The Long-term Period Changes and Evolution of V1, a W Virginis Star in the Globular Cluster M12

Pradip Karmakar
Department of Mathematics, Madhyamgram High School (H.S.), Madhyamgram, Sodepur Road, Kolkata 700129, India; pradipkarmakar39@gmail.com

Horace A. Smith
Department of Physics and Astronomy, Michigan State University, East Lansing, MI 48824; smith@pa.msu.edu

Nathan De Lee
Department of Physics, Geology, and Engineering Technology, Northern Kentucky University, Highland Heights, KY 41099; deleenm@nku.edu

Received May 29, 2019; revised June 19, July 8, 2019; accepted July 10, 2019

Abstract We present new *B*, *V*, and *I* band photometry of the W Virginis-type variable star V1 in the globular cluster M12. Observations made from 1916 through 2018 show that during this interval the period of V1 has not shown a constant rate of period change, as might be expected were evolution alone responsible for the period changes. It has, however, shown period changes that appear to have a more abrupt character, probably both increases and decreases.

1. Introduction

Type II Cepheids are believed to be evolved low-mass pulsating variable stars, located within the instability strip at a level brighter than the RR Lyrae stars (e.g. Percy 2007; Catelan and Smith 2015). Type II Cepheids are often divided into two subgroups, depending upon their period. Those with periods shorter than 4 to 8 days are often termed BL Her stars, whereas those of longer period are often denoted W Virginis stars. Metal-poor BL Her variables are believed to be stars that have evolved from the blue horizontal branch, and which are now passing through the instability strip heading toward the asymptotic red giant branch (see, for example, Neilson *et al.* 2016; Osborn *et al.* 2019). The evolution of W Virginis stars poses greater difficulties. W Virginis stars have been sometimes considered to be stars undergoing thermal pulse instabilities that cause them to loop to the blue from the asymptotic giant branch. As they loop to the blue, they enter the instability strip at luminosities brighter than those of BL Her variables. However, some recent theoretical calculations indicate that thermal pulses are not sufficient to create such blue loops (Bono *et al.* 2016), opening once more the question of the nature of such stars. Studies of the long-term period changes of W Virignis stars have the potential to shed light upon this perplexing problem. The periods of pulsating stars are often known to greater accuracy than any other property, and changes in pulsation period may reveal the direction and speed of stellar evolution through the instability strip. If W Virginis stars are undergoing loops into the instability strip, the pulsation equation tells is that we should expect to detect some stars with increasing periods (those on the redward portion of the loop) and some with decreasing periods (those on the blueward part of the loop). The rates of period change should be consistent with the theoretical predictions of the durations of blue loops.

V1 in the globular cluster M12 (NGC 6218) was discovered by Sawyer (1938a), and, with a period of 15.5 days, it is classified as a W Virginis star. Clement *et al.* (1988) used photographic observations to study its period between 1916 and 1985. They concluded that during this interval the pulsation period of V1 underwent both increases and decreases. In this paper we examine the light curve and period of V1 using observations obtained between 2002 and 2018, with the goal of detecting any long-term period change that might be attributed to evolution.

2. The observations and data set

We obtained new *B*, *V*, and Cousins *I* band images of M12 between 2006 and 2011 using the 0.6-m telescope of the Michigan State University campus observatory with an Apogee Alta U47 CCD camera (0.6 arc-second pixel, 10×10 arcmin field of view). Bias and dark images were subtracted in the conventional way, and twilight images were used as flat field images. Exposures were about 1 minute long, varying somewhat with sky conditions.

As noted by Sawyer (1938b) and Clement *et al.* (1988), photometry of V1 is made difficult by the presence of a neighboring star with a blue photographic magnitude near 14.0. Taking advantage of excellent seeing, Klochkova *et al.* (2003) were able to analyze the spectra both of V1 and its close companion, although they found the companion to be less than an arc second from V1. They found the companion star to have an effective temperature of 4200 K, making it considerably cooler than V1. As the seeing at the Michigan State University observatory is typically 3 or 4 arc seconds, the companion is deeply imbedded within the image of the brighter V1. The companion is also blended with V1 in the ASAS-SN data, which we discuss below.

We considered which of three methods of reduction might be best under this circumstance: aperture photometry, profile fitting photometry, and image differencing photometry. We decided against aperture photometry because the crowded nature of the M12 field means that V1 has other neighbors, somewhat more distant than the close companion, but near enough to fall within any large aperture or sky annulus. Although profile fitting photometric routines, such as DAOPHOT/ALLSTAR (Stetson 1987, 1994), can mitigate the effect of blends, in this case the blended companion was too deeply within the profile of V1 for that mitigation to be successful. Nor did image differencing (Alard and Lupton 1998) succeed in separating V1 from its close companion. In the end, we decided to proceed with profile fitting photometry using the DAOPHOT and ALLSTAR routines as in Rabidoux *et al.* (2010). Because of the near neighbor we will, however, emphasize period determination for V1 rather than details of the light curve in this paper.

Instrumental magnitudes obtained from DAOPHOT were transformed to the standard system as in Rabidoux *et al.* (2010), applying color terms as in equations 1, 2, and 3 of that paper. We used seven uncrowded local standards with APASS magnitudes (data release 10; Henden *et al.* 2018) to set the magnitude zero-points for *B* and *V*. APASS provides Sloan i' magnitudes, whereas we used a Cousins *I*-band filter. Because of this, and because of the unknown effect of the close companion on the *I* photometry, we could not use the APASS stars to calibrate our *I*-band photometry. Peter Stetson has created local standard stars in and near M12, which can be found at the website: http://www.cadc-ccda.hia-iha.nrc-cnrc.gc.ca/en/community/STETSON/standards/. Although most of the Stetson standards are too faint and too crowded for our use, a few are sufficiently bright and uncrowded to provide a check on our calibration. Because the Stetson standards include Cousins I-band photometry, we used three uncrowded Stetson local standards to set the *I* calibration. However, the circumstance that the star blended with V1 is cooler than V1 means that it will cause our *I* photometry to be too bright by an even greater amount than in *B* or *V*. Thus, particular caution attaches to any use of the *I* light curve. The Michigan State University (MSU) CCD photometry for V1 is listed in Table 1.

ASAS-SN observations (Shappee *et al.* 2014; Kochanek *et al.* 2017) of V1 were downloaded from the Sky Patrol option on the ASAS-SN webpage, using the position for V1 from the Clement *et al.* (2001) catalogue of variable stars in globular clusters. Only *V*-band observations are available for most of the 2012–2018 time period, though *g*-band data are more recently available. Because the *g* data gave period results identical to those from *V*, we include only our analyses for the *V* periods in Table 2. Few ASAS-SN observations of V1 for 2012 are available, but the number increases in later years. The large ASAS-SN pixels mean, however, that the 1,090 *V* data points that we found useful show the effects of blending even more seriously than is the case for the MSU CCD observations. Thus, the ASAS-SN observations are valuable mainly for the investigation of period changes, but they are very important for that purpose.

A search was made for additional observations of V1 in the Harvard DASCH photometry (http://dasch.rc.fas.harvard.edu/project.php), but no useful observations of V1 were found.

3. Period determinations for V1

We have used two period-finding routines to search for periodicities in the V1 data, PERIOD04 (Lenz and Berger 2005) and a date-compensated discrete Fourier transform, as implemented in PERANSO 2.0 (Vanmunster 2006). Period searches were carried out, with the results shown in Table 2. The searches were carried out for the MSU *B* and *V* data, for infrared *K*-band observations of V1 by Matsunaga *et al.* (2006), and for ASAS-SN *V* observations. The column headed N(obs) indicates the number of observations.

The primary periods found by PERIOD04 and PERANSO 2.0 agree well. For PERIOD04, the listed uncertainties derive from the least squares fitting routine. For the PERANSO 2.0 results, uncertainties depend upon the noise in the amplitude spectrum, which we estimated independently of the default values in the PERANSO routine. Clement *et al.* (1988) derived the periods at the beginning of their Table IV from their phase shift diagram for V1, but gave no uncertainties for the derived periods. The N(obs) values in Table 2 for Clement *et al.* (1988) do not double-count the photographic observations but, of course, since Clement *et al.* (1988) derived periods from phase shifts, they actually use observations from more than one time interval in deriving periods. As a check on the Clement *et al.* (1988) periods, and to verify that our period determinations are consistent with theirs, we reanalyzed the Clement *et al.* (1988) data using PERIOD04 and PERANSO 2.0. However, only three of the data subsets used by Clement *et al.* (1988) contained enough observations and were sufficiently free of cycle-count uncertainties to permit the application of our techniques. We have included our results directly beneath the Clement *et al.* (1988) periods in Table 2. The two approaches generally agree to within the uncertainties. The referee was concerned by the large 1916–1938 interval for which Clement *et al.* (1988) derived their initial period. We therefore selected a subset of those data covering just the 1931–1938 interval for an additional period search, with the result shown in Table 2.

W Virginis itself is known to show multiple periods (Templeton and Henden 2007). The MSU CCD observations are too sparse to effectively search for a second, weaker, period but the 2013–2018 ASAS-SN V-band observations were prewhitened in PERIOD04 by removing the main frequency and four higher harmonics. A period search was conducted on the residuals from this prewhitening. Although a weak signal was found for a period of 15.296 days (full amplitude of 0.02 mag compared to 0.18 mag for the 15.544-day period), we do not regard this detection as significant. The supposed secondary period and the 15.544-day period would beat with an interval about as long as the time interval of the entire ASAS-SN dataset. To illustrate the ASAS-SN period search results, we show the PERIOD04 amplitude spectrum for the ASAS-SN 2013–2014 V-data in Figure 1. The peaks on either side of the main peak are one cycle per year aliases of the main peak. We find no significant secondary period.

The MSU *B*, *V*, and Cousins *I* phased light curves for V1 are shown in Figures 2–4, with approximate fits to guide the eye. Fits were made by applying the routine from Kovács and Kupi (2007). The ASAS-SN 2013–2018 *V* light curve for V1 is shown in Figure 5.

Table 1. The MSU CCD Photometry.

B observations			*V observations*			*I observations*		
HJD	*B(V1)*	*Error*	*HJD*	*V(V1)*	*Error*	*HJD*	*I(V1)*	*Error*
2453892.6375	12.730	0.03	2453892.6402	11.318	0.02	2455714.6071	10.391	0.04
2453895.6427	13.051	0.03	2453895.6457	11.609	0.02	2455714.6346	10.398	0.04
2453903.6287	12.248	0.02	2453906.6322	11.174	0.02	2453892.6354	10.240	0.04
2453906.6298	12.461	0.02	2453907.6292	11.237	0.02	2453895.6395	10.462	0.04
2453907.6268	12.642	0.03	2453910.6360	11.580	0.03	2453895.6409	10.467	0.04
2453910.6383	13.055	0.04	2453935.6369	11.050	0.02	2453899.6313	10.486	0.04
2453935.6394	12.279	0.03	2453936.6407	11.092	0.02	2455726.6587	10.560	0.04
2453936.6420	12.406	0.03	2454360.5477	11.672	0.03	2455727.6430	10.600	0.04
2453937.6245	12.480	0.03	2455706.6150	11.583	0.02	2453903.6274	10.177	0.04
2453943.6165	12.911	0.03	2455714.6104	11.306	0.02	2453935.6435	10.190	0.04
2454357.5254	12.941	0.04	2455714.6373	11.284	0.02	2453906.6273	10.072	0.04
2454360.5476	13.016	0.04	2455714.6392	11.251	0.02	2453906.6284	10.101	0.04
2454649.6132	12.476	0.02	2455718.6231	11.339	0.02	2455739.6460	10.532	0.04
2454653.6415	12.980	0.03	2455726.6506	11.648	0.02	2455741.6244	10.590	0.04
2455706.6114	13.013	0.03	2455726.6548	11.636	0.02	2453937.6269	10.199	0.05
2455714.6086	12.460	0.03	2455727.6387	11.524	0.02	2453943.6192	10.614	0.04
2455714.6356	12.529	0.03	2455739.6406	11.803	0.03	2453943.6205	10.555	0.04
2455718.6213	12.626	0.03	2455741.6154	11.666	0.02	2455749.6379	10.191	0.04
2455726.6568	12.881	0.03	2455749.6324	11.244	0.02	2454357.5276	10.067	0.04
2455727.6409	12.684	0.03				2454360.5495	10.415	0.04
2455739.6441	13.148	0.04				2453910.6357	10.316	0.04
2455741.6225	12.886	0.03				2454649.6072	10.113	0.04
2455749.6360	12.506	0.02				2455706.6040	10.386	0.04

Table 2. Period determinations for V1 for the years 1916–2018.

Years	*period*	*Uncertainty*	*N(obs)*	*Reference*
1916–1938	15.50	—	56	Clement *et al.* 1988
1916–1938	15.51	0.01	56	PERANSO result for Clement *et al.* 1988 data
1916–1938	15.47	0.01	56	PERIOD04 result for Clement *et al.* 1988 data
1931–1938	15.50	0.02	43	PERANSO result for Clement *et al.* 1988 data
1931–1938	15.50	0.01	43	PERIOD04 result for Clement *et al.* 1988 data
1938–1946	15.55	—	20	Clement *et al.* 1988
1938–1946	15.54	0.02	20	PERANSO result for Clement *et al.* 1988 data
1938–1946	15.53	0.02	20	PERIOD04 result for Clement *et al.* 1988 data
1946–1962	15.51	—	19	Clement *et al.* 1988
1962–1970	15.54	—	19	Clement *et al.* 1988
1970–1975	15.51	—	40	Clement *et al.* 1988
1970–1975	15.51	0.03	40	PERANSO result for Clement *et al.* 1988 data
1970–1975	15.52	0.02	40	PERIOD04 result for Clement *et al.* 1988 data
1975–1985	15.57 (or 15.49)	—	14	Clement *et al.* 1988
2002–2005	15.47	0.03	19	PERANSO result for Matsugana *et al.* 2006 data
2002–2005	15.47	0.01	19	PERIOD04 result for Matsunaga *et al.* 2006 data
2006–2011	15.486	0.02	23	PERANSO result for MSU B data
2006–2011	15.484	0.006	23	PERIOD04 result for MSU B data
2006–2011	15.487	0.02	19	PERANSO result for MSU V data
2006–2011	15.489	0.009	19	PERIOD04 result for MSU V data
2013–2018	15.544	0.008	1059	PERANSO result for ASAS-SN data
2013–2018	15.542	0.001	1059	PERIOD04 result for ASAS-SN data
2012	15.623	0.30	14	PERANSO result for ASAS-SN data
2012	15.623	0.12	14	PERIOD04 result for ASAS-SN data
2013	15.597	0.19	63	PERANSO result for ASAS-SN data
2013	15.597	0.12	63	PERIOD04 result for ASAS-SN data
2014	15.606	0.016	140	PERANSO result for ASAS-SN data
2014	15.606	0.009	140	PERIOD04 result for ASAS-SN data
2015	15.555	0.015	225	PERANSO result for ASAS-SN data
2015	15.555	0.008	225	PERIOD04 result for ASAS-SN data
2016	15.575	0.014	218	PERANSO result for ASAS-SN data
2016	15.575	0.008	218	PERIOD04 result for ASAS-SN data
2017	15.504	0.013	267	PERANSO result for ASAS-SN data
2017	15.495	0.008	267	PERIOD04 result for ASAS-SN data
2018	15.566	0.016	163	PERANSO result for ASAS-SN data
2018	15.565	0.011	163	PERIOD04 result for ASAS-SN data

Clement *et al.* (1988) divided their photographic photometry of V1 into discrete groups, comparing the light curves for those groups to the light curve they obtained from observations obtained in 1970 to derive the phase-shift diagram shown in their Figure 2. A mean period of 15.527 days was used in their analysis. It was from their phase shift diagram that they determined the periods in their Table IV. However, they ran into an ambiguity in interpreting the phase-shift diagram of observations obtained *in 1985*, which showed a very large jump in phase. Should that point be plotted at a phase shift of +0.48 or –0.52? They chose to just plot the +0.48 value in Figure 2, but they noted that, depending upon which choice of phase shift was made, the resultant period could be either 15.57 or 15.49 days. Because the 1975–1985 observations are composed of two sets of observations separated by a long gap in time, a direct period determination for this interval does not resolve the ambiguity, but fits both of the Clement *et al.* (1988) alternatives.

We attempted to add our MSU *B* and *V* observations, as well as the annual ASAS-SN observations between 2014 and 2018 to the Clement *et al.* (1988) Figure 2 diagram (ASAS-SN data for 2012 and 2013 being fewer in number). We determined a phase shift for the recent observations by comparing them to the Clement *et al.* (1988) 1970 light curve, following the procedure they adopted. The 1970 curve has significant gaps, however, introducing some uncertainty in the size of the shift. In deriving these shifts, we scaled our light curves in amplitude and applied zero-point shifts to better match the Clement *et al.* (1988) photographic observations. Infrared data were not included in this comparison because the time of maximum in *I* and longer wavelength bands can show significant shifts with respect to *B* or *V* (see, for example, Osborn *et al.* 2019). Our phase shifts are shown in Table 3, where the reference times are the approximate mid-points of each set of observations. In interpreting our results, we quickly ran into a problem with deciding upon the correct cycle-count between observed epochs.

In Figure 6, we plot the phase shifts from Clement *et al.* (1988) as filled points, with the exception of the point for their 1985 data, which is plotted twice, once with at +0.48 (filled point) and again at –0.52 (cross). We then plotted two alternatives for the more recent data, with the crosses shifted one cycle lower in phase.

How do we tell which, if either, alternative in Figure 6 correctly plots the phase differences between the 1970 light curve and the later observations? At first, one might think that the crosses indicate that all except the 1985 point could be fit by a period near 15.527 days. However, that is not the case. A period near 15.527 days fits neither the MSU *B* and *V* light curves nor the ASAS-SN data, as was found in our period searches. It is apparent that gaps in time coverage and the jumps in phase shown in Figure 6 are too large for us to determine recent periods from the phase diagram alone. The data are continuous enough, however, for the restricted time interval covered by the ASAS-SN data for the phase shifts for those observations alone to yield a period of 15.543 days, consistent with the direct period determinations in Table 2.

4. Conclusions

What can we conclude from both the phase diagram and the direct period determinations? As shown in Figure 7, an increase in period to 15.54 days in the 2013–2018 time interval from a period near *15.48 days in 2002–2011 seems certain from the direct period determinations*. This would thus appear to be a relatively abrupt period increase. In their Table IV, Clement *et al.* (1988) used their phase-shift diagram to determine periods for V1 between 15.50 and 15.57 days (or 15.49 and 15.55 days, depending upon the interpretation of the 1975–1985 data). Can we establish the reality of the Clement *et al.* (1988) period jumps independently of their phase-shift diagram? The answer is yes but to a limited degree. For the 1938–1946 time interval, our PERANSO and PERIOD04 period determinations of 15.53 and 15.54 days are in reasonable agreement with the Clement *et al.* (1988) value of 15.55 days. For the 1970–1975 data, our periods of 15.51 or 15.52 days agree with the Clement *et al.* (1988) value of 15.51 days. It remains true, however, that most of the periods in Table IV of Clement *et al.* (1988) depend mainly upon the interpretation of the phase-diagram in their Figure 2. It is nonetheless not possible to revise the cycle-counting used in creating their Figure 2 without introducing alternative jumps in period. We therefore conclude that the period of V1 was not constant nor did it change at a constant rate between 1916 and 2018. We consider the Clement *et al.* (1988) interpretation that V1 has undergone both increases and decreases in period to be likely.

The referee asked whether the simpler interpretation of a period slightly smaller than 15.527 days might approximately explain all of the phase shift points after about 1930, except for that of the 1985 observations, assuming the lower crosses to be correct for the post-2000 observations. The answer would be yes, if the phase shift points were all that needed to be considered. However, as noted above, we see problems with such an interpretation. A period of 15.527 days or smaller produces a light curve for the ASAS-SN observations with much more scatter than the 15.542 day period. The direct period determinations in Table 2 for the post-2000 observations indicate periods near either 15.48 days (for the Matsunaga et al. 2006 and MSU CCD data) or 15.54 days (for the ASAS-SN 2013–2018 data). The average is 15.51 days, but the average period would not produce the best light curve for any of the post-2000 datasets. Moreover, with a gap of some two decades between the Clement *et al.* (1988) 1985 data and the MSU CCD observations, periods of 15.48 or 15.54 days would lead to different cycle counts across the gap. We thus prefer not to rely upon the phase-shift diagram in deriving periods for this interval, placing greater trust in the period determinations listed in Table 2.

Instead of finding that V1 in M12 underwent a constant rate of increase or decrease in period over the past century, as might have been expected from stellar evolution theory, our preferred interpretation is that its period changed in a more variable way. It fluctuated in period between about 15.47 and 15.57 days. If there is any long-term evolutionary period change happening in V1, it is effectively masked by more abrupt jumps in period.

Not all W Virginis variables show period fluctuations similar to those of V1. For example, Templeton and Henden

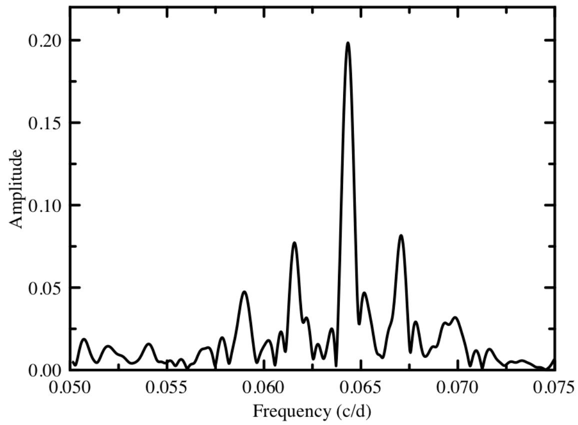

Figure 1. The amplitude spectrum from the PERIOD04 period search of the ASAS-SN data.

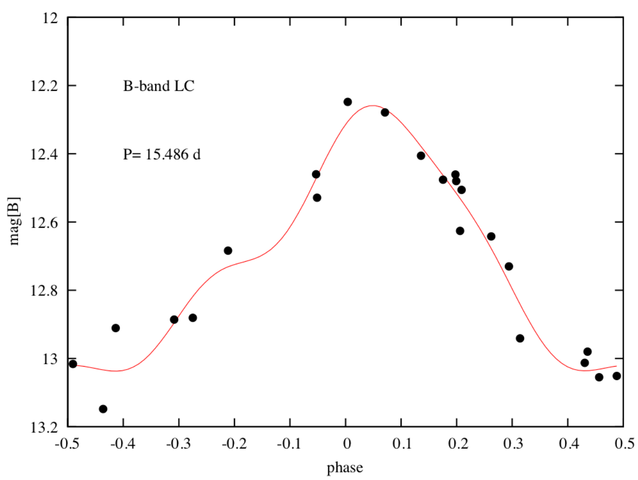

Figure 2. The light curve of V1 from the MSU *B* data, phased with a period of 15.486 days.

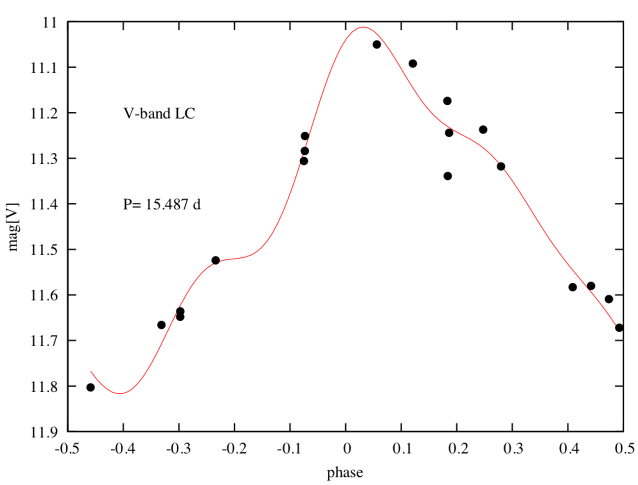

Figure 3. The light curve of V1 from the MSU *V* data, phased with a period of 15.487 days.

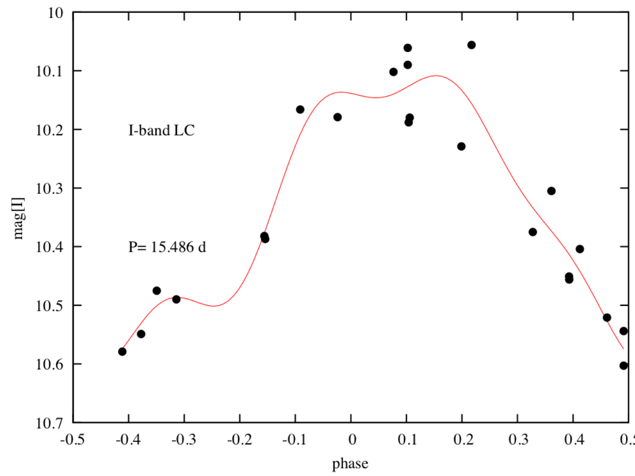

Figure 4. The light curve of V1 from the MSU *I* data, phased with a period of 15.486 days.

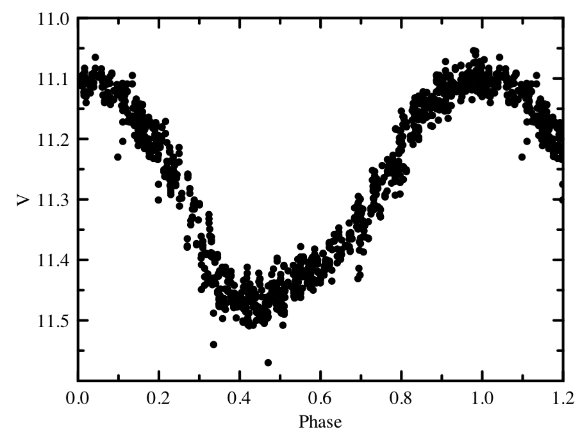

Figure 5. The light curve of V1 from the 2013–2018 ASAS-SN *V* observations, phased with a period of 15.542 days.

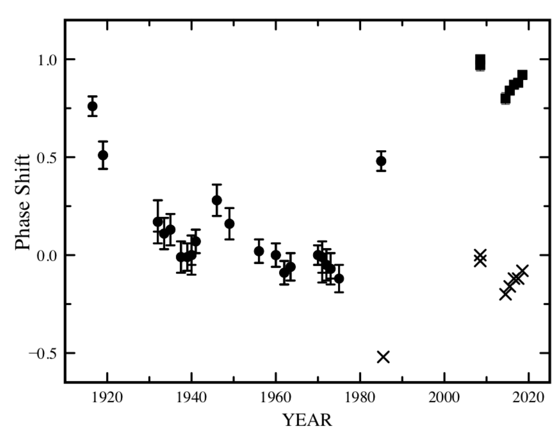

Figure 6. Recent observations are added to the phase-shift diagram of Clement *et al.* (1988). This cannot be done unambiguously, and alternative phase shifts are presented as squares and crosses. The two alternatives for the 1985 Clement *et al.* data are plotted as a circle and a cross.

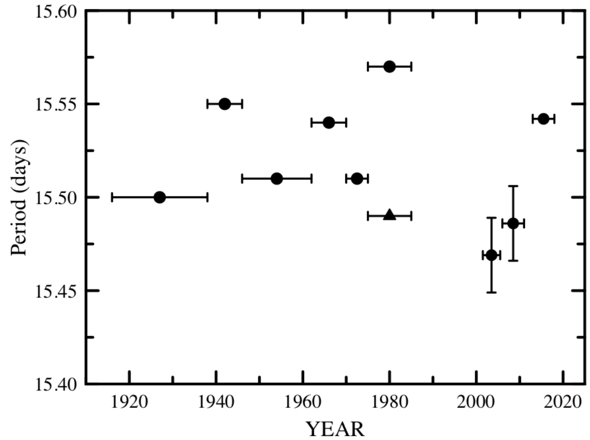

Figure 7. The changing period of V1 over time, from Clement *et al.* (1988) and from this paper. The triangle is an alternative period from Clement *et al.* (1988) for their 1975–1985 data. The Clement *et al.* (1988) periods are derived from phase shifts, and do not have associated error bars, but see Table 2.

Table 3. Phase Shift determinations for V1 for the years 2008–2018.

Epoch	*Phase Shift*	*Uncertainty*	*Dataset*
2008.5	–0.03	0.05	MSU V
2008.5	0.00	0.05	MSU B
2014.5	0.80	0.04	ASAS-SN
2015.5	0.84	0.04	ASAS-SN
2016.5	0.87	0.04	ASAS-SN
2017.5	0.88	0.04	ASAS-SN
2018.5	0.92	0.04	ASAS-SN

(2007) found a long-term period decrease for W Virginis itself. RV Tauri variables have been observed to show random changes of period (e.g. Percy and Coffey 2005), which can be superposed upon long-term period changes. However, RV Tauri behavior is typically seen at longer periods than the 15.5 days of V1. It may be noted that the long-term period changes of relatively few W Virginis variables have been studied. The relatively small number of W Virginis variables with long-term period studies, and the variety of period change behavior observed among those stars that have been studied (Rabidoux *et al.* 2010; Neilson *et al.* 2016) are an encouragement to further study the periods of variables such as V1.

5. Acknowledgements

We thank Charles Kuehn, who obtained three of the CCD images used in this study. We thank Christine Clement for her help and advice. We thank the referee for constructive advice that led us to make significant revisions in this paper.

References

Alard, C., and Lupton, R. H. 1998, *Astrophys. J.*, **503**, 325.

Bono, G. *et al.* 2016, *Mem. Soc. Astron. Ital.*, **87**, 358.

Catelan, M., and Smith, H. A. 2015, *Pulsating Stars*, Wiley-VCH, Weinheim, Germany, 183.

Clement, C. M., Hogg, H. S., and Yee, A. 1988, *Astron. J.*, **96**, 1642.

Clement, C. M., *et al.* 2001, *Astron. J.*, **122**, 2587.

Henden, A., Levine, S.,; Terrell, D., Welch, D. L., Munari, U., and Kloppenborg, B. K. 2018, *Amer. Astron. Soc.*, AAS Meeting No. 232, i.d. 223.06.

Klochkova, V. G., Panchuk, V. E., Tavolganskaya, N. S., and Kovtyukh, V. V. 2003, *Astron. Lett.*, **29**, 748.

Kochanek, C. S., *et al.* 2017, *Publ. Astron. Soc. Pacific*, **129**, 104502.

Kovẃacs, G., and Kupi, G. 2007, *Astron. Astrophys.*, **462**, 1007.

Lenz, P., and Breger, M. 2005, *Commun. Asteroseismology*, **146**, 53.

Matsunaga, N., *et al.* 2006, *Mon. Not. Roy. Astron. Soc.*, **370**, 1979.

Neilson, H. R., Percy, J. R., and Smith, H. A. 2016, *J. Amer. Assoc. Var. Star Obs.*, **44**, 179.

Osborn, W., *et al.* 2019, *Acta Astron.*, in press.

Percy, J. R. 2007, *Understanding Variable Stars*, Cambridge University Press, Cambridge.

Percy, J. R., and Coffey, J. 2005, *J. Amer. Assoc. Var. Star Obs.*, **33**, 193.

Rabidoux, K., *et al.* 2010, *Astron. J.*, **139**, 2300.

Sawyer, H. B. 1938a, *Publ. Dom. Astrophys. Obs.*, **7**, 121.

Sawyer, H. B. 1938b, *Publ. David Dunlap Obs.*, **1**, 57.

Shappee, B. J., *et al.* 2014, *Astrophys. J.*, **788**, 48.

Stetson, P. B. 1987, Publ. Astron. Soc. Pacific, **99**, 191.

Stetson, P. B. 1994, in *Astronomy with the CFHT Adaptive Optics Bonnette*, ed. R. Arsenault, Canada-France-Hawaii Telescope Corp., Kamuela, HI, 72.

Templeton, M. R., and Henden, A. A. 2007, *Astron. J.*, **134**, 1999.

Vanmunster, T. 2006, light curve and period analysis software PERANSO 2.0, CBA Belgium (http://www.cbabelgium.com/peranso).