Light Curve Analysis of 33 Pulsating Red Giant Stars

Kate M. Blackham

Swinburne University of Technology, John Street, Hawthorn, Victoria, Australia 3122; 100627441@student.swin.edu.au

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Abstract This project sought to use the AAVSO VSTAR software package to confirm the classification and periodicity of 33 pulsating red giant stars listed within the AAVSO International Database and look for changes in their periodicity over time. V826 Cas and V451 Cep were found to be semiregular with periods of 419 and 385.4 days, respectively. There was no evidence that GO Peg is semiregular; the data are consistent with its being an irregular star. BR CVn has a period of 860 days. V854 Cas, TU CVn, and μ Cep show variations to their periodicity that, while within the few percentage wanderings typical of these stars, may be the beginnings of longer-term changes and warrant further investigation.

1. Introduction

Stars that have exhausted the hydrogen in their cores evolve off the main sequence. Red giants burn the hydrogen found in a shell around their cores, and their outer layers cool and expand. Later, the star begins to burn helium, first in its core and later in shells around that core. The presence of a helium-burning shell within a star causes its surface to expand and cool again. Stars with helium-burning shells are found in a region asymptotic to the red giant branch of the Hertzsprung–Russell (H–R) diagram, known as the asymptotic giant branch (AGB).

Many pulsating stars are found in the 600–1100 K wide instability strip that includes the Cepheid and RR Lyrae variables, but stars at many other locations on the H–R diagram can also become variable. It was suggested at the beginning of the last century that Cepheid and RR Lyrae stars might behave as thermodynamic heat engines and this would drive pulsations within their atmospheres, leading to variability in the light observed on Earth (Shapley 1914; Eddington 1918, 1919). However, the reason for the variability in the pulsating red giant, AGB, and supergiant star populations that exist outside the instability strip is not fully understood and therefore worth further investigation. One possible theory is that oscillations in semiregular and OGLE small amplitude red giant (OSARG) variables are broadly similar to the oscillations of Sun-like stars, which display stochastic excitation due to convection near to their surfaces; red giants with their increased luminosity and convection strength would therefore have a much increased oscillation amplitude (Christensen-Dalsgaard *et al.* 2001; Takayama *et al.* 2013). A competing theory is that highly luminous red giants act like Miras: their convection and oscillations are coupled in a complex manner that causes the turbulence to act at one time as a damping effect and later as a cause of excitation (Xiong and Deng 2013; Xiong *et al.* 2018).

Following a convention used by Percy (2007) all pulsating red giant, AGB, and supergiant stars are referred to as pulsating red giants (PRG) when discussed as a group within this project, although PRG is not a classification that occurs in the *General Catalogue of Variable Stars* (GCVS; Samus *et al.* 2017). This is not without justification; the classification of stars into groups such as Mira, semiregular, and irregular is arbitrary and there is no discontinuity as you cross between groups of the physical properties of period, amplitude, and degree of periodicity. The distinction between semiregular stars and irregular is particularly ill-defined. Semiregular stars, by their nature, display less predictable periodicity than other pulsating variables and SRB stars in particular may enter periods of irregular pulsation. The irregular category is therefore a mixture of truly irregular stars with no discernible periodicity and other variable stars for which observations have not yet been made in the quantity and over sufficient time to detect their true category.

PRGs, like all stars, evolve and change over time. The evolution from one type to another is rarely obvious over the course of a human lifetime, but with records for some stars dating back over 100 years (Mattei 1997; Henden 2013; AAVSO 2018) it is possible to identify changes that take place over many decades. For example, one hundred years' worth of data on V725 Sagittarii was sufficient for Henrietta Swope to identify that it had evolved from a type II Cepheid to a semiregular PRG (Swope 1937; Percy 2010, 2020). However, sometimes radical changes are obvious within a lifespan; T UMi was considered to be a Mira until around 1970 when its period began to change (Templeton *et al.* 2005) and it now displays an interesting and complicated periodicity, with periods of about 201 days and 109 days that are probably its fundamental and first overtone modes, respectively (Molnár *et al.* 2019).

While changing between one variable star classification and another requires significant changes to the physical properties of a star, more subtle changes also occur. The periods of PRGs can "wander," that is, increase or decrease by several percent over the course of several decades (Eddington and Plakidis 1929). The physical processes that are involved in PRGs' periods and their variation are poorly understood and since their periods last months to years, to stand any chance of detecting long-term changes to a PRG's period, amplitude, or mean magnitude, it is important to analyze long-term data that have been collected over many decades.

2. Project aims

The project was undertaken as part of a master's degree in astronomy and sought to investigate the long-term data held in the AAVSO's International Database, the AID (AAVSO 2013), on a group of 33 PRGs. All 33 PRGs were included in North (2004) as being suitable for amateur observation.

An initial list of 15 stars was explicitly chosen because they are poorly observed stars and of questionable classification. Four were thought to be Miras (V2582 Oph, V409 Per, V418 Cas, and V854 Cas), five were thought to be semiregular (EV Aqr, W Boo, RY Dra, UX Dra, and V336 Vul), and six were thought to be irregular (V826 Cas, V451 Cep, T Cyg, RU Crt, NSV 12441, and NSV 24346). However, the paucity of data meant that these were far from certain and North (2004) suggested that observers adopt them into their observing plans.

The remaining 17 PRGs were all well observed with many decades of observations held in the AID. They were selected from North's lists of 134 Miras, 104 semiregulars, and 11 irregular stars, excluding those stars for which similar studies (e.g. Percy and Qiu 2019) had already investigated any possible long-term changes.

No doubt thanks in part to North's plea, there was a noticeable increase in observations for the 16 questionably classified stars post-2004. While there are often insufficient data on these stars to observe any long-term changes, these stars were more likely to have been misclassified in 2004 and a reassessment of the properties of their light curves was therefore important.

For each star, an attempt was made to ascertain each star's period of variability or, in the case of irregular stars, verify that they do indeed display no periodicity. It is common for newly discovered or poorly observed stars to be misidentified due to lack of evidence. Stars originally believed to be irregular may be semiregular and vice versa. Confirmation of the classification of each PRG from its light curve was also attempted.

3. Equipment

3.1. AID

The American Association of Variable Star Observers (AAVSO) holds more than 42,000,000 variable star observations from over 5,000 observers dating back, for some stars, more than one hundred years in its AAVSO International Database, the AID (AAVSO 2013). The AID includes observations not just from AAVSO members, but from other amateur variable star observation groups internationally, including the British Astronomical Association. Hence the AID is the best resource available for analysis of long-term data of variable stars (Kafka 2019).

3.2. SIMBAD

SIMBAD is an astronomical database for over 11 million objects outside of the solar system. It includes basic data (including equatorial and galactic coordinates, proper motions, redshift, parallax, and spectral type), cross-identifiers for each object, and over 375,000 bibliographic references. The SIMBAD database can be queried not only by an object's name or identifier but also by its coordinates. SIMBAD is also linked to a number of other powerful tools including VizieR's 20,035 astronomical catalogues (including the GCVS), the Aladin Sky Atlas, and the CDS xMatch service.

3.3. VSX

The International Variable Star Index (VSX; Watson *et al.* 2014) was originally created using the entire Combined GCVS 4.2 with additional data from the Northern Sky Variability Survey (NSVS), the third All Sky Automated Survey (ASAS), the *Information Bulletin on Variable Stars* (IBVS), and new variable stars discovered by the Optical Gravitational Lensing Experiment (OGLE-II) and the Robotic Optical Transient Search Experiment (ROTSE-I) (VSXweb). A team of moderators update the VSX with the latest peer reviewed information. The page for each variable star holds information on its position, aliases, variability type, spectral type, magnitude range, and period, as well as links to relevant academic references.

3.4. VSTAR

Analysis was performed using the AAVSO's VSTAR data visualisation and analysis tool (Benn 2012). VSTAR is a free, open source application that runs on any operating system that supports JAVA. VSTAR can be used for any time-series data, but its origin as an AAVSO Citizen Sky Project means that it is designed primarily for the analysis of variable stars. The application will accept a wide variety of files as input, including AID files, .csv, .dat, .tsv, and .txt file types, as well as the files released from satellite missions such as Hipparcos and Kepler. VSTAR allows the user to plot light curves and phase diagrams based on the main period specified in the AAVSO's own Variable Star Index (VSX) or a user-supplied value, and can also display the data in table form. Data can be filtered in a variety of ways including by observation band (i.e. visual, Johnson and Cousins filters, etc.) and observer.

Light curves produced in VSTAR can be analyzed using its Fourier transform routine, which produces a power spectrum and table of what are referred to as "top hits." The Fourier transform routine allows users to find the most probable period, but also helps to identify potential secondary periods. Many PRGs exhibit secondary periods, but their cause is not currently known (Wood 2000; Percy and Qiu 2019). If multiple periods are found, the CLEANest algorithm (Foster 1995) can be applied to help create a model.

VSTAR also provides time-frequency analysis of data via an integrated wavelet analysis routine, which allows users to detect any long-term changes in a light curve.

4. Experimental technique

The analysis in this project relies on several algorithms integrated into VSTAR. Although the AID holds records of observations of many types including charge-coupled devices (CCD), to ensure that long-term changes are detected for the most part only visual data are considered in this project.

4.1. DC DFT

Fourier analysis is used to find periodic behavior from data. Assuming the best fit to the data has a sinusoidal shape, you can determine the period by testing trial periods (represented mathematically by frequencies). Then for each trial frequency you can calculate its power, which measures the statistical significance of that fit to the data. The frequency can then be plotted against the power in a graph called a periodogram or power spectrum (Figure 1). The highest peaks are therefore statistically significant and provide clues as to the true period.

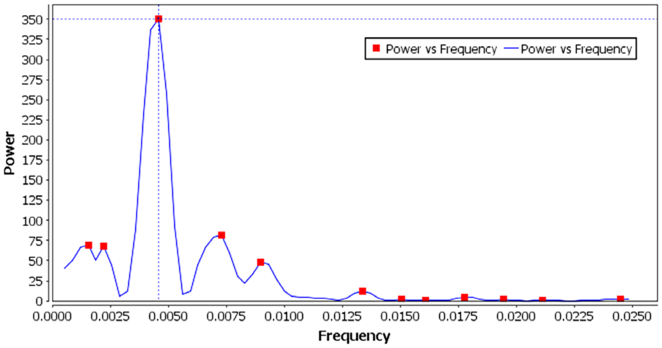

Figure 1. A periodogram for R Boo. Top hits are highlighted in VSTAR by red boxes. Note the statistically significant top hit at the center of the cross-hairs.

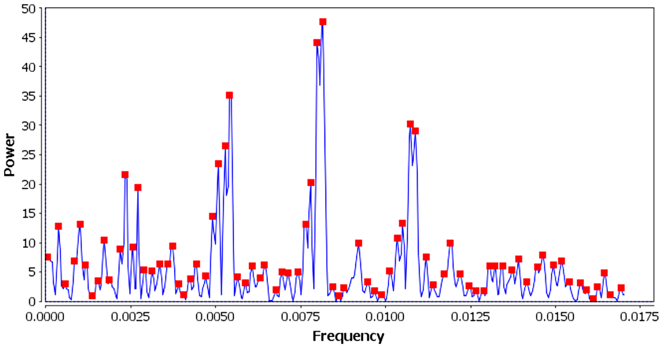

Figure 2. Power spectrum for the SRa star EV Aqr. Alias peaks are located at frequencies of approximately 0.0025, 0.0053, and 0.0107.

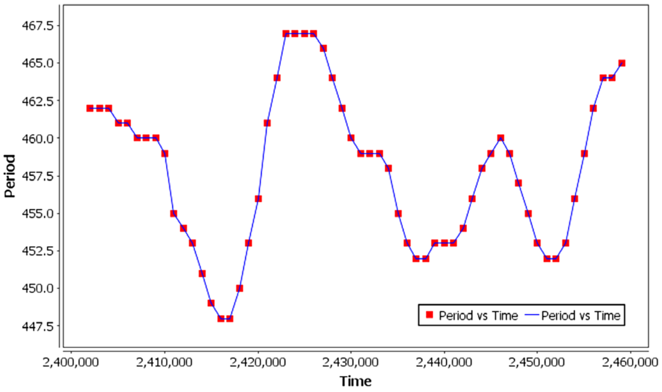

Figure 3. A plot of R Aur's period vs. time shows that its period has wandered within the expected several percent throughout the 154 years for which observations have been recorded.

Other peaks occur for a variety of reasons including the presence of noise. In Figure 1, the top hit represents a period of 219 days.

There are various algorithms that can be used to perform Fourier analysis on a data set, the most widely used being discrete Fourier transform (DFT) and fast Fourier transform (FFT). Both DFT and FFT rely on data being equally spaced with respect to time, an invalid assumption to make with astronomical data. Date-compensated discrete Fourier transform (DC DFT) was designed by Ferraz-Mello (1981) to be a more accurate Fourier transform method with data sets that have uneven spacing of observation dates.

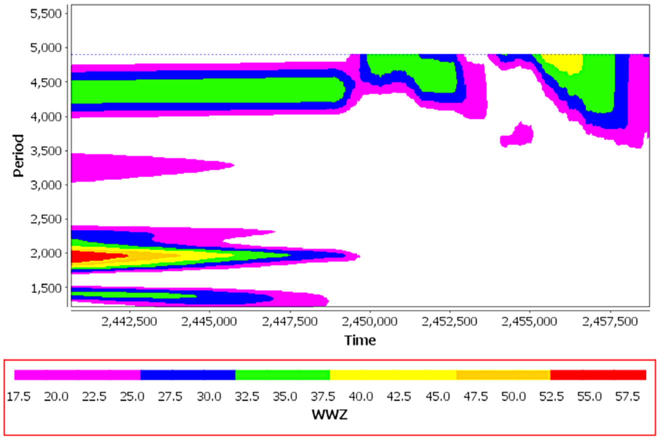

Figure 4a. A contour plot of period vs. time vs. the WWZ F-statistic for μ Cep from 13 Apr 1970 – 14 Aug 2019.

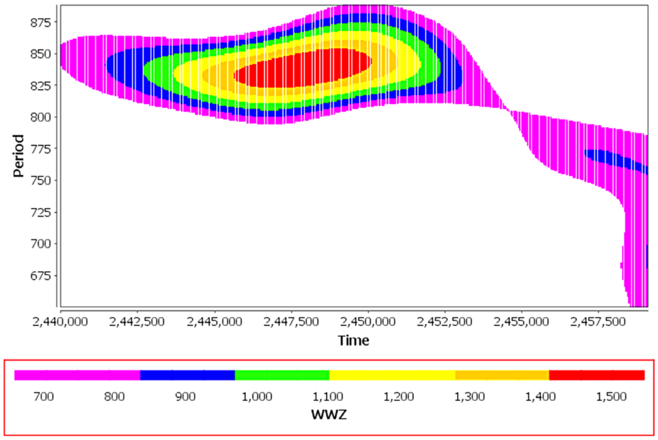

Figure 4b. A contour plot of period vs. time vs. the WWZ F-statistic for μ Cep from 6 Aug 1968 – 8 Sep 2020 showing clear variations in the light curve on time scales of hundreds of days.

4.2. CLEANest

False peaks can occur in power spectrums due to a star's location near the ecliptic, which causes recurrent seasonal gaps to appear in the star's light curve. This produces "alias" peaks in the power spectrum appear at frequencies governed by the equation:

$$\text{frequency} = \text{true frequency} \pm \mathrm{N}/365.25 \qquad (1)$$

where N is an integer and the strongest alias peak is found where N = 1 (Percy and Qiu 2019).

These alias peaks (see Figure 2) arise because when the Sun is in the star's constellation, it is difficult, if not impossible, for astronomers to undertake an observation of it.

The CLEANest algorithm was developed to help remove false peaks from a power spectrum. It is also beneficial for detecting and describing light curves composed of more than one period (Foster 1995).

4.3. WWZ

The AID holds data on in excess of 52,000 variable stars, with most of them added in recent decades (Schweitzer undated). While the majority of variable stars will probably not have enough observations to allow for the detection of long-term changes that accompany stellar evolution, as long as a variable has been regularly observed for approximately 20 years its light curve may profit from undergoing time-frequency analysis.

The analysis technique for detecting evolution over time of parameters such as a light curve's period, amplitude, or phase is known as wavelet analysis. Exactly as the Fourier transform algorithm used in VSTAR allows for uneven spacing of observations over time, so must the wavelet analysis algorithm. Wavelet analysis of light curves is included in the VSTAR application via the weighted wavelet Z-transform (WWZ) algorithm (Foster 1996).

The WWZ algorithm creates plots for a variable star's period vs. time (see Figure 3), frequency vs. time, semi-amplitude vs. time, as well as the period vs. time vs. WWZ F-statistic as three-dimensional rotatable graph and as a contour plot (see Figure 4).

The contour plot uses regular x- and y-axes for time and period, respectively, with colors to represent the third dimension, the WWZ F-statistic, creating an image not unlike a topographic map. The WWZ F-statistic gives an indication as to the strength of the periodicity at that point in time (Foster 2010; AAVSO 2018).

5. Results

The overall results of my investigation of the 33 PRGs surveyed are summarized in Table 1, with the values shown ascertained from light curves, phase diagrams, power spectrums, and WWZ plots. For the sake of brevity, only the most distinctive or interesting figures are included in this paper, but other figures are available upon request.

5.1. Notes on individual stars in Table 1

The magnitude of BZ And is tending to decrease. It also shows a very clear peak in its Fourier spectrum very close to P = 365 with an amplidude of 0.1. This might be a spurious result caused by some annual variation in observation.

There are insufficient data to confirm the periods of W Boo, RU Crt, MV Del, NSV 12441, and NSV 24346.

Using visual data from pre-2007 gives a period of 334 days for V854 Cas.

Harmonics can be detected for a number of the stars in this paper. R Cam has a harmonic of 134.99 days. V Cam has a harmonic of 261.69 days. W Cas has a harmonic of 203.02 days (and other periods of 135.34 and 101.51 days). BR CVn has a harmonic of 431 days. TU CVn has an overtone period of 22 days and WWZ suggests its period has tended towards lengthening after around Julian Day 2455000 (i.e. June 2009). R Lyr has a harmonic of 23.27 days. V2582 Oph has a harmonic of 130.91 days. V409 Per has a harmonic of 172.32 days. The first period of V336 Vul (131.7 days) has a harmonic of 65.8 days and a harmonic of 43.9 days.

RU Crt has a complicated variability pattern that appears to be the result of a pulsation period of approximately 60 days and a long secondary period of approximately 620 days.

GO Peg has reported periods of 79.3 and 65 days. However, neither period could be confirmed. GO Peg appears to be an irregular star according to the data held in the AID.

The data held in the AID confirm that TV Psc has a period of 55.1 days, but did not confirm that it also has a period of 49.1 days.

Table 1. Summary of the 33 PRGs surveyed during this project.

Name	*GCVS Type*	*VSX Period (days)*	*Type*	*Period (days)*	*Range (mag)*
BZ And	LB	—	LB	—	1.3
W And	M	397.3	M	396.16	9
EV Aqr	SRA	124.2	SRA	122.55	2.8
R Ari	M	185.67	M	186.90	7.5
R Aur	M	457.51	M	457.52	7.5
X Aur	M	163.79	M	164.15	6
R Boo	M	223.4	M	223.58	7.5
W Boo	SRB	25.51	SRB	—	1.8
R Cam	M	270.22	M	269.99	7
V Cam	M	522.45	M	523.38	8.5
W Cas	M	405.57	M	406.03	5
V418 Cas	M	480	M	479.46	6.5
V826 Cas	LB	730	L	419	3
V854 Cas	M	332	M	329.72	5.5
μ Cep	SRC	835 (also 390)	SRC	825	2
V451 Cep	LB	—	SR	385.4 & 415.8	4
RU Crt	SRB	60.85	SRB	620	2.3
T Cyg	LB	—	LB	—	2.3
BR CVn	SRB	—	SRB	860	2
TU CVn	SRB	44.2	SRB	44.846	1.8
MV Del	SRB	25.099	SRB	—	1.2
RY Dra	SRB	300	SRB	276.7	2.2
UX Dra	SRB	175	SRB	177.34	1
R Lyr	SRB	46	SRB	46.54	1.2
V2582 Oph	M	262	M	261.82	5
V409 Per	M	355	M	344.64	2.5
GO Peg	SRB	79.3 (also 65)	L	—	1.9
KK Per	LC	—	LC	—	2
TV Psc	SR	49.1	SR	55.1	0.9
TV UMa	SRB	53.74	SRB	50–60, 170–200	2.5
V336 Vul	SRC	131.6 (also 113.8)	SRC	131.7 and 115.4	2.25
NSV 12441	LB	—	LB	—	0.5
NSV 24346	LB	583	LB	—	1.8

For TV UMa, WWZ suggests that possible periods are in the range 50–60 days and 170–200 days.

V826 Cas has a 3-magnitude slow variation.

5.2. V854 Cas

The data held in AID show that V854 Cas has a range of magnitude 11–16.5. Analysis via a DC DFT Standard Scan gives a top hit for a period of 333.77 days on the visual data and 329.72 on the more recent Johnson V data. V854 Cas is a poorly observed variable, however, no doubt due to its being a dimmer star and hence impossible to see with the naked eye. In 2007 there was a switch in the observations from being exclusively visual to exclusively Johnson V. Hence we can clearly see that V854 Cas's period has reduced in the more recent data (Figure 5); the slight difference in periods have caused the visual and Johnson V observations to be offset in time.

5.3. TU CVn

A DC DFT Standard Scan with CLEANest on the much more abundant visual data for TU CVn gives a period 44.846 days with a harmonic of 22 days. However, looking at the phase plot with the DC DFT values, it becomes immediately apparent that the Johnson V data mostly lie above the model. If the phase plot is repeated (Figure 6) but without the visual data this is

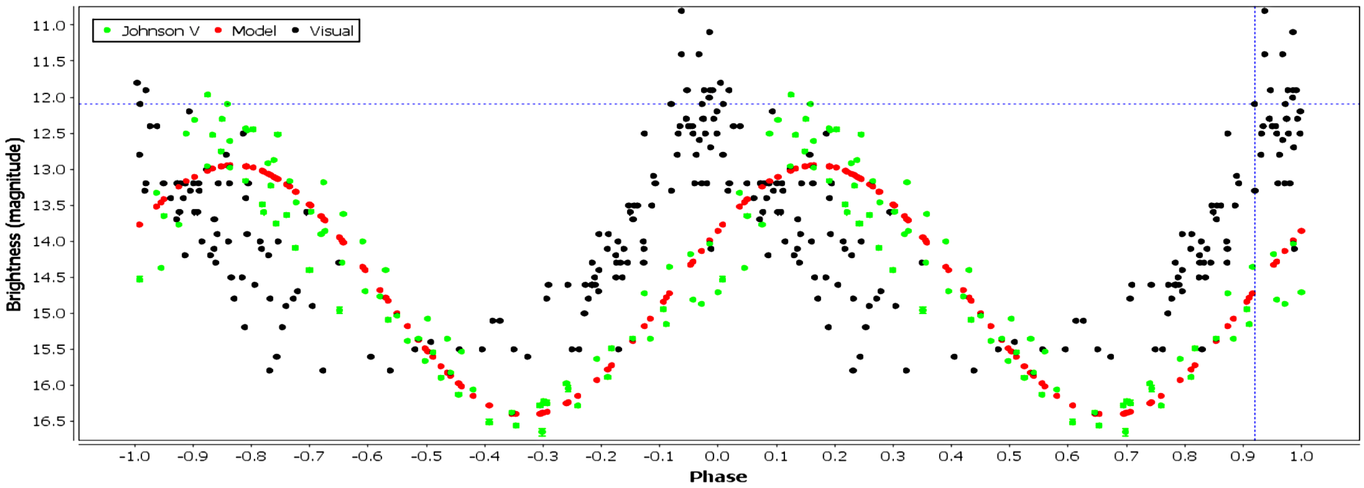

Figure 5. V854 Cas's wandering period is clear from its phase plot. The model is based on the Johnson V data.

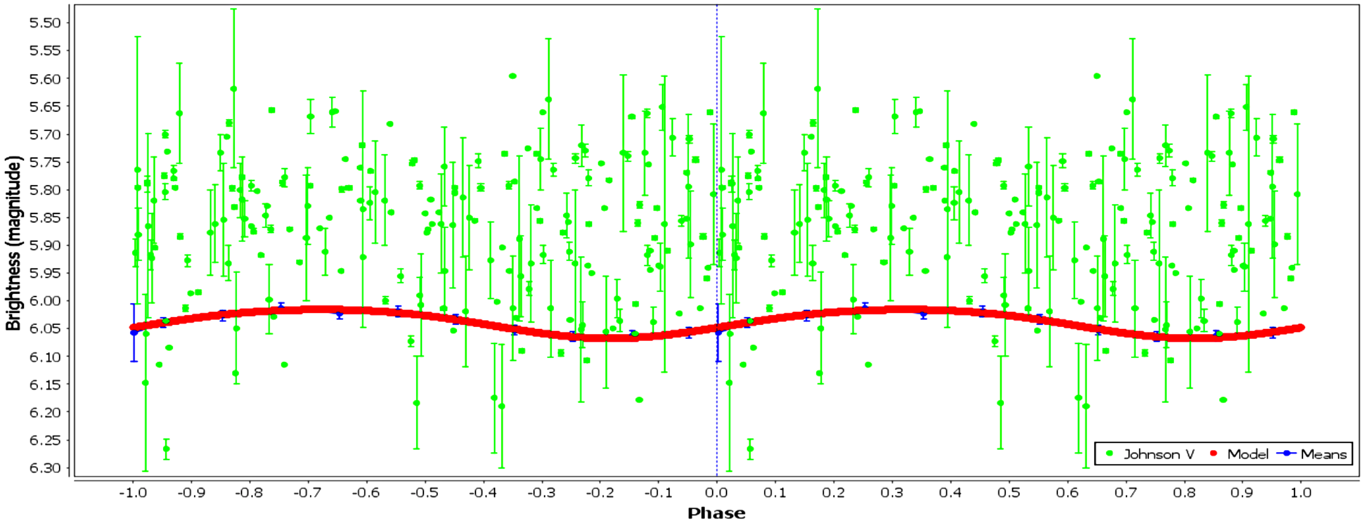

Figure 6. Phase plot of TU CVn with period of 44.846 and harmonic 22 days. The model is based on the visual data; note how the Johnson V observations are on average brighter.

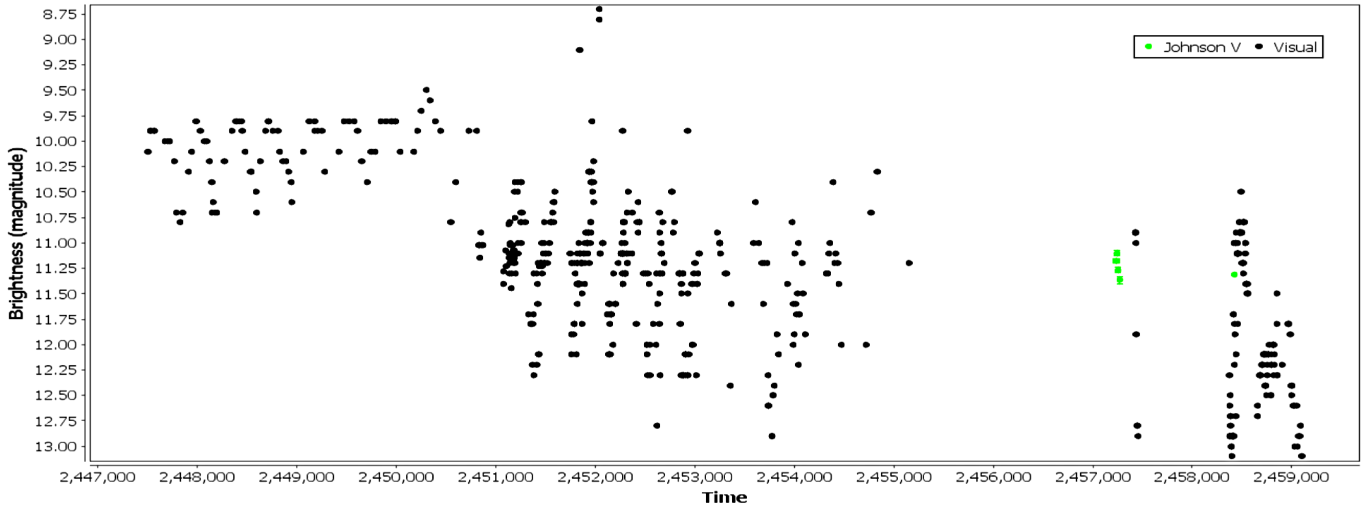

Figure 7. Light curve for V451 Cep between 5 Dec 1988 and 8 Sep 2020.

even more obvious (note that the model is a very good fit for the means of the visual data (the blue points)).

The Johnson V data held for TU CVn in AID span from June 1972 until August 2019. However, the oldest visual data were recorded in March 1955, with the most recent data recorded in October 2019. The differences between the position of the means in the visual and Johnson V datasets suggests that TU CVn has brightened. This is somewhat apparent in the raw data plot, where the mean brightness has increased from around magnitude 6.1 to magnitude 5.9. Note that the bandpasses of the Johnson V and visual are not identical so it is possible that the result shown here is due to a previous bias towards underestimating the brightness of TU CVn due to its spectrum (SIMBAD records TU Cvn as being a red star of spectral type M5). The difference in brightness between visual and V could well be the result of differences in the comparison stars used for visual versus for V; there is often an offset between visual and V in the AAVSO data, especially in the earlier years.

5.4. V451 Cep

The AID data suggest that V451 Cep has a period of 385.4 (and 415.8) days. Unfortunately the data only go back as far as 1988, but it appears that there was an abrupt change in the magnitude of V451 Cep during 1998. For the ten years preceeding 1998 its brightness only varied between magnitude 9.75–10.75, more recently it has been as bright as magnitude 10 and as dim as magnitude 13 (Figure 7). It is unclear whether this change in the magnitude range has an astronomical cause or not; it is possible that the sole early observer was using different comparison stars to later observers.

6. Discussion

As expected, many of the PRGs showed signs of having periods that wandered within a few percent, as can be seen from Table 1 and some of the WWZ plots such as Figure 3. Unfortunately, none of the selected stars displayed significant long-term changes to their periodicity. There are several instances where a model produced on the more recent Johnson V data has a different period and/or average magnitude from that produced by the generally older visual data, such as V854 Cas and TU CVn. Unfortunately, only time will tell whether these are at the beginning of a long-term change or if the differences are purely due to PRG wanderings. If these are the beginnings of long-term changes then it is thought that these are driven by a thermal pulse (Templeton *et al.* 2008).

7. Conclusion

This project aimed to confirm the classification and periodicity of a group of 33 PRGs and to look for any evidence of long-term changes in their periods.

Within the irregular stars group, I was able to detect a period of 385.4 (and 415.8) days for V451 Cep. I therefore tentatively suggest that it be updated to the SR classification instead. The relatively recent apparent abrupt change in magnitude range suggests that V451 Cep warrants close attention by amateur astronomers

Within the semiregular stars group, I was unable, despite a wealth of data, to confirm that GO Peg was an SRb star. In fact, all the evidence within the AID data points towards it being an irregular star instead.

Many of the semiregular and Mira stars had periods different from those published, but within the range expected due to PRGs' proclivity for their periods to wander by several percent.

The VSX lists BR CVn as an SRb with unknown period, however the AID data suggest a good fit for a period of 860 days with a harmonic of 431.4 days.

While I was unable to detect any long-term changes in the variability of the PRGs studied, three variables in particular warrant further investigation: V854 Cas, a Mira that appears to have decreased in period since 2007; TU CVn, an SRb for which WWZ analysis suggests a lengthening of its period around June 2009; and μ Cep, an SRc with a particularly complicated periodicity for which the contour plot (Figure 4a) suggests an event around JD 2450000 (9 October 1995) that has left its periodicity dominated largely by its 4,000-day period created by its non-spherical circumstellar nebula (De Wit *et al.* 2008), while is approximately 800-day pulsation period shows large variations (Figure 4b).

As is always the case with an observational science, more data would be beneficial, but this is especially the case with: NSV 12441, NSV 24346, W Boo, RU Crt, and MV Del. However, it is important not just that more data are collected, but the right type of data. W Boo, for example, has 10,607 observations stored in AID. Unfortunately, as a short-period PRG, W Boo would benefit greatly from observers prepared to adopt it in a regular observation program on a near-nightly basis.

8. Acknowledgements

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