 10^{-6}$—were: β Lyr (12.94), SW Cyg (4.57), TY Peg (3.09), TX UMa (3.06), XZ Aql (2.14), U CrB (3.45), ST Per (2.65), and U Cep (2.50); the numbers in brackets are the periods in days. Large slopes are associated with longer periods.

Some stars with large ranges (2.5 or greater) and/or faint minima had large slopes and small intercepts, though they tended to be stars that did not fit the Eddington-Plakidis model.

The most interesting correlation was that the stars with the largest slopes tended to be the stars with longest periods. The most extreme case was β Lyr with a period of 12.94 days, and a slope of 1.6×10^{-4}. Other examples are listed above.

The Eddington-Plakidis algorithm assumes that there are fixed values of α and ε. For some stars, the average error of measurement of the times of minima decreases with time, especially as CCD techniques have begun to replace visual techniques. (Almost all of the minima that we analyzed were measured visually.) An inspection of the (O–C) diagrams shows no obvious relation between the properties of the $<u(x)>^2$ diagram and any systematic decreases in the scatter in the corresponding (O–C) diagram (Nelson, undated).

The values of ε / P tend to cluster into two groups: $3–16 \times 10^{-5}$ and $25–45 \times 10^{-5}$. These values are much smaller than in the pulsating variables discussed by Turner *et al.* 2009.

4. Discussion

The results of our study raise several questions: (1) What is the nature and cause of the cycle-to-cycle period fluctuations which are present in some of the stars? (2) Why do many of the stars show no fluctuations, with zero slope in the $<u(x)>^2$ diagram? (3) Why do some stars have $<u(x)>^2$ diagrams which do not fit the Eddington-Plakidis model, but curve upward as in Figure 2?

One way to approach question (3) is to look at the nature of the most extreme cases, those mentioned above. For instance: β Lyr, the most extreme case, is known to have an (O–C) diagram that is highly parabolic, indicative of a large, constant rate of period change (e.g. Wilson and Van Hamme 1999). The same is true of U Cep (e.g. Mansoori 2008). The two stars with the most prominently curved $<u(x)>^2$ diagrams, AB Cas and RT Per, also have (O–C) diagrams with distinct curvature. SW Cyg, TY Peg, TX UMa, and possibly ST Per have (O–C) diagrams which appear as a broken straight line, indicative of an abrupt period change between two intervals of constant period (Nelson, undated). According to Batten (1973), this is the most common form of period change in eclipsing variables: abrupt changes which are well separated in time. XZ Aql has an (O–C) diagram which appears parabolic, with a possible 36.7-year periodic component. The (O–C) diagram of U CrB appears straight, but van Gent (1982) suggested that there was (one) 160-year cycle present. Such periodic or possibly periodic components would be explainable by the presence of a third star in the system, with a very long orbital period. Many binary systems are actually triple (or more). Apsidal motion, with a time scale of tens of years, is another possible cause (Batten 1973). The (O–C) diagram of Algol (β Per), for instance, shows cycles of 1.8 years due to a third star, 33 years due to apsidal motion, and a time scale of 175 years ("the great inequality") of unknown cause. The (O–C) diagrams that are either parabolic or broken straight lines are assumed to be due to either continuous or episodic mass transfer or loss in the system. We conclude that the stars in our sample whose $<u(x)>^2$ diagrams do not fit the Eddington-Plakidis model are those which, because of the system configuration, undergo continuous or episodic mass transfer or loss.

As for the random cycle-to-cycle period fluctuations seen in many stars, these may be due to random distortions in the light curve minima. Kwee (1958) studied sixteen eclipsing variables, and concluded that there were "small and rapid fluctuations...near the limit of what can be shown by present observations." He drew this conclusion because the residuals in the (O–C) diagram were two to three times larger than the computed mean errors of the observations—typically 0.001 to 0.003 day. He and van Woerden (1957) both attributed such fluctuations to distortions in the light curves.

These distortions of the light curve, during minima, could be due to spots on the eclipsed star, if its rotation period was less than the binary period. This might explain why fluctuations are greater in binaries with longer orbital periods. The distortions could also be due to small oscillations of one of the stars, or due to disks or streams of gas. It is unlikely that they are due to actual changes in the orbital period of the system, caused by mass loss or transfer, because fluctuations of the order of $\times 10^{-4}$ would require much larger mass loss or transfer than is reasonable.

When the seven stars with largest slopes are compared with a sample of stars with slopes close to zero, it turns out that all of the large-slope stars have primaries with spectral type A5 or hotter, whereas the small-slope stars have primaries which are cooler. There is no obvious physical difference between the stars that show small but significant random cycle-to-cycle period fluctuations, and those that do not. Both groups include all types of eclipsing binaries, and a mixture of A, F, and G spectral types.

5. Conclusions

We have analyzed AAVSO timings of the minima of 100 eclipsing variable stars, to look for evidence of random, cycle-to-cycle period fluctuations which may explain some of the features of the (O–C) diagrams of these stars. About 25–35% of our sample show little or no evidence of these; 40–50% show evidence of small but significant fluctuations, and 20–30% show evidence that the main features of their (O–C) diagrams are not due to such fluctuations. We describe some processes which might possibly explain the fluctuations, though the precise cause of the fluctuations in any one star has yet to be determined.

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Table 1. Analysis of Stars on the AAVSO Eclipsing Binary Program, Group 1.

Name	*Type*	*Period (d)*	*Max*	*Min*	*Range*	*Slope*	*Error*	*Intercept*	*Error*	α	ε	*ε/Period*
RT AND	EA/RS	6.29E–01	9	9.8	0.8 (V)	1.40E–09	5.77E–10	4.16E–05	2.49E–06	4.56E–03	3.74E–05	5.94E–05
WZ AND	EB	6.96E–01	11.2	12	0.8 (V)	7.75E–10	2.75E–09	1.19E–04	1.04E–05	7.71E–03	2.78E–05	4.00E–05
XZ AND	EA	1.36E+00	9.9	12.5	2.6 (V)	4.24E–08	2.10E–09	–4.87E–06	4.34E–06	7.07E–04*	2.06E–04	1.52E–04
AB AND	EW	3.32E–01	9.5	10.5	1 (V)	1.18E–09	2.52E–10	4.81E–05	2.04E–06	4.90E–03	3.43E–05	1.03E–04
CX AQR	EB	5.56E–01	10.6	11.8	1.2 (V)	–6.39E–10	2.26E–10	2.06E–05	1.14E–06	3.21E–03	N/A	N/A
OO AQL	EW/DW	5.07E–01	9.2	9.9	0.7 (V)	1.45E–10	4.37E–10	7.12E–05	2.56E–06	5.97E–03	1.21E–05	2.38E–05
V346 AQL	EA/SD	1.11E+00	9	10.1	1.1 (p)	–1.45E–09	9.57E–10	3.24E–05	2.49E–06	4.02E–03	N/A	N/A
WW AUR	EA/DM	2.53E+00	5.8	6.5	0.7 (V)	1.11E–08	1.16E–08	1.39E–04	1.23E–05	8.33E–03	1.05E–04	4.18E–05
SV CAM	EA/DW/RS	5.93E–01	8.4	9.1	0.7 (V)	7.43E–08	1.48E–09	–8.13E–05	6.79E–06	2.59E–03*	2.73E–04	4.59E–04
RZ CAS	EA/SD	1.20E+00	6.2	7.7	1.5 (V)	1.27E–08	8.74E–10	2.76E–05	2.20E–06	3.71E–03	1.13E–04	9.42E–05
TV CAS	EA/SD	1.81E+00	7.2	8.2	1 (V)	1.94E–08	1.49E–08	1.95E–04	1.89E–05	9.87E–03	1.39E–04	7.69E–05
AB CAS	EA	1.37E+00	10.1	11.9	1.8 (V)	1.07E–07	2.40E–09	–6.63E–05	5.12E–06	3.54E–03*	3.28E–04	2.40E–04
IR CAS	EB	6.81E–01	10.8	12.3	1.5 (p)	6.83E–08	2.34E–09	–2.40E–05	8.83E–06	4.55E–03*	2.61E–04	3.84E–04
IV CAS	EA	9.99E–01	11.2	12.4	1.2 (p)	–3.28E–09	2.13E–09	8.09E–05	5.96E–06	6.36E–03	N/A	N/A
U CEP	EA/SD	2.49E+00	6.8	9.2	2.4 (V)	1.26E–06	1.54E–08	–3.78E–04	1.79E–05	3.00E–03*	1.12E–03	4.49E–04
ZZ CEP	EA/DM	2.14E+00	8.6	9.6	1 (V)	4.62E–09	1.88E–08	2.52E–04	2.41E–05	1.12E–02	6.79E–05	3.17E–05
EG CEP	EB	5.45E–01	9.3	10.2	0. 9 (V)	–2.96E–09	6.90E–10	8.54E–05	3.30E–06	6.54E–03	N/A	N/A
RW COM	EW/KW	2.37E–01	11	11.7	0.7 (V)	9.31E–09	6.03E–10	1.78E–05	7.34E–06	2.99E–03	9.65E–05	4.06E–04
W CRV	EB/KW	3.88E–01	11.2	12.5	1.3 (V)	2.09E–09	5.26E–10	3.56E–05	3.38E–06	4.22E–03	4.57E–05	1.18E–04
ZZ CYG	EA/SD	6.29E–01	10.6	11.7	1.1 (V)	2.85E–08	8.83E–10	–1.19E–05	3.64E–06	1.53E–03*	1.69E–04	2.69E–04
BR CYG	EA	1.33E+00	9.4	10.6	1.2 (V)	2.26E–09	1.63E–09	3.69E–05	3.30E–06	4.30E–03	4.76E–05	3.57E–05

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Table 1. Analysis of Stars on the AAVSO Eclipsing Binary Program, Group 1, cont.

Name	*Type*	*Period (d)*	*Max*	*Min*	*Range*	*Slope*	*Error*	*Intercept*	*Error*	α	ε	*ε/Period*
CG CYG	EA/SD/RS	6.31E–01	9.7	10.9	1.2 (V)	6.11E–08	1.95E–09	–5.42E–05	7.60E–06	5.30E–03*	2.47E–04	3.92E–04
V387 CYG	EA/K	6.41E–01	11.5	12.3	0.8 (V)	7.85E–10	4.14E–10	2.31E–05	1.62E–06	3.40E–03	2.80E–05	4.38E–05
TY DEL	EA/SD	1.19E+00	9.7	10.9	1.2 (V)	1.77E–07	5.90E–09	–4.96E–05	1.24E–05	2.36E–03*	4.21E–04	3.53E–04
YY DEL	EA	7.93E–01	11.3	12	0.7 (p)	–2.09E–09	1.71E–09	6.49E–05	4.67E–06	5.69E–03	N/A	N/A
FZ DEL	EA/SD	7.83E–01	10.2	11.3	1.1 (p)	6.72E–08	1.84E–09	–5.40E–05	6.14E–06	0.00E+00*	2.59E–04	3.31E–04
Z DRA	EA/SD	1.36E+00	10.8	14.1	3.3 (p)	9.29E–07	2.69E–08	–5.06E–04	5.18E–05	1.41E–03*	9.64E–04	7.10E–04
RZ DRA	EB/SD	5.51E–01	10.1	11	0.9 (V)	6.53E–08	2.28E–09	–5.00E–05	1.12E–05	6.13E–03*	2.56E–04	4.64E–04
AI DRA	EA/SD	1.20E+00	7.1	8.1	1 (V)	1.17E–08	3.79E–09	7.96E–05	7.94E–06	6.31E–03	1.08E–04	9.03E–05
YY ERI	EW/KW	3.21E–01	8.1	8.8	0.7 (V)	1.08E–07	1.76E–09	–1.99E–04	1.58E–05	7.07E–04*	3.29E–04	1.02E–03
SZ HER	EA/SD	8.18E–01	9.9	11.9	2 (V)	–5.54E–08	2.62E–08	5.64E–04	9.09E–05	1.68E–02	N/A	N/A
AV HYA	EB/KE	6.83E–01	10.2	10.8	0.6 (V)	7.96E–08	6.21E–09	4.99E–05	2.32E–05	5.00E–03	2.82E–04	4.13E–04
SW LAC	EW/KW	3.21E–01	8.5	9.4	0.9 (V)	2.08E–08	7.84E–10	8.92E–06	6.71E–06	2.11E–03	1.44E–04	4.49E–04
VX LAC	EA/SD	1.07E+00	10.9	13	2.1 (p)	1.47E–08	8.13E–10	4.93E–08	1.94E–06	1.57E–04	1.21E–04	1.13E–04
Y LEO	EA/SD	1.69E+00	10.1	13.2	3.1 (V)	4.28E–09	1.49E–09	1.77E–05	2.38E–06	2.97E–03	6.54E–05	3.88E–05
EW LYR	EA/SD	1.95E+00	11.2	13.5	2.3 (V)	1.88E–08	7.27E–09	6.35E–05	1.01E–05	5.63E–03	1.37E–04	7.04E–05
ER ORI	EW/KW	4.23E–01	9.3	10	0.7 (V)	2.17E–09	7.04E–10	9.04E–05	4.72E–06	6.72E–03	4.66E–05	1.10E–04
U PEG	EW/KW	3.75E–01	9.2	10.1	0.9 (V)	5.83E–08	2.09E–09	–3.78E–05	1.63E–05	4.95E–03*	2.41E–04	6.44E–04
RT PER	EA/SD	8.49E–01	10.5	11.7	1.2 (V)	1.25E–07	2.31E–09	–9.59E–05	7.65E–06	3.36E–03*	3.54E–04	4.17E–04
XZ PER	EA/SD	1.15E+00	11.4	13.4	2 (p)	3.80E–08	3.94E–09	7.09E–05	9.32E–06	5.95E–03	1.95E–04	1.69E–04
RW TAU	EA/SD	2.77E+00	8	11.6	3.6 (V)	8.00E–07	4.64E–08	–1.47E–04	4.41E–05	0.00E+00*	8.94E–04	3.23E–04
EQ TAU	EW/KW	3.41E–01	10.5	11	0.5 (V)	1.51E–08	9.70E–10	2.32E–05	7.99E–06	3.41E–03	1.23E–04	3.59E–04

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Table 1. Analysis of Stars on the AAVSO Eclipsing Binary Program, Group 1, cont.

Name	*Type*	*Period (d)*	*Max*	*Min*	*Range*	*Slope*	*Error*	*Intercept*	*Error*	α	ε	*ε/Period*
V TRI	EB/SD	5.85E–01	10.7	11.8	1.1 (V)	4.74E–09	6.03E–10	2.26E–05	2.53E–06	3.36E–03	6.89E–05	1.18E–04
X TRI	EA/SD	9.72E–01	8.6	11.3	2.7 (V)	2.49E–08	1.10E–09	5.76E–06	3.00E–06	1.70E–03	1.58E–04	1.62E–04
RV TRI	EA/SD	7.54E–01	11.5	13.3	1.8 (p)	7.73E–08	2.21E–09	–5.21E–05	7.41E–06	2.47E–03*	2.78E–04	3.69E–04
W UMA	EW/KW	3.34E–01	7.8	8.5	0.7 (V)	–5.28E–10	6.07E–10	7.94E–05	4.74E–06	6.30E–03	N/A	N/A
VV UMA	EA/SD	6.87E–01	10.1	10.9	0.8 (V)	–2.08E–09	1.82E–09	7.68E–05	6.45E–06	6.20E–03	N/A	N/A
XZ UMA	EA/SD	1.22E+00	10.1	11.7	1.6 (p)	1.42E–07	5.97E–09	–8.44E–05	1.26E–05	2.83E–03*	3.77E–04	3.08E–04
RU UMI	EB/DW	5.25E–01	10	10.7	0.7 (V)	6.78E–09	1.21E–09	4.50E–05	5.15E–06	4.74E–03	8.24E–05	1.57E–04
BU VUL	EA/SD	5.69E–01	10.6	11.4	0.8 (p)	9.62E–10	2.61E–10	2.40E–05	1.23E–06	3.47E–03	3.10E–05	5.45E–05

Note: Any alpha (α) value with an asterisk (*) is calculated from the $< u(x) >^2$ value with the smallest x.

Table 2. Analysis of Stars on the AAVSO Eclipsing Binary Program, Group 2.

Name	*Type*	*Period (d)*	*Max*	*Min*	*Range*	*Slope*	*Error*	*Intercept*	*Error*	α	ε	*ε/Period*
RT AND	EA/RS	6.29E–01	9	9.8	0.8 (V)	1.40E–09	5.77E–10	4.16E–05	2.49E–06	4.56E–03	3.74E–05	5.94E–05
BX AND	EB	6.10E–01	8.87	9.53	0.66 (V)	8.02E–09	3.39E–09	9.35E–05	1.25E–05	6.84E–03	8.96E–05	1.47E–04
XZ AQL	EA/SD	2.14E+00	10.1	11.4	1.3 (p)	2.13E–06	7.68E–08	–7.71E–04	1.06E–04	6.36E–03*	1.46E–03	6.83E–04
V343 AQL	EA/SD	1.84E+00	10.6	12.3	1.7 (p)	2.99E–08	9.44E–09	6.31E–05	1.51E–05	5.62E–03	1.73E–04	9.37E–05
EP AUR	EB	5.91E–01	11.2	11.8	0.66 (V)	–1.18E–08	6.33E–09	3.16E–04	3.13E–05	1.26E–02	N/A	N/A
AL CAM	EA/SD	1.33E+00	10.5	11.3	0.8 (p)	9.07E–08	5.36E–09	–2.98E–05	1.14E–05	4.95E–03*	3.01E–04	2.27E–04
R CMA	EA/SD	1.14E+00	5.7	6.34	0.64 (V)	4.05E–07	2.32E–08	–8.22E–05	5.72E–05	5.66E–03*	6.36E–04	5.60E–04
XZ CMI	EA	5.79E–01	9.7	10.4	0.72 (V)	1.22E–08	3.86E–09	1.08E–04	1.91E–05	7.34E–03	1.10E–04	1.91E–04
RZ CAS	EA/SD	1.20E+00	6.18	7.72	1.54 (V)	1.59E–09	1.11E–08	3.32E–05	4.77E–06	4.07E–03	3.98E–05	3.33E–05
MM CAS	EA/SD	1.16E+00	12	13.1	1.1 (p)	–5.08E–09	8.46E–09	1.81E–04	1.94E–05	9.52E–03	N/A	N/A
OR CAS	EA/SD	1.25E+00	11.4	12.4	1 (p)	2.99E–08	4.46E–09	5.54E–05	1.01E–05	5.27E–03	1.73E–04	1.39E–04
XX CEP	EA/SD	2.34E+00	9.2	10.3	1.12 (V)	–1.26E–08	8.02E–09	1.12E–04	9.69E–06	7.49E–03	N/A	N/A
DV CEP	E	1.16E+00	11.4	12.2	0.8 (p)	1.05E–08	1.82E–09	2.79E–05	3.76E–06	3.74E–03	1.02E–04	8.80E–05
RZ COM	EW/KW	3.39E–01	10.4	11.1	0.71 (V)	2.18E–08	2.29E–09	2.99E–05	1.87E–05	3.86E–03	1.48E–04	4.37E–04
U CRB	EA/SD	3.45E+00	7.66	8.79	1.13 (V)	1.65E–06	9.95E–08	–2.29E–04	9.42E–05	9.55E–03*	1.28E–03	3.72E–04
SW CYG	EA/SD	4.57E+00	9.24	11.8	2.59 (V)	8.13E–06	4.99E–07	–1.37E–03	3.08E–04	3.96E–03*	2.85E–03	6.24E–04
WW CYG	EA/SD	3.32E+00	10	13.3	3.24 (V)	1.21E–07	1.39E–08	4.91E–06	9.84E–06	1.57E–03	3.48E–04	1.05E–04
DK CYG	EW/D	4.71E–01	10.4	10.9	0.56 (V)	1.99E–07	9.35E–09	–2.54E–04	5.81E–05	2.83E–03*	4.46E–04	9.47E–04
KR CYG	EB	8.45E–01	9.19	10	0.81 (V)	–1.11E–08	3.79E–09	5.63E–05	5.24E–06	5.31E–03	N/A	N/A
V477 CYG	EA/DM	2.35E+00	8.5	9.34	0.84 (V)	1.87E–08	6.51E–09	6.05E–05	8.45E–06	5.50E–03	1.37E–04	5.82E–05

table continued on following pages

Table 2. Analysis of Stars on the AAVSO Eclipsing Binary Program, Group 2, cont.

Name	*Type*	*Period (d)*	*Max*	*Min*	*Range*	*Slope*	*Error*	*Intercept*	*Error*	α	ε	*ε/Period*
V704 CYG	EW	5.71E–01	13.6	14.6	1 (p)	5.50E–08	4.80E–09	1.44E–06	2.46E–05	8.49E–04	2.35E–04	4.11E–04
BH DRA	EA/SD	1.82E+00	8.38	9.27	0.89 (V)	5.50E–09	1.23E–08	1.66E–04	1.61E–05	9.11E–03	7.42E–05	4.08E–05
TU HER	EA/SD	2.27E+00	10.9	13.7	2.82 (V)	7.42E–07	4.57E–08	–3.09E–04	5.38E–05	1.59E–03*	8.61E–04	3.80E–04
CT HER	EA/SD	1.79E+00	10.6	11.7	1.1 (p)	3.48E–08	9.95E–09	8.22E–05	1.55E–05	6.41E–03	1.87E–04	1.04E–04
CM LAC	EA/DM	1.60E+00	8.18	9.15	0.97 (V)	4.54E–10	1.71E–09	3.62E–05	3.20E–06	4.25E–03	2.13E–05	1.33E–05
UV LEO	EA/DW	6.00E–01	8.9	9.56	0.66 (V)	1.64E–08	2.41E–09	2.28E–05	1.07E–05	3.38E–03	1.28E–04	2.13E–04
RY LYN	EA/SD	1.43E+00	11.4	13.3	1.9 (p)	1.40E–07	1.07E–08	–1.54E–05	2.10E–05	7.78E–03*	3.74E–04	2.61E–04
FL LYR	EA/DM	2.18E+00	9.27	9.89	0.62 (V)	2.39E–08	8.18E–09	6.41E–05	1.12E–05	5.66E–03	1.54E–04	7.09E–05
BETA LYR	EB	1.29E+01	3.25	4.36	1.11 (V)	1.60E–04	3.96E–05	4.76E–02	7.48E–03	1.54E–01	1.26E–02	9.77E–04
RW MON	EA/SD	1.91E+00	9.26	11.5	2.25 (V)	4.16E–08	5.52E–09	2.26E–05	8.18E–06	3.36E–03	2.04E–04	1.07E–04
U OPH	EA/DM	1.68E+00	5.84	6.56	0.72 (V)	4.76E–08	1.53E–08	1.49E–04	3.02E–05	8.64E–03	2.18E–04	1.30E–04
EQ ORI	EA/SD	1.75E+00	10.2	13.3	3.1 (p)	2.23E–07	1.28E–08	–2.66E–05	1.62E–05	2.47E–03*	4.72E–04	2.70E–04
FL ORI	EA/SD	1.55E+00	11.4	14.6	3.2 (p)	1.92E–08	1.16E–08	1.60E–04	2.34E–05	8.95E–03	1.39E–04	8.94E–05
TY PEG	EA/SD	3.09E+00	10.1	12	1.9 (V)	4.76E–06	3.16E–07	–1.35E–03	2.79E–04	3.77E–03*	2.18E–03	7.05E–04
BB PEG	EW/KW	3.62E–01	10.8	11.5	0.68 (p)	6.29E–09	2.08E–09	1.02E–04	1.63E–05	7.14E–03	7.93E–05	2.19E–04
BX PEG	EW/KW	2.80E–01	11	11.7	0.69 (V)	1.24E–08	1.14E–09	–1.08E–05	1.11E–05	7.07E–04*	1.11E–04	3.97E–04
DI PEG	EA/SD	7.12E–01	9.38	10.5	1.1 (V)	6.39E–09	1.28E–09	3.35E–05	4.75E–06	4.09E–03	8.00E–05	1.12E–04
GP PEG	EA	9.76E–01	10.2	11	0.8 (p)	–9.69E–09	2.10E–08	1.59E–04	2.46E–05	8.92E–03	N/A	N/A
RV PER	EA/SD	1.97E+00	10.3	12.7	2.4 (V)	4.33E–09	3.94E–09	3.06E–05	5.05E–06	3.91E–03	6.58E–05	3.33E–05
ST PER	EA/SD	2.65E+00	9.52	11.4	1.88 (V)	1.42E–06	9.19E–08	–3.49E–04	8.77E–05	3.77E–03*	1.19E–03	4.50E–04
BETA PER	EA/SD	2.87E+00	2.12	3.39	1.27 (V)	5.40E–07	3.80E–08	–7.06E–05	4.35E–05	4.95E–03*	7.35E–04	2.56E–04

table continued on following page

Table 2. Analysis of Stars on the AAVSO Eclipsing Binary Program, Group 2, cont.

Name	*Type*	*Period (d)*	*Max*	*Min*	*Range*	*Slope*	*Error*	*Intercept*	*Error*	α	ε	ε/*Period*
U SGE	EA/SD	3.38E+00	6.45	9.28	2.83 (V)	2.59E–08	7.75E–09	3.53E–05	6.39E–06	4.20E–03	1.61E–04	4.76E–05
V505 SGR	EA/SD	1.18E+00	6.46	7.51	1.05 (V)	1.73E–08	6.02E–09	7.84E–05	1.53E–05	6.26E–03	1.32E–04	1.11E–04
RZ TAU	EW/KW	4.16E–01	10.1	10.7	0.63 (V)	3.02E–08	3.12E–09	1.11E–04	2.23E–05	7.45E–03	1.74E–04	4.18E–04
CT TAU	EW/KW	6.67E–01	10.3	11.1	0.78 (V)	1.10E–08	6.63E–09	1.74E–04	2.74E–05	9.34E–03	1.05E–04	1.57E–04
HU TAU	EA/SD	2.06E+00	5.85	6.68	0.83 (V)	3.93E–08	1.22E–08	1.29E–04	1.79E–05	8.02E–03	1.98E–04	9.65E–05
TX UMA	EA/SD	3.06E+00	7.06	8.8	1.74 (V)	3.63E–06	2.52E–07	–6.11E–04	2.12E–04	2.36E–03*	1.91E–03	6.22E–04
UX UMA	EA/WD+NL	1.97E–01	12.6	14.2	1.58 (V)	3.76E–11	1.87E–11	3.17E–06	3.23E–07	1.26E–03	6.14E–06	3.12E–05
AZ VIR	EW/KW	3.50E–01	10.7	11.4	0.63 (V)	9.79E–09	1.01E–09	2.55E–05	8.14E–06	3.57E–03	9.89E–05	2.83E–04
BO VUL	EA/SD	1.95E+00	10.5	13.3	2.8 (p)	3.41E–07	1.20E–08	–1.13E–04	1.93E–05	1.72E–03*	5.84E–04	3.00E–04
BS VUL	EB/KW	4.76E–01	10.9	11.6	0.7 (V)	–1.27E–09	1.59E–09	9.84E–05	9.79E–06	7.02E–03	N/A	N/A

Note: Any alpha (α) value with an asterisk (*) is calculated from the $< u(x) >^2$ *value with the smallest x.*

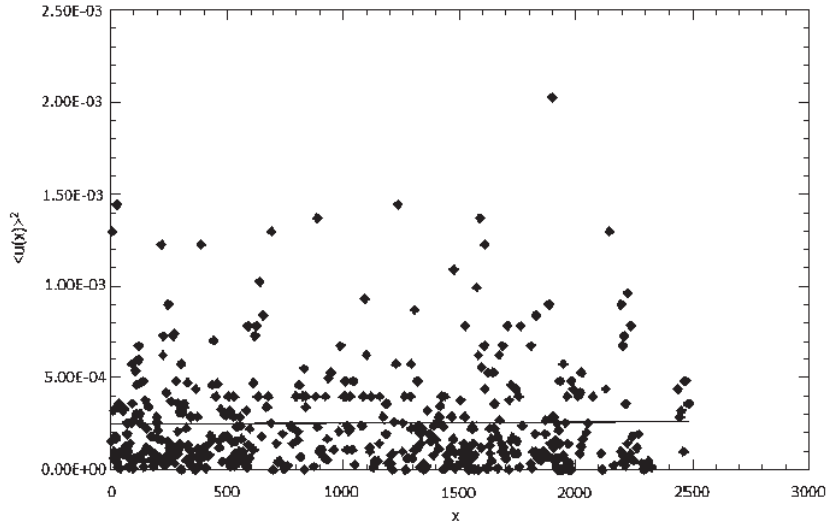

Figure 1. $<u(x)>^2$ diagram for ZZ Cep, a representative variable with no significant random cycle-to-cycle period fluctuations. The diagram is linear and its slope is close to zero.

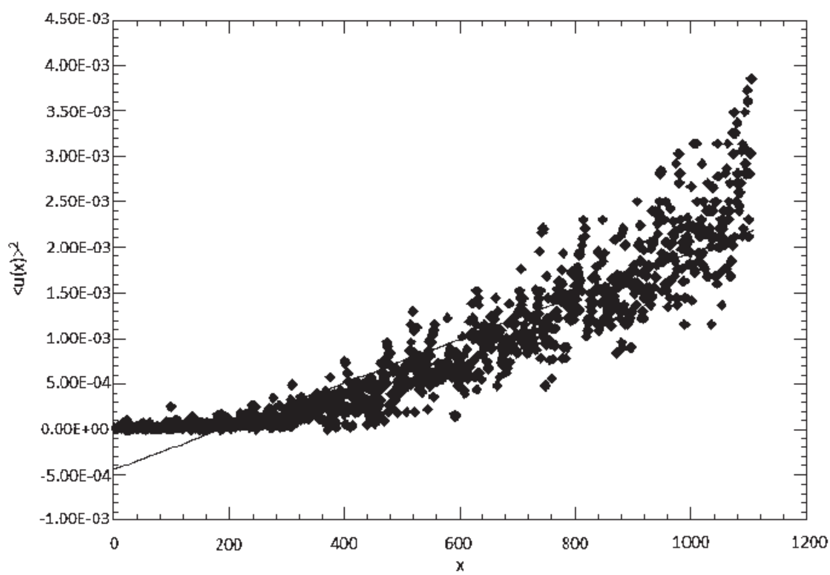

Figure 2. $<u(x)>^2$ diagram for U Cep, one of several variables whose diagram is not a straight line, but curves upward. A linear fit gives a positive slope and a negative intercept.

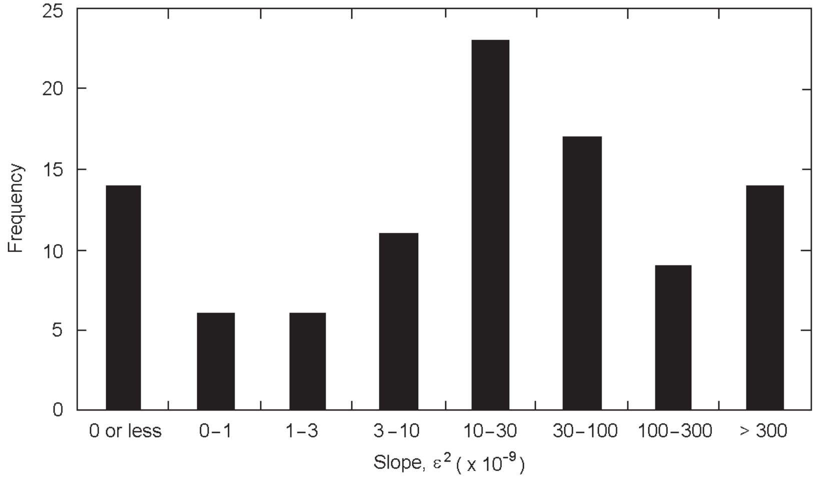

Figure 3. Histogram of slopes of the $<u(x)>^2$ diagrams of the 100 stars in Tables 1 and 2.