Analysis of the Petersen Diagram of Double-Mode High-Amplitude δ Scuti Stars

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Abstract I created the Petersen diagram relative to all the Double Mode High Amplitude δ Scuti stars listed in the AAVSO's International Variable Star Index up to date December 29, 2015. For the first time I noticed that the ratio between the two periods P1/P0 seems in evident linear relation with the duration of the period P0, a finding never explicitly described in literature regarding this topic.

1. Introduction

Within the wide range of pulsating variables there is a group with a very large population: the δ Scuti stars. It is a very heterogeneous group composed of stars with radial and non-radial pulsations and normally small amplitudes variations. Nevertheless a small group of stars represents a special subtype: the Double-Mode High-Amplitude δ Scuti stars (from now HADS(B)), characterized by pulsations of higher intensity and a ratio between the period of the fundamental mode (P0) and first overtone (P1) around the value 0.77. The ratio between the two periods has been for a long time the main parameter to identify a HADS(B) compared to a simple HADS. But is there a more complex relation between P0 and P1/P0? In recent years the two major contributions that have attempted to find an extensive model capable to explain the link between the period of the fundamental mode (usually between 0.05 and 0.25 day) and the period ratio P1/P0 were those by Petersen and Christensen-Dalsgaard (1996) and by Poretti *et al.* (2005).

It should be noted, however, that only 7 HADS(B) were used for the model validation in the paper published by Petersen and Christensen-Dalsgaard, instead of 25 stars used in the paper by Poretti *et al.*

With regard to the relationship between the duration of the fundamental mode (P0) and the ratio between this and the first overtone (P1/P0), the first cited paper presents a model (their Figure 5) that predicts a peak value of the ratio around 0.774, which corresponds to log P0 = –0.9. For both increasing and decreasing duration of P0, the model predicts a lower ratio that becomes equal to 0.764 for values of log P0 = –0.55 and equal to 0.770 for log P0 = –1.1. In extreme simplification the model is similar to a downward parabola shape with the vertex (high ratio) for HADS with log P0 = –0.9.

In the paper published by Poretti *et al.* (their Figure 4) the model predicts a ratio characterized by a long standstill between log P0 = –1.30 and log P0 = –0.90. For the shortest period the ratio is increasing (0.778 for SX Phe itself) while for longer periods the ratio is decreasing (0.765 for GSC 04257-00471). In extreme synthesis the result of this model is to identify a direct relationship between the duration of the period and ratio (short periods have higher ratios while long periods have lower ratios) but considering the ratio stable for values between log P0 = –1.30 and log P0 = –0.90. The ratio's variability is mainly explained by lower metallicity for the higher values and in lower masses for the lower values.

On the other side I think it is important to mention the work of Pigulski *et al.* (2006) concerning the analysis of the data obtained from the OGLE-II (Udalski *et al.* 1997) and MACHO (Allsman and Axelrod 2001) projects. In this paper, the authors identify several other HADS(B) and publish a much more detailed Petersen diagram of the work here previously mentioned. However, while deciding to put in direct relation the duration of P0 and the ratio P1 / P0 (not a common choice as the diagram is normally realized with the logarithm of the fundamental period), they do not give any observations concerning a possible linear relation between the data nor, of course, its computed equation.

2. The Petersen diagram of HADS(B) stars

I decided to create a new Petersen's diagram using data from the AAVSO International Variable Star Index (Watson *et al.* 2014) related to 85 HADS(B), many of them completely unknown only five years ago. The period ratio was calculated by the author when not present in literature or simply reported when present. From these HADS(B) 8 stars were excluded for the following reasons:

- V798 Cyg: first and second overtone pulsator (Musazzi *et al.* 1998)
- V1719 Cyg: first and second overtone pulsator (Musazzi *et al.* 1998)
- VZ Cnc: first and second overtone pulsator (Fu and Jiang 1999)
- 1SWASP J211253.68+331734.3: probable second and third overtone pulsator (Khruslov 2014)
- ASAS J205850+0854.1: probable second and third overtone pulsator (Khruslov 2011)
- V1553 Sco: probable second and third overtone pulsator (Khruslov 2009)
- V526 Vel: probable second and third overtone pulsator (Khruslov 2011)

- V823 Cas: anomalous HADS because "the periods of the stars are in a transient, resonance affected state, thus do not reflect the true parameters of the object that is in effect a triple-mode pulsating variable." (Jurcsik *et al.* 2006)

The full list of stars used (as well as those not included) to create the Petersen's diagram is presented in Table 1, ordered by increasing P0.

The resulting diagram is shown in Figure 1, where the x-axis represents the log of P0 and the y-axis the ratio between P1 and P0.

Although the purpose of creating a new model capable of predicting the variation of the period and the ratio on the basis of the physical parameters of the star is outside the scope of this work, in observing the Petersen's diagram relative to all the HADS considered we may notice that the relationship between P0 and the period ratio does not appear as predicted by Petersen and Christensen-Dalsgaard (1996) and does not present even the long standstill described by Poretti *et al.* (2005).

Passing from an x-axis expressed as log P0 to an axis simply expressed in days, we notice that in fact the data seem well fitted by a straight line (red line), suggesting a possible linear relationship between the two factors as presented in Figure 2.

A greater scattering is certainly evident for the shortest periods and some stars are markedly outside the line of fit. However, considering the number of stars used for this plot

Table 1. List of stars used (as well as those not included) to create the Petersen's diagram, ordered by increasing P0.

Name	*R. A. (J2000)*			*Dec. (J2000)*			*P0 duration (d)*	*P1 duration (d)*	*P1/P0 ratio*
	h	*m*	*w*	°	′	″			
2MASS J06451725+4122158	06	45	17.25	+41	22	15.9	0.0500071	0.0386898	0.77369
LINEAR 9328902	13	35	49.76	+26	55	16.7	0.05174768	0.04046822	0.78203
[SIG2010] 3269918	20	59	27.28	–01	13	49.0	0.052376	0.040885	0.78061
NSVS 10590484	15	13	22.01	+18	15	58.3	0.0541911	0.0419105	0.77338
USNO–B1.0 0961-0254829	15	52	51.38	+06	06	06.1	0.05492	0.042667	0.77689
SSS_J095657.2-231722	09	56	57.19	–23	17	22.9	0.0566708	0.0442543	0.78090
V879 Her	17	31	12.72	+28	03	16.8	0.0568926	0.044128	0.77564
[MHF2014] J336.0969-15.6349	22	24	23.25	–15	38	05.5	0.057182	0.044534	0.77881
TSVSC1 TN-N231330220-6-67-2	08	58	54.72	+15	22	09.7	0.0576289	0.044596	0.77385
ASAS J061518+0604.2	06	15	17.73	+06	04	12.6	0.0580806	0.044828	0.77182
GSC 02008-00003	14	22	31.21	+24	34	57.0	0.059596	0.046136	0.77415
SDSS J151253.97+231748.4	15	12	53.99	+23	17	48.3	0.06001381	0.0467412	0.77884
GSC 07243-00871	12	08	49.77	–36	33	11.1	0.060031	0.04648	0.77427
BPS BS 16084-151	16	29	40.31	+57	20	33.3	0.06114265	0.0475034	0.77693
LINEAR 1683151	11	32	05.40	–03	48	27.5	0.0618462	0.04820869	0.77949
CSS_J213533.0+124341	21	35	32.99	+12	43	41.3	0.0630537	0.0487775	0.77359
NSVS 2577931	10	55	02.50	+61	42	17.2	0.06404409	0.0496142	0.77469
NSV 7805	16	32	20.12	–02	12	08.3	0.064604	0.050699	0.78477
OGLE BW2 V142	18	02	18.04	–30	08	11.4	0.066041	0.051404	0.77836
NSVS 2684702	13	45	21.66	+54	11	51.2	0.06794351	0.0526002	0.77418
SSS_J095011.1-244057	09	50	11.12	–24	40	58.0	0.0683901	0.0530193	0.77525
SEKBO 112944.737	20	10	22.51	–23	10	59.7	0.0688009	0.0532926	0.77459
LINEAR 16586778	16	13	57.55	+28	28	57.2	0.070751	0.055701	0.78728
V803 Aur	06	12	13.90	+31	48	24.4	0.0710556	0.0550312	0.77448
FASTT 8	00	39	09.42	+00	40	12.1	0.0730198	0.0571184	0.78223
V1392 Tau	04	26	05.90	+01	26	26.2	0.07443025	0.05790307	0.77795
KID 2857323	19	29	49.16	+38	01	21.7	0.07618	0.05897	0.77409
CSS_J214745.8+122726	21	47	45.78	+12	27	26.6	0.07820144	0.06062011	0.77518
[SIG2010] 2345453	21	29	52.69	–01	10	18.9	0.080586	0.0624379	0.77480
OGLE BW1 V207	18	02	14.98	–29	54	08.8	0.085601	0.066234	0.77375
MACHO 116.24384.481	18	13	16.45	–29	49	27.0	0.086914	0.06716	0.77272
GSC 07460-01520	20	33	38.54	–32	55	03.6	0.087011	0.068152	0.78326
NSVS 7293918	07	44	38.60	+29	12	22.8	0.088535	0.068501	0.77372
GSC 03693-01705	02	12	19.83	+57	00	16.4	0.09108389	0.0704693	0.77367
MACHO 115.22573.263	18	09	00.48	–29	14	30.9	0.091754	0.070871	0.77240
RV Ari	02	15	07.46	+18	04	28.0	0.0931281	0.0719466	0.77256
QS Dra	15	21	34.64	+61	29	22.7	0.09442318	0.07304432	0.77358
LINEAR 2653935	11	59	42.51	+06	08	22.0	0.09520999	0.07460334	0.78357
GSC 03949-00386	20	19	44.95	+58	29	20.0	0.095783796	0.073937974	0.77193
ASAS J094303-1707.3	09	43	02.81	–17	07	15.9	0.0991782	0.07651564	0.77150
USNO–A2.0 1425-12623576	21	59	23.24	+59	24	56.9	0.1027306	0.079165	0.77061
MACHO 114.19969.980	18	02	52.20	–29	30	24.5	0.103272	0.079811	0.77282
MACHO 119.19574.1169	18	02	00.37	–29	48	43.2	0.1068464	0.082722	0.77421
GSC 03887-00087	17	08	14.77	+52	53	53.4	0.107183	0.082932	0.77374
ASAS J182536-4213.6	18	25	36.26	–42	13	35.8	0.1071934	0.0821611	0.76648
[SIG2010] 2196466	21	36	30.17	–00	21	27.6	0.107404	0.083675	0.77907
BP Peg	21	33	13.53	+22	44	24.3	0.109543375	0.08451	0.77148
V899 Car	11	09	52.24	–60	57	56.7	0.1108014	0.0858512	0.77482

(Table 1 continued on next page)

Table 1. List of stars used (as well as those not included) to create the Petersen's diagram, ordered by increasing P0, cont.

Name	*R. A. (J2000)* *h m w*	*Dec. (J2000)* *° ′ ″*	*P0 duration (d)*	*P1 duration (d)*	*P1/P0 ratio*
MACHO 162.25343.874	18 15 16.33	–26 35 40.2	0.111281	0.085905	0.77196
AI Vel	08 14 05.15	–44 34 32.9	0.11157411	0.08620868	0.77266
ASAS J231801-4520.0	23 18 01.14	–45 19 55.0	0.1150105	0.0889176	0.77313
2MASS J18294745+3745005	18 29 47.55	+37 45 01.5	0.116576	0.090297	0.77458
V1393 Cen	13 57 15.60	–52 55 22.6	0.1177831	0.0908322	0.77118
NSV 9856	17 56 00.20	–30 42 46.6	0.118488	0.0912733	0.77032
MACHO 128.21542.753	18 06 35.93	–28 39 31.3	0.120052	0.09254	0.77083
BPS BS 16553-0026	10 52 48.49	+41 54 35.3	0.125508	0.096953	0.77248
MACHO 114.19840.890	18 02 31.85	–29 27 03.9	0.125566	0.096789	0.77082
ASAS J152315-5603.7	15 23 15.43	–56 03 43.2	0.1267467	0.0976718	0.77061
GSC 04757-00461	05 23 54.48	–03 07 32.3	0.1325305	0.1019376	0.76916
GSC 02860-01552	03 16 02.70	+43 20 34.3	0.13831414	0.10675322	0.77182
V1384 Tau	03 54 07.27	+07 59 15.4	0.1397914	0.1073918	0.76823
V575 Lyr	18 29 43.24	+28 09 54.6	0.1455591	0.1115016	0.76602
ASAS J192227-5622.5	19 22 27.39	–56 22 28.1	0.1490898	0.1127701	0.75639
V703 Sco	17 42 16.81	–32 31 23.6	0.1499615	0.11521772	0.76832
ASAS J062542+2206.4	06 25 41.61	+22 06 19.5	0.1526484	0.117307	0.76848
V403 Gem	06 44 01.06	+22 44 31.7	0.15338	0.117698	0.76736
NSV 14800	00 01 16.22	–60 36 57.1	0.1578385	0.122071	0.77339
USNO–B1.0 1329-0132547	04 44 37.78	+42 54 34.4	0.16189	0.12413	0.76676
GSC 03949-00811	20 26 01.74	+59 30 53.5	0.169751	0.1300791	0.76629
GSC 04257-00471	21 26 01.11	+64 30 57.5	0.173799	0.133084	0.76574
V542 Cam	04 53 46.52	+68 28 26.5	0.174773	0.133986	0.76663
DO CMi	07 12 19.41	+09 21 02.7	0.194506	0.14862	0.76409
ASAS J194803+4146.9	19 48 02.92	+41 46 55.8	0.203636	0.155488	0.76356
VX Hya	09 45 46.85	–12 00 14.3	0.2233889	0.17272	0.77318
V733 Pup	08 18 06.98	–22 14 07.7	0.2287147	0.1742342	0.76180
AG Aqr	22 05 31.82	–22 30 00.7	0.291736	0.2222	0.76165
V829 Aql	19 46 57.29	+03 30 28.5	0.292444	0.220972	0.75560
Stars excluded					
V798 Cyg	19 38 06.90	+30 54 33.5			
V1719 Cyg	21 04 32.92	+50 47 03.3			
VZ Cnc	08 40 52.12	+09 49 27.2			
V823 Cas	00 05 42.38	+63 24 14.2			
1SWASP J211253.68+331734.3	21 12 53.69	+33 17 34.3			
ASAS J205850+0854.1	20 58 49.64	+08 54 05.3			
V1553 Sco	16 20 21.77	–35 41 16.0			
V526 Vel	09 03 13.34	–52 02 28.7			

(almost 3 times compared to the works of Poretti *et al.*) we can actually note that the greatest number of stars lie along the path of the fit line. This evidence is also highlighted by looking at the ratio residuals compared to the best-fit linear regression (the computed equation is Y = –0.084809X +0.782048) as presented in Figure 3.

In this case even a second and third polynomial fit of the residuals (red and blue lines) shows a substantial absence of trend, suggesting that a linear interpretation of the relation is possible, a fact that argues for a more accurate revision of HADS stellar models than so far proposed in the literature.

From a purely theoretical point of view I suggest this interpretion: in short period stars the metallicity could vary greatly simply because double mode pulsators with such short periods are, for example, characteristic of double-mode SX Phe stars, characterized precisely by low metallicity as explained in McNamara (2000). In other words the area with period shorter than 0.1 day is probably a transition area with stars of population I and II mixed together, and thus the stellar parameters are less homogeneous than in typical double-mode HADS. This could result in a stronger scattering that does not, however, affect the linear relation suggested.

Finally, the two stars with the highest residuals in the diagram relative to period > 0.1 day (ASAS J192227-5622.5 and VX Hya) could be stars for which the pure nature of double-mode HADS should be evaluated more carefully, as was the case for the previously cited V798 Cyg. For example, VX Hya was involved in a careful analysis by an AAVSO campaign in 2006 and 2007 and the data obtained (Templeton *et al.* 2009) showed that it is certainly an HADS(B) but characterized by unusual and not fully explained long-term amplitude variations.

The linear relationship proposed in this paper could then more easily show which stars belong to a pure type HADS(B): the presence of unusual peculiarity immediately puts the star clearly outside the best-fit line.

Of course the presence of a greater number of stars identifiable as Double-Mode HADS could significantly improve the results of this work in determining the correct parameters of this possible linear relation. I believe that much work can be done from the large amount of data collected from large

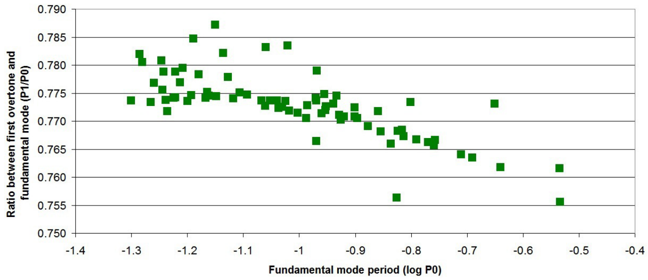

Figure 1. Double Mode HADS Petersen diagram with fundamental mode period expressed as log P0.

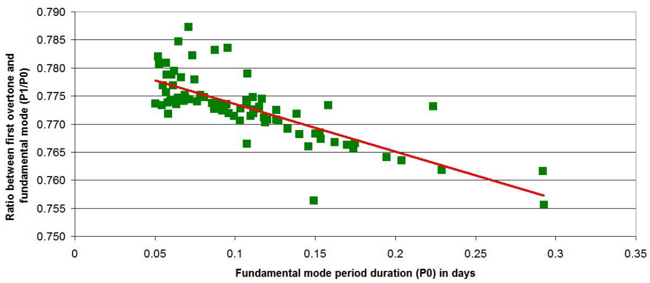

Figure 2. Double Mode HADS Petersen diagram with fundamental mode period expressed in days. The red line represent the best-fit linear regression.

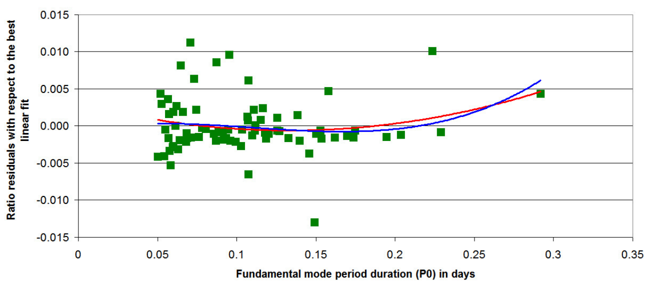

Figure 3. P1/P0 ratio residuals for the best-fit linear regression vs period duration. The red and blue lines represent, respectively, a second and third degree polynomial fit showing no relevant residual trends of the data.

photometric surveys of recent years (OGLE, SuperWASP, ASAS, and so on) and from the data of exceptional quality obtained from the Kepler satellite. This can certainly be a stimulus for new and extensive works.

3. Fit line parameters and statistical correlation evidence

The calculated equation for the best-fit line presented in Figure 2 is:

$$Y = -0.084809\ (\pm 0.008298)X + 0.782048\ (\pm 0.000995) \quad (1)$$

with an RMS error = 0.003765 and a correlation coefficient = 0.762926.

4. Acknowledgements

This work has made use of the International Variable Star Index (VSX; Watson *et al.* 2014) operated by the AAVSO, Cambridge, Massachusetts, USA. The best fit line and relative errors were calculated with the Nonlinear Least Squares Regression (Curve Fitter) courteously provided by Prof. John C. Pezzullo at http://statpages.org/nonlin.html.

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