AL Pictoris and FR Piscium: Two Regular Blazhko RR Lyrae Stars

Pierre de Ponthière
15 Rue Pré Mathy, Lesve, Profondeville 5170, Belgium; address email correspondence to pierredeponthiere@gmail.com

Franz-Josef (Josch) Hambsch
12 Oude Bleken, Mol, 2400, Belgium

Kenneth Menzies
318A Potter Road, Framingham, MA 01701

Richard Sabo
2336 Trailcrest Drive, Bozeman, MT 59718

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Abstract The results presented are a continuation of observing campaigns conducted by a small group of amateur astronomers interested in the Blazhko effect of RR Lyrae stars. The goal of these observations is to confirm the RR Lyrae Blazhko effect and to detect any additional Blazhko modulation which cannot be identified from all sky survey data-mining. The Blazhko effect of the two observed stars is confirmed, but no additional Blazhko modulations have been detected.

The observation of the RR Lyrae star AL Pictoris during 169 nights was conducted from San Pedro de Atacama (Chile). From the observed light curve, 49 pulsation maxima have been measured. Fourier analyses of (O–C), magnitude at maximum light (M_{max}), and the complete light curve have provided a confirmation of published pulsation and Blazhko periods, 0.548622 and 34.07 days, respectively. The second multi-longitude observation campaign focused on the RR Lyrae star FR Piscium and was performed from Europe, the United States, and Chile. Fourier analyses of the light curve and of 59 measured brightness maxima have improved the accuracy of pulsation and Blazhko periods to 0.45568 and 51.31 days, respectively. For both stars, no additional Blazhko modulations have been detected.

1. Introduction

The data-mining of automated sky surveys like the All Sky Automated Survey—ASAS (Wils and Sódor 2005) and the Northern Sky Variability Survey—NSVS (Wils *et al.* 2006) is frequently used to discover new RR Lyrae stars, to measure their pulsation periods, and for some of them to detect and measure Blazhko modulation. Automated sky surveys with their low sampling frequencies (at best one sample per night) generate sparse datasets. As a result, spectral analysis of the datasets is not always fully reliable. Some Blazhko modulation periods are published as uncertain and multiple Blazhko modulations are not always detected. To overcome those shortcomings, more observations of identified RR Lyrae stars are required. Detailed study of long time-series observations allows the identification of individual brightness maxima of the light curve and other light curve details. Spectral analysis of the rich time-series is able to characterize the Blazhko modulation(s) in detail.

The results of observing campaigns presented herein are related to two RR Lyrae stars exhibiting the Blazhko effect, AL Pic and FR Psc, discovered from ASAS and NSVS surveys, respectively (Wils and Sódor 2005 and Wils *et al.* 2006).

The designation of AL Pic appeared in the *General Catalogue of Variable Stars* with the 79th Name List of Variable Stars (Kazarovets *et al.* 2008); previously this star was identified as GSC 8082-0469, NSV 1700, and ASAS J044131-5216.6. From the All Sky Automated Survey database, Wils and Sódor (2005) measured a pulsation period of 0.54861 day and also a Blazhko period of 34.0 days. The results presented herein are derived from data gathered during 169 nights between July 17, 2012, and February 1, 2013. A total of 17,416 magnitude measurements covering 5.8 Blazhko cycles were collected. All the observations were made by Franz-Josef Hambsch, remotely using a 40-cm f/6.8 telescope located in San Pedro de Atacama (Chile).

The designation of FR Psc appeared in the *General Catalogue of Variable Stars* with the first part of 80th Name List of Variable Stars (Kazarovets *et al.* 2011). This star was previously identified as GSC 0607-0591 and NSVS 9149730. From the Northern Sky Variability Survey data (Wozniak *et al.* 2004), Wils *et al.* (2006) have measured a pulsation period of 0.45570 day and an uncertain Blazhko period of 55 days. Our observations were made between August 18, 2012, and December 30, 2013. During the 126 observation nights, a total of 12,653 observations were made by Franz-Josef Hambsch remotely from Cloudcroft (New Mexico) and San Pedro de Atacama (Chile), by Richard Sabo from Bozeman (Montana), Kenneth Menzies from Framingham (Massachusetts) and Pierre de Ponthière from Lesve (Belgium). The numbers of observations for the different locations are 3597, 7272, 1106, 146, and 532, respectively.

For images of both stars, dark and flat field corrections were performed with MAXIMDL software (Diffraction Limited 2004), and aperture photometry was performed using LESVEPHOTOMETRY (de Ponthière 2010), a custom software which also evaluates the SNR and estimates magnitude errors. The photometric observations of both stars are available in the AAVSO International Database (AAVSO 2014).

AL Pic observations were performed with only a V filter and are not transformed to the standard system. The comparison stars are given in Table 1;

their coordinates and magnitudes in B and V bands were obtained from the AAVSO's Variable Star Database (VSD). The observations have been reduced with C1 as a magnitude reference and C2 as a check star.

The observations of FR Psc were also performed with a V filter and are not transformed to the standard system.

The FR Psc comparison star coordinates and magnitudes in B and V bands were extracted from the AAVSO APASS survey and are given in Table 8. All observations, except from Cloudcroft, were reduced with C1 as magnitude reference and C2 as check star. Cloudcroft observations were reduced with C2 and C3 as reference and comparison stars. A correction of 0.021 magnitude has been applied to the Cloudcroft observations. This correction has been calculated from the magnitude difference of C3 when measured with C1 and C2 as magnitude references.

2. Light curve maxima analyses

The times of maxima of the light curves have been evaluated with custom software (de Ponthiere 2010), fitting the light curve with a smoothing spline function (Reinsch 1967).

2.1. AL Pic

A total of 49 maxima have been observed for AL Pic. Table 2 provides the list of the observed maxima and Figure 1 shows the (O–C) and M_{max} (magnitude at maximum) values. A linear regression of all available (O–C) values has provided a pulsation period of 0.548549 d (1.822858 d^{-1}). The (O–C) values have been re-evaluated with this new pulsation period and the pulsation ephemeris origin has been set to the highest recorded brightness maximum: HJD 2456154.7560. The new derived pulsation elements are:

$$HJD_{Pulsation} = (2456154.7560 \pm 0.0080) + (0.548549 \pm 0.000044)\, E_{Pulsation} \quad (1)$$

The derived pulsation period is in good agreement with the value of 0.54861 published by Wils and Sódor (2005). The folded light curve on the newly determined pulsation period is shown in Figure 2.

To determine the Blazhko period, Fourier analyses and sine-wave fittings of the (O–C) values and M_{max} (magnitude at maximum) values were performed with PERIOD04 (Lenz and Breger 2005). These analyses were limited to the three first harmonic components and are given in Table 3. The frequency uncertainties have been evaluated from the Monte Carlo simulation module of PERIOD04. The Blazhko periods obtained from (O–C) and M_{max} are 34.23 ± 0.12 and 34.03 ± 0.07 days, respectively, which are in reasonable agreement. On this basis the best Blazhko ephemeris is

$$HJD_{Blazhko} = 2456154.75 + (34.03 \pm 0.07)\, E_{Blazhko} \quad (2)$$

The origin has been selected as the epoch of the highest recorded maximum. The (O–C) and M_{max} curves folded with this Blazhko period are given in Figure 3. The variations of (O–C) and M_{max} over the Blazhko cycles are 0.097 day and 0.639 magnitude, respectively.

Table 1. AL Pic comparison stars.

Identification	*R.A. (2000)* *h m s*	*Dec. (2000)* *° ′ ″*	*B*	*V*	*B–V*	*Reference Star*
GSC 8082-440	04 40 50.7	–52 22 27.4	12.608	12.006	0.602	C1
GSC 8082-564	04 42 14.3	–52 23 12.9	13.252	12.590	0.662	C2

Table 2. AL Pic measured maxima.

Maximum HJD	*Error*	*O–C (day)*	*E*	*Magnitude (V)*	*Error*
2456144.8755	0.0060	–0.0059	–18	13.2840	0.015
2456149.8190	0.0027	0.0003	–9	12.9820	0.012
2456150.9137	0.0020	–0.0022	–7	12.9020	0.010
2456154.7560	0.0030	0.0000	0	12.8060	0.011
2456155.8556	0.0026	0.0024	2	12.8710	0.011
2456160.7891	0.0040	–0.0014	11	13.1290	0.012
2456161.8840	0.0048	–0.0037	13	13.1720	0.013
2456167.8557	0.0049	–0.0664	24	13.3320	0.014
2456172.7790	0.0085	–0.0804	33	13.3760	0.019
2456177.7657	0.0059	–0.0310	42	13.3250	0.015
2456182.7317	0.0030	–0.0023	51	13.0730	0.012
2456183.8278	0.0043	–0.0034	53	13.0180	0.026
2456188.7707	0.0035	0.0022	62	12.8800	0.014
2456189.8658	0.0020	0.0001	64	12.8810	0.013
2456221.6849	0.0028	0.0010	122	12.8660	0.015
2456222.7875	0.0022	0.0065	124	12.8970	0.013
2456226.6283	0.0038	0.0071	131	13.0180	0.015
2456233.7125	0.0038	–0.0403	144	13.2920	0.016
2456234.8038	0.0068	–0.0462	146	13.3650	0.021
2456239.7063	0.0053	–0.0810	155	13.3470	0.017
2456240.8087	0.0079	–0.0758	157	13.4140	0.018
2456244.7041	0.0071	–0.0205	164	13.4160	0.017
2456245.8124	0.0048	–0.0094	166	13.3930	0.014
2456249.6738	0.0057	0.0119	173	13.1860	0.025
2456250.7666	0.0029	0.0075	175	13.1350	0.015
2456254.6044	0.0028	0.0052	182	12.8370	0.014
2456255.7034	0.0029	0.0070	184	12.8530	0.017

Table continued on next page

Table 2. AL Pic measured maxima, cont.

Maximum HJD	*Error*	*O–C (day)*	*E*	*Magnitude (V)*	*Error*
2456256.8042	0.0023	0.0106	186	12.8680	0.02
2456259.5492	0.0034	0.0127	191	12.9610	0.018
2456261.7435	0.0029	0.0126	195	13.0910	0.018
2456265.5621	0.0043	–0.0089	202	13.2590	0.014
2456266.6495	0.0055	–0.0187	204	13.2920	0.02
2456272.6187	0.0052	–0.0839	215	13.3530	0.015
2456273.7370	0.0085	–0.0628	217	13.4330	0.021
2456277.6083	0.0064	–0.0316	224	13.4250	0.013
2456278.7047	0.0043	–0.0324	226	13.4300	0.009
2456283.6765	0.0031	0.0021	235	13.1350	0.016
2456288.6169	0.0024	0.0052	244	12.8560	0.007
2456289.7104	0.0019	0.0015	246	12.8240	0.015
2456293.5592	0.0018	0.0102	253	12.9820	0.006
2456294.6572	0.0026	0.0110	255	13.0200	0.007
2456295.7542	0.0068	0.0108	257	13.1420	0.033
2456299.5782	0.0050	–0.0053	264	13.2820	0.016
2456300.6648	0.0034	–0.0159	266	13.2850	0.008
2456301.7665	0.0112	–0.0114	268	13.3850	0.021
2456305.5711	0.0153	–0.0469	275	13.4250	0.025
2456307.7435	0.0092	–0.0688	279	13.4450	0.015
2456317.6883	0.0043	0.0014	297	13.1820	0.013
2456321.5373	0.0040	0.0102	304	12.9370	0.021

Table 3. AL Pic Blazhko spectral components from light curve maxima.

From	*Frequency (cycle/days)*	*σ(d^{-1})*	*Period (days)*	*σ(d)*	*Amplitude*	*Φ (cycle)*	*SNR*
(O–C) values	0.02922	10×10^{-5}	34.23	0.12	0.037 day	0.758	21.1
M_{max} values	0.02939	6×10^{-5}	34.025	0.07	0.270 mag.	0.207	36.9

Table 4. AL Pic triplet component frequencies and periods.

Component	*Derived from*	*Frequency (d^{-1})*	*σ (d^{-1})*	*Period (d)*	*σ (d)*
f_0		1.822749	4.5×10^{-6}	0.548622	1.4×10^{-6}
f_B	$f_0 + f_B$	0.029353	13×10^{-6}	34.07	0.02

Table 5. AL Pic multi-frequency fit results.

Component	*f(d^{-1})*	*σ(f)*	*A_i (mag.)*	*σ(A_i)*	*Φ_i (cycle)*	*σ(Φ_i)*	*SNR*
f_0	1.822749	4.5×10^{-6}	0.2790	0.0007	0.5947	0.0005	99.8
$2f_0$	3.645498		0.0972	0.0008	0.5489	0.0013	38.5
$3f_0$	5.468248		0.0512	0.0007	0.4973	0.0022	22.3
$4f_0$	7.290997		0.0321	0.0008	0.4488	0.0033	14.5
$5f_0$	9.113746		0.0197	0.0007	0.4206	0.0065	11.2
$6f_0$	10.936495		0.0117	0.0008	0.3931	0.0102	7.4
$7f_0$	12.759244		0.0075	0.0007	0.3313	0.0167	5.3
$8f_0$	14.581993		0.0051	0.0007	0.2765	0.0215	4.3
f_0+f_b	1.852103	13×10^{-6}	0.1090	0.0007	0.9100	0.0012	39.1
f_0-f_b	1.793396		0.1006	0.0007	0.9945	0.0012	35.9
$2f_0+f_b$	3.674852		0.0797	0.0007	0.9162	0.0015	31.5
$2f_0-f_b$	3.616145		0.0703	0.0007	0.9495	0.0017	27.9
$3f_0+f_b$	5.497601		0.0504	0.0007	0.9529	0.0023	22.0
$3f_0-f_b$	5.438894		0.0460	0.0008	0.9602	0.0027	20.0
$4f_0+f_b$	7.320350		0.0271	0.0007	0.9422	0.0044	12.3
$4f_0-f_b$	7.261643		0.0233	0.0007	0.9605	0.0053	10.5
$5f_0+f_b$	9.143099		0.0129	0.0007	0.8866	0.0100	7.3
$5f_0-f_b$	9.084393		0.0142	0.0008	0.8988	0.0083	8.0
$6f_0+f_b$	10.965848		0.0102	0.0008	0.8041	0.0125	6.5
$6f_0-f_b$	10.907142		0.0107	0.0008	0.8459	0.0125	6.8
$7f_0+f_b$	12.788598		0.0073	0.0007	0.7938	0.0157	5.1

Table 6. AL Pic harmonic, triplet amplitudes, ratios, and asymmetry parameters.

i	*A_i/A_1*	*A_i^+/A_1*	*A_i^-/A_1*	*R_i*	*Q_i*
1	1.00	0.39	0.36	1.08	0.04
2	0.35	0.29	0.25	1.13	0.06
3	0.18	0.18	0.16	1.10	0.05
4	0.12	0.10	0.08	1.16	0.08
5	0.07	0.05	0.05	0.91	–0.05
6	0.04	0.04	0.04	0.95	–0.03
7	0.03	0.03	—	—	—
8	0.02	—	—	—	—

Table 7. AL Pic Fourier coefficients over Blazhko cycle.

Ψ (cycle)	A_1 (mag.)	A_2 (mag.)	A_3 (mag.)	A_4 (mag.)	Φ_1 (rad.)	Φ_{21} (rad.)	Φ_{31} (rad.)	Φ_{41} (rad.)
0.0–0.1	0.444	0.212	0.134	0.071	3.297	2.501	5.225	1.596
0.1–0.2	0.376	0.167	0.086	0.049	3.593	2.543	5.340	1.978
0.2–0.3	0.286	0.120	0.063	0.036	3.504	2.522	5.232	1.625
0.3–0.4	0.235	0.095	0.034	0.021	4.462	2.798	5.394	2.124
0.4–0.5	0.209	0.096	0.041	0.021	4.446	2.433	5.305	1.913
0.5–0.6	0.235	0.078	0.032	0.023	4.554	2.482	5.319	1.765
0.6–0.7	0.205	0.083	0.035	0.015	3.459	2.204	4.660	1.182
0.7–0.8	0.285	0.125	0.072	0.028	3.877	2.177	4.841	1.023
0.8–0.9	0.362	0.192	0.116	0.080	3.422	2.508	5.310	1.685
0.9–1.0	0.421	0.223	0.156	0.099	3.350	2.461	5.217	1.623

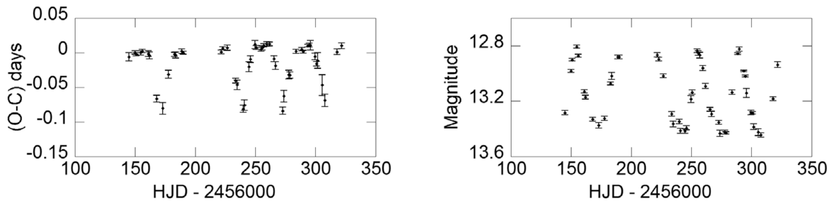

Figure 1. AL Pic O–C (days, left plot) and magnitude at maximum (right plot).

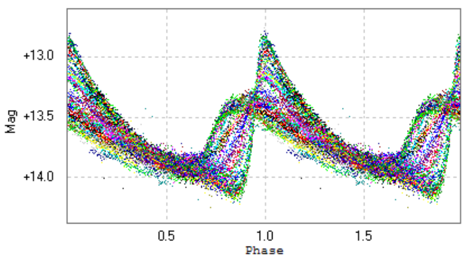

Figure 2. AL Pic light curve folded with pulsation period.

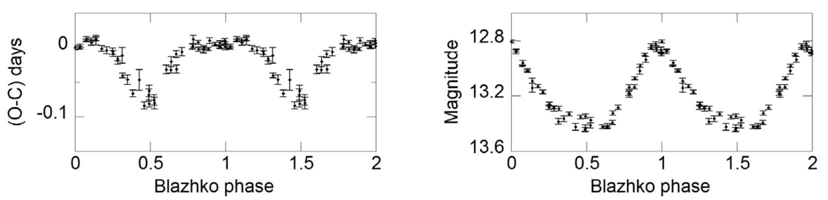

Figure 3. AL Pic O–C (left plot) and Magnitude at maximum (right plot) folded with Blazhko period.

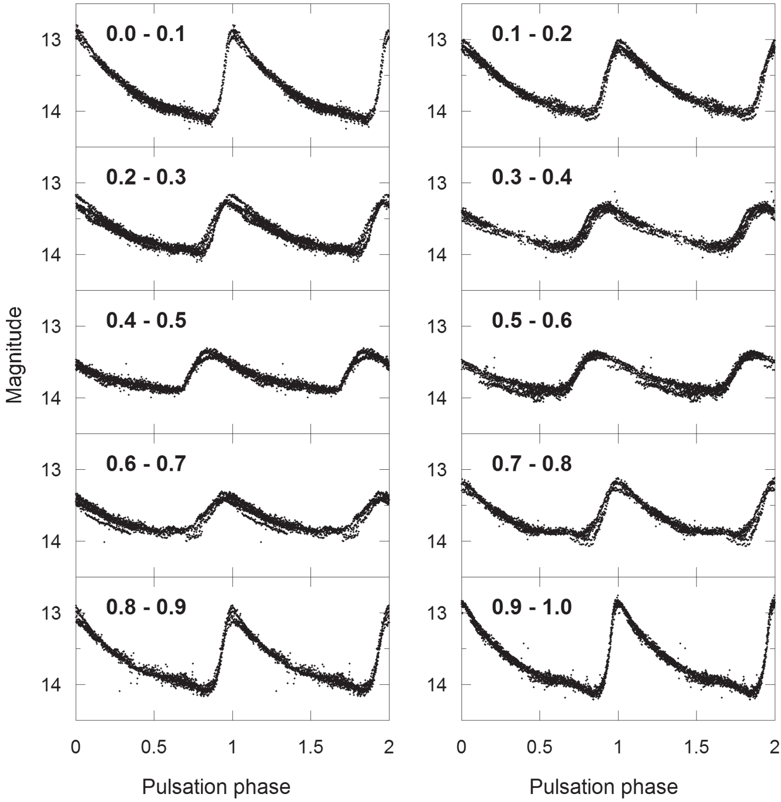

Figure 4. AL Pic light curves (magnitude vs. pulsation phase) for the ten temporal subsets based on Blazhko period.

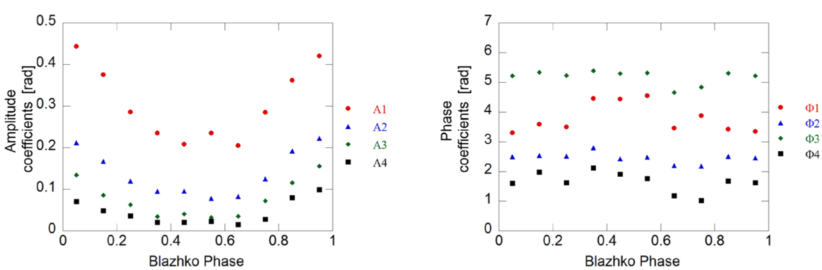

Figure 5. Al Pic Fourier amplitude (left plot) and phase coefficients (mag., right plot) for the ten temporal subsets based on a Blazhko period.

2.2. FR Psc

For FR Psc, the 59 observed maxima are listed in Table 9 and the graphs of (O–C) and M_{max} values versus time (HJD) are given in Figure 6. A pulsation period of 0.4555800 d (2.1945223 d^{-1}) was derived from a linear regression of the (O–C) values. The (O–C) values have been re-evaluated with this new pulsation period and the pulsation ephemeris origin has been set to the highest recorded brightness maximum: HJD 2456631.5796. The new derived pulsation elements are:

$$HJD_{Pulsation} = (2456631.5796 \pm 0.0021) + (0.4556800 \pm 0.0000042)\ E_{Pulsation} \quad (3)$$

The calculated pulsation period is very close to the value of 0.45570 day published by Wils *et al.* (2006). The Blazhko amplitude and phase modulations are clearly visible in Figure 7, which presents the graph of the light curve folded with the pulsation period.

The Blazhko period has been measured with PERIOD04 in the same way as for AL Pic. The results are given in Table 10. The Blazhko periods, from (O–C) and M_{max} are 51.32 ± 0.05and 51.35 ± 0.09 days, respectively, and agree within the errors. The Blazhko period of 55 days reported as uncertain by Wils *et al.* (2006) is slightly longer and not compatible with our improved results. The best Blazhko ephemeris is:

$$HJD_{Blazhko} = 2456631.58 + (51.32 \pm 0.05)\ E_{Blazhko} \quad (4)$$

The (O–C) and M_{max} curves folded with the Blazhko period are given in Figure 8. The (O–C) and M_{max} curves are anti-correlated and their shapes are very similar to the corresponding curves of AL Pic. The variations of (O–C) and M_{max} over the Blazhko cycles are 0.042 day and 0.396 magnitude, respectively.

Table 8. FR Psc comparison stars (AAVSO chart 8256CED).

Identification	*R.A. (2000)* h m s	*Dec. (2000)* ° ′ ″	*B*	*V*	*B–V*	*Reference Star*
GSC 607-409	00 47 35.6	11 47 09.1	12.347	11.657	0.690	C1
GSC 607-679	00 47 54.0	11 42 16.4	14.017	13.310	0.707	C2
GSC 607-799	00 47 57.4	11 41 45.2	14.837	14.237	0.600	C3

Table 9. FR Psc measured maxima.

Maximum HJD	Error	O–C (day)	E	Magnitude (V)	Error	Location*
2456158.5795	0.0020	–0.0042	–1038	11.639	0.010	1
2456159.4882	0.0045	–0.0069	–1036	11.629	0.013	1
2456192.7269	0.0051	–0.0328	–963	11.691	0.009	4
2456195.9113	0.0048	–0.0382	–956	11.682	0.045	4
2456202.7678	0.0055	–0.0169	–941	11.738	0.017	4
2456203.6806	0.0026	–0.0154	–939	11.708	0.016	4
2456205.9714	0.0049	–0.0030	–934	11.703	0.017	4
2456213.7200	0.0024	–0.0010	–917	11.477	0.014	4
2456214.6314	0.0014	–0.0010	–915	11.414	0.011	4
2456218.7341	0.0028	0.0006	–906	11.360	0.036	4
2456219.6433	0.0019	–0.0015	–904	11.343	0.010	4
2456259.7407	0.0031	–0.0040	–816	11.635	0.026	4
2456261.5653	0.0023	–0.0021	–812	11.564	0.014	4
2456264.7575	0.0024	0.00034	–805	11.493	0.017	4
2456265.6685	0.0019	–0.00002	–803	11.456	0.013	4
2456270.6801	0.0018	–0.00090	–792	11.357	0.011	4
2456540.4398	0.0014	–0.00379	–200	11.503	0.008	1
2456546.8092	0.0013	–0.01391	–186	11.659	0.005	2
2456546.8095	0.0016	–0.01361	–186	11.685	0.009	5
2456547.7192	0.0023	–0.01527	–184	11.678	0.006	2
2456551.8039	0.0040	–0.03169	–175	11.721	0.014	2
2456556.8141	0.0040	–0.03397	–164	11.724	0.017	5
2456560.4719	0.0037	–0.02161	–156	11.713	0.009	1
2456566.8696	0.0021	–0.00343	–142	11.668	0.010	5
2456567.7806	0.0035	–0.00379	–140	11.636	0.005	2
2456568.6969	0.0025	0.00115	–138	11.607	0.006	2
2456571.8873	0.0003	0.00179	–131	11.538	0.004	5
2456572.8007	0.0013	0.00383	–129	11.484	0.004	2
2456573.7104	0.0014	0.00217	–127	11.471	0.005	2
2456574.6216	0.0018	0.00201	–125	11.418	0.005	2
2456578.7206	0.0010	–0.00011	–116	11.374	0.005	2
2456588.7447	0.0011	–0.00098	–94	11.441	0.005	2
2456589.6550	0.0013	–0.00204	–92	11.480	0.005	2
2456589.6557	0.0007	–0.00134	–92	11.507	0.005	5
2456592.8414	0.0012	–0.00540	–85	11.584	0.005	5
2456593.7524	0.0015	–0.00576	–83	11.574	0.006	2
2456594.6623	0.0013	–0.00722	–81	11.623	0.006	3
2456594.6635	0.0022	–0.00602	–81	11.587	0.008	2
2456595.5729	0.0021	–0.00798	–79	11.636	0.003	2
2456599.6659	0.0016	–0.01610	–70	11.685	0.004	2

Table continued on next page

Table 9. FR Psc measured maxima, cont.

Maximum HJD	Error	O–C (day)	E	Magnitude (V)	Error	Location*
2456600.5695	0.0019	–0.02386	–68	11.694	0.004	2
2456604.6661	0.0038	–0.02838	–59	11.703	0.005	2
2456605.5741	0.0025	–0.03174	–57	11.704	0.005	2
2456607.3937	0.0035	–0.03486	–53	11.711	0.007	1
2456609.6729	0.0030	–0.03406	–48	11.713	0.008	2
2456614.7092	0.0021	–0.01024	–37	11.708	0.010	2
2456615.6255	0.0030	–0.00530	–35	11.709	0.010	2
2456616.5376	0.0024	–0.00456	–33	11.681	0.010	2
2456619.7317	0.0008	–0.00022	–26	11.614	0.005	5
2456620.6441	0.0019	0.00082	–24	11.575	0.009	2
2456621.5584	0.0029	0.00376	–22	11.556	0.013	2
2456625.6577	0.0012	0.00194	–13	11.414	0.006	2
2456626.5695	0.0017	0.00238	–11	11.420	0.009	2
2456631.5796	0.0012	0.00000	0	11.342	0.015	2
2456636.5929	0.0016	0.00082	11	11.373	0.011	2
2456641.6030	0.0014	–0.00156	22	11.491	0.008	2
2456647.5211	0.0014	–0.00730	35	11.628	0.006	2
2456657.5222	0.0027	–0.03116	57	11.738	0.008	3

** Locations: 1—Lesve (Belgium); 2—Bozeman (Montana); 3—Framingham (Massachusetts); 4—Cloudcroft (New Mexico); 5—San Pedro de Atacama (Chile).*

Table 10. FR Psc Blazhko spectral components from light curve maxima.

From	Frequency (cycle/days)	$\sigma(d^{-1})$	Period (days)	$\sigma(d)$	Amplitude	Φ (cycle)	SNR
(O–C) values	0.01948	2×10^{-5}	51.324	0.053	0.015 day	0.948	33.1
M_{max} values	0.01947	3×10^{-5}	51.353	0.087	0.193 mag.	0.428	21.0

Table 11. FR Psc Triplet component frequencies and periods.

Component	Derived from	Frequency (d^{-1})	σ (d^{-1})	Period (d)	σ (d)
f_0		2.1945066	1.3×10^{-6}	0.4556833	2.7×10^{-7}
f_B	f_0+f_B	0.019490	6.1×10^{-6}	51.31	0.016

Table 12. FR Psc multi-frequency fit results.

Component	$f(d^{-1})$	$\sigma(f)$	A_i (mag.)	$\sigma(A_i)$	Φ_i (cycle)	$\sigma(\Phi_i)$	SNR
f_0	2.194507	6.7×10^{-7}	0.4261	0.0011	0.1956	0.0004	252.2
$2f_0$	4.389014		0.1768	0.0011	0.7527	0.0011	139.5
$3f_0$	6.583521		0.0901	0.0010	0.3300	0.0019	77.9
$4f_0$	8.778028		0.0379	0.0010	0.9201	0.0039	34.8
$5f_0$	10.972536		0.0121	0.0009	0.3977	0.0138	11.9
$6f_0$	13.167043		0.0111	0.0008	0.8574	0.0147	13.1
$7f_0$	15.361550		0.0115	0.0010	0.3822	0.0175	16.3
$8f_0$	17.556057		0.0103	0.0010	0.9469	0.0177	17.0
$9f_0$	19.750564		0.0087	0.0010	0.5409	0.0182	13.2
f_0+f_b	2.213999	30×10^{-7}	0.0731	0.0011	0.7259	0.0019	43.7
f_0-f_b	2.175015		0.0664	0.0010	0.3003	0.0026	39.2
$2f_0+f_b$	4.408506		0.0627	0.0009	0.2775	0.0025	49.4
$2f_0-f_b$	4.369522		0.0591	0.0011	0.8392	0.0026	46.5
$3f_0+f_b$	6.603013		0.0608	0.0010	0.8647	0.0024	52.6
$3f_0-f_b$	6.564030		0.0543	0.0011	0.4355	0.0034	46.9
$4f_0+f_b$	8.797520		0.0391	0.0009	0.4688	0.0048	35.8
$4f_0-f_b$	8.758537		0.0357	0.0011	0.0446	0.0045	32.7
$5f_0+f_b$	10.992027		0.0264	0.0011	0.0512	0.0061	26.1
$5f_0-f_b$	10.953044		0.0216	0.0010	0.6616	0.0069	21.3
$6f_0+f_b$	13.186535		0.0161	0.0010	0.6491	0.0090	19.0
$6f_0-f_b$	13.147551		0.0122	0.0010	0.2430	0.0158	14.4
$7f_0+f_b$	15.381042		0.0087	0.0011	0.2217	0.0168	12.4
$7f_0-f_b$	15.342058		0.0065	0.0010	0.8239	0.0263	9.2
$8f_0+f_b$	17.575549		0.0040	0.0011	0.7335	0.0424	6.6
$8f_0-f_b$	17.536565		0.0022	0.0011	0.3048	0.0793	3.7

Table 13. FR Psc harmonic, triplet amplitudes, ratios, and asymmetry parameters.

i	A_i/A_1	A_i^+/A_1	A_i^-/A_1	R_i	Q_i
1	1.00	0.17	0.16	1.10	0.05
2	0.41	0.15	0.14	1.06	0.03
3	0.21	0.14	0.13	1.12	0.06
4	0.09	0.09	0.08	1.09	0.05
5	0.03	0.06	0.05	1.23	0.10
6	0.03	0.04	0.03	1.32	0.14
7	0.03	0.02	0.02	1.34	0.14
8	0.02	0.01	0.01	1.80	0.29
9	0.02	—	—	—	—

Table 14. FR Psc Fourier coefficients over Blazhko cycle.

Ψ (cycle)	A_1 (mag.)	A_2 (mag.)	A_3 (mag.)	A_4 (mag.)	$Φ_1$ (rad.)	$Φ_{21}$ (rad.)	$Φ_{31}$ (rad.)	$Φ_{41}$ (rad.)
0.0–0.1	0.496	0.234	0.159	0.084	3.977	2.276	4.728	1.092
0.1–0.2	0.462	0.211	0.134	0.068	4.149	2.311	4.792	1.151
0.2–0.3	0.441	0.179	0.106	0.054	3.766	2.357	4.850	1.197
0.3–0.4	0.409	0.173	0.093	0.044	4.379	2.322	4.822	1.201
0.4–0.5	0.394	0.170	0.098	0.050	4.540	2.324	4.897	1.207
0.5–0.6	0.390	0.161	0.094	0.047	4.646	2.309	4.934	1.222
0.6–0.7	0.402	0.163	0.082	0.038	4.437	2.282	4.929	1.409
0.7–0.8	0.418	0.180	0.087	0.035	4.200	2.304	4.767	1.021
0.8–0.9	0.456	0.212	0.132	0.068	4.082	2.238	4.629	0.921
0.9–1.0	0.491	0.240	0.170	0.108	3.979	2.273	4.712	1.046

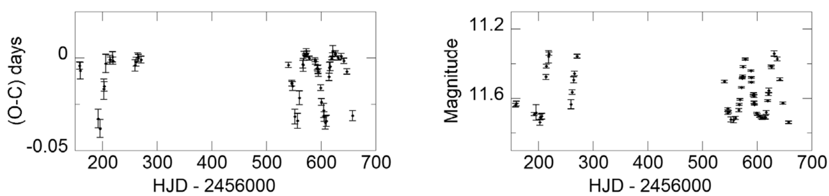

Figure 6. FR Psc O–C (days, left plot) and magnitude at maximum (right plot).

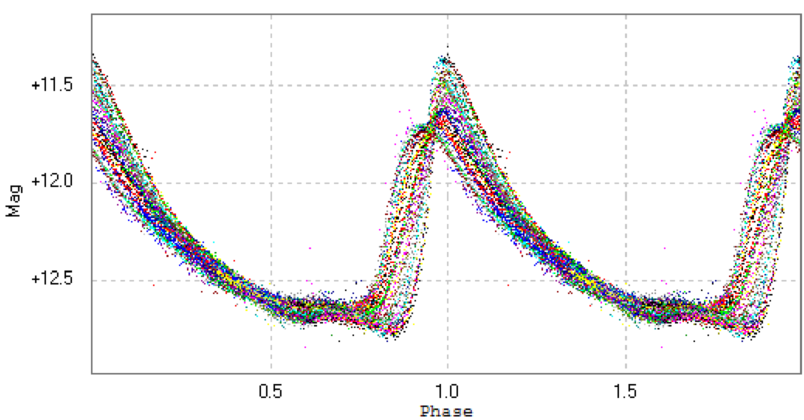

Figure 7. FR Psc light curve folded with pulsation period.

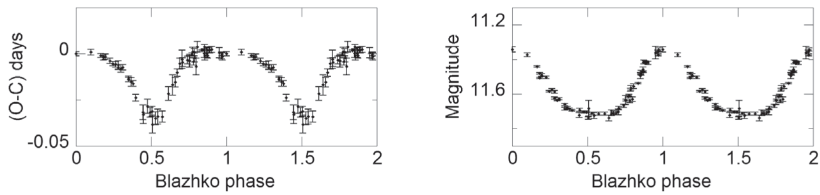

Figure 8. FR Psc O–C (left plot) and magnitude at maximum (right plot) folded with Blazhko period.

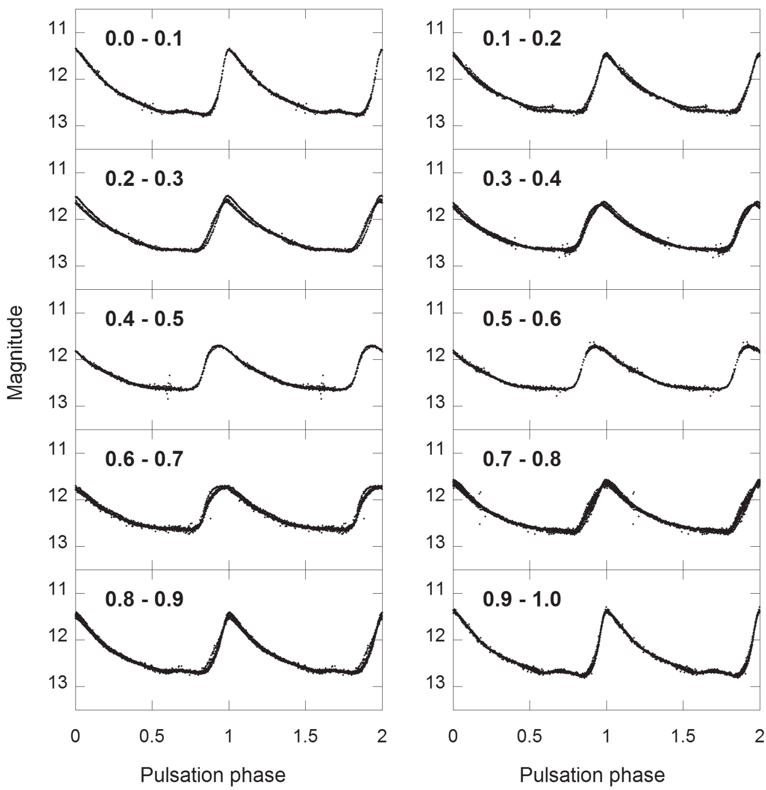

Figure 9. FR Psc light curves (magnitude vs. pulsation phase) for the ten temporal subsets based on Blazhko period.

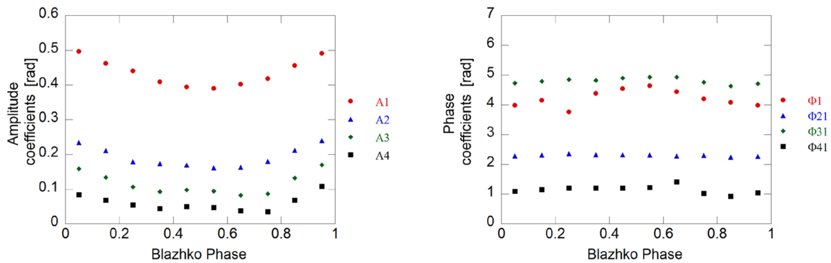

Figure 10. FR Psc Fourier amplitude (left plot) and phase coefficients (mag., right plot) for the ten temporal subsets based on a Blazhko period.

3. Frequency spectrum analyses of the light curve

For both AL Pic and FR Psc, the primary pulsations and Blazhko frequency have been derived from M_{max} and (O–C) analyses. The Blazhko modulation frequencies appear clearly in the spectrum of the complete light curves. The method used for spectrum analysis has already been detailed in other observation reports (de Ponthiere *et al.* 2014), but the method is nevertheless briefly recalled here.

The spectrum of a signal modulated in amplitude and phase is characterized by a pattern of peaks called multiplets at the positions $kf_0 \pm nf_B$, with k and n being integer numbers corresponding, respectively, to the harmonic and multiplet orders. The frequencies, amplitudes, and phases of the multiplets have been obtained with PERIOD04 by performing a succession of Fourier analyses, pre-whitenings, and sine-wave fittings. Only the harmonic and multiplet components having a signal to noise ratio (SNR) greater than 4 have been retained as significant signals. Tables 5 and 12, for AL Pic and FR Psc, respectively, provide the complete lists of Fourier components with their amplitudes, phases, and uncertainties. For both stars, besides the pulsation frequency f_0 and harmonics nf_0, one series of triplets $nf_0 \pm f_B$ based on the Blazhko frequency f_B has been found. The Blazhko frequencies and corresponding periods are tabulated in Tables 4 and 11 with their uncertainties. These Blazhko periods are close to the values obtained with the M_{max} analysis given in Tables 3 and 10.

During the sine-wave fitting, the fundamental frequencies f_0 and triplet $f_0 + f_B$ have been left unconstrained and the other frequencies have been entered as combinations of these two frequencies. The uncertainties of frequencies, amplitudes, and phases have been estimated by Monte Carlo simulations. The amplitude and phase uncertainties have been multiplied by a factor of two as it is known that the Monte Carlo simulations underestimate these uncertainties (Kolenberg *et al.* 2009). Tables 6 and 13 list for each harmonic the amplitude ratios A_i/A_1 and the ratios usually used to characterize the Blazhko effect, that is, A_i^+/A_1 ; A_i^-/A_1 ; $R_i = A_i^+/A_i^-$; and asymmetries $Q_i = (A_i^+ - A_i^-) / (A_i^+ + A_i^-)$.

In the present cases the side lobe amplitudes A_i^- and A_i^+ are similar, which leads to small values for the Q_i asymmetry ratios. Except for some higher order triplets of AL Pic, the asymmetry ratios are positive ($A_i^+ > A_i^-$). For the majority of Blazhko stars, the Q_i asymmetry ratios are larger and generally lie between 0.1 and 0.5, but smaller and negative values are not unusual (Alcock *et al.* 2003). The fact that first triplet values (A_i^+, A_i^-) are larger for AL Pic (0.39 and 0.36) than for FR Psc values (0.17 and 0.16) is due to the relative strength of Blazhko modulations and is consistent with the corresponding variations of the magnitudes at maximum light provided in the preceding section (0.639 and 0.396 magnitude).

4. Light curve variations over Blazhko cycle

Subdividing the data set into temporal subsets is a classical method to visualize and analyze the light curve variations over the Blazhko cycle. For both stars, ten temporal subsets corresponding to the different Blazhko phase intervals Ψ_i (i = 0 , 9) have been created using the epochs of the highest recorded maxima as the origins of the first subset. The folded light curves for the ten subsets are presented in Figures 4 and 9. For AL Pic, the data cover around six successive Blazhko cycles and are relatively well distributed over the different temporal subsets and Blazhko cycles. The population percentage of data points varies between 7.7% and 12% for the ten temporal subsets and between 8.4% and 30% for the six Blazhko cycles. The first and second observation seasons of FR Psc cover around 2.6 and 2.3 Blazhko cycles, respectively. The observation data are also well distributed over the temporal subsets, the population varying between 6.4% and 16%.

From a visual inspection of Figures 4 and 9, it is clear that the light curves are only affected by a small scatter during the successive Blazhko cycles. This fact is not surprising, indeed for both stars only one Blazhko modulation frequency has been detected in the spectrum analysis. Fourier analyses and Least-Square fittings have been performed on the different temporal subsets. For the fundamental frequency and the first four harmonics the amplitudes A_i and the epoch-independent phase differences ($\Phi_{k1} = \Phi_k - k\Phi_1$) are given in Tables 7 and 14 and plotted in Figures 5 and 10. The differences between maximal and minimal values of A_1 over the Blazhko cycle for AL Pic and FR Psc are 0.235 and 0.106 magnitude, respectively. The larger A_i value for AL Pic is a confirmation that the Blazhko modulation is stronger for AL Pic than for FR Psc.

5. Conclusions

The two analysis methods, maximum brightness and light curve Fourier analyses, have provided similar results for both stars. No multiple or irregular

Blazhko modulations have been detected and for the two stars the light curves repeat from one cycle to another. The FR Psc Blazhko period was previously published as uncertain. The new measured period value of 51.31 ± 0.02 days removes the uncertainty. The new period of 34.07 ± 0.02 days for AL Pic is in agreement with the previously published value. The objective of this small group of amateur astronomers is to observe and to analyze Blazhko RR Lyrae stars with the hope of finding stars affected by irregular or multiple Blazhko modulations. These two coordinated multi-longitude campaigns have not revealed such multiple modulations. However, observers should continue their regular and coordinated multi-longitude observations to precisely characterize Blazhko modulations in other RR Lyrae stars.

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