Systematic Effects in the Visual Estimation of the Brightness of Red Variable Stars

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Abstract Visual brightness estimates for red variable stars are complicated by a number of effects. These phenomena have been investigated by comparing visual estimates with V-band photometric measurements. The differences between these quantities for individual stars often vary with the V magnitude at the time of the measurement in a way that is different from the collective trend for many stars and corresponds to an underestimate of the full amplitude of variation by visual observers. This may result from biases introduced in the estimation process, specifically in the choice of comparison stars and in the interpolation of the brightness of the variable between them. These results may shed some light on the factors affecting the transformation between visual estimates and photometric V values for red stars and provide some guidance in the use of that transformation. They also provide insight into the visual estimation process itself.

1. Introduction

The extensive long-term monitoring of variable stars by thousands of visual observers coordinated by the American Association of Variable Star Observers (AAVSO) has produced a database that is a valuable resource for professional astronomers. While these estimates are generally reliable, there has been a persistent question regarding the relationship between visual estimates (m_v) and photometric magnitudes (V) for red stars (Percy *et al.* 1993). This relationship, in the form of the difference m_v – V, has been quantified at various times and in various ways. The work described here evolved from such an effort (Cadmus 2021). A general discussion of the procedure followed in that work as well as a discussion of the history of such determinations is included there. The result was the determination of the dependence of m_v – V on V alone, on the B–V color alone, and on V and B–V together. The purpose of this paper is to explore specific issues associated with the visual estimating process that arose in that project. The focus here is on this group of red stars; although some of the insight may have more general applicability, that has not been confirmed.

2. Observations

The determination of the transformation between visual brightness estimates and photometric magnitudes reported by Cadmus (2021) involved the comparison of visual estimates for 38 mostly semiregular variable stars from the AAVSO International Database (Henden 2014; Kafka 2015–2020) with photometric observations made with the 0.61-m telescope at Grinnell College's Grant O. Gale Observatory. The measurements were made in the V and B bands using a photoelectric photometer incorporating an uncooled 1P21 photomultiplier and processed using conventional methods (see Cadmus 2015 and Cadmus 2021). They cover almost three decades from roughly May 1984 to December 2013 (JD 2445849 to 2456657), depending on the star. These stars are pulsating red giants and some are carbon stars, which are very red. They were selected on the basis of their likelihood to experience episodes of very reduced amplitude (see Cadmus 2015). The V data have been fit with a spline curve to facilitate matching the dates to those of the visual data, which are averages in 20-day bins. The shapes of the V light curves span the range from nearly sinusoidal to highly erratic. There is relatively little variation in B–V for most of the stars but for the carbon stars the B–V variation is very similar to the V variation, with large B–V associated with large V.

For reasons to be discussed in section 5.2, one would expect the m_v – V transformation to be color-dependent. On the other hand, because the estimates are made by observers using a variety of instruments providing images with a range of brightness for the same star at the same time, one would be surprised to find that the transformation depends in a significant way on stellar brightness. The transformation does, in fact, behave as expected. The overall transformation for the entire collection of stars is nearly flat when plotted vs. V and sloped when plotted vs. B–V. These plots are shown in Figure 1 (adapted from Cadmus 2021).

As one can see from Figure 1, the distributions of data points are not as tidy as one might hope. The use of 38 stars gives coverage over a wide range of V and B–V, with each star contributing its part to the distributions. One might expect that the data for each star would simply be a chunk of the complete coherent distribution but this is not the case, leading to the clumpy appearance of the distributions in Figure 1. The vertical lines in panel b are for those stars that have so little variation in B–V that spline fits to those photometric data were not helpful or for S Aur, which is faint and very red, resulting in poor quality B data. The trends are clear and can be fit separately with straight lines as shown in Figure 1 or collectively with a function of both V and B–V to get equations that transform the m_v estimates to V values (Cadmus 2021). The fit to m_v – V as a function of both V and B-V is:

$$m_v - V = 0.13 - 0.02\ V + 0.18\ (B-V). \qquad (1)$$

The uncertainties in the coefficients in this transformation are primarily determined not by the uncertainties in the data or in the fitting process, but by the substantial differences in the behavior of the data for different stars that give widely

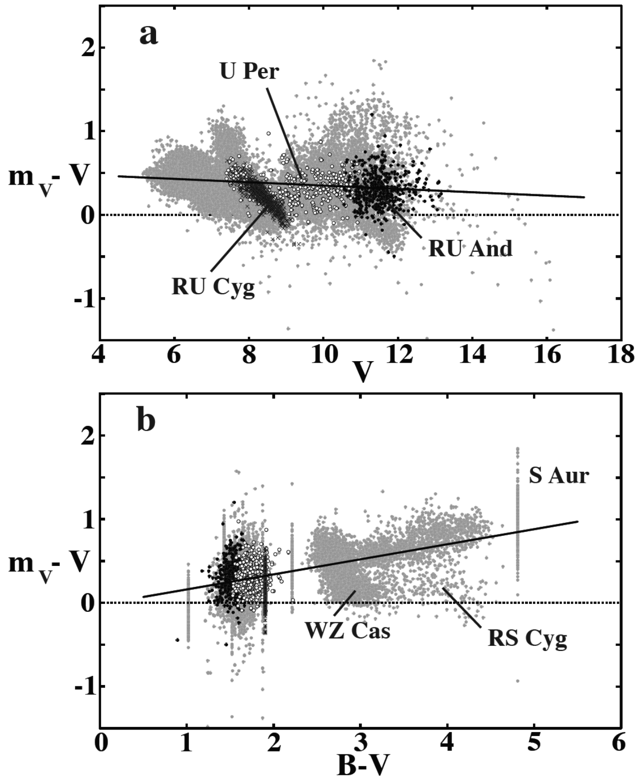

Figure 1. The visual / photometric difference m_v – V plotted vs. both V (panel a) and B–V (panel b). Each point corresponds to a single value in the binned visual light curve. The RU And data are represented by solid black points, the RU Cyg data by crosses, and the U Per data by open points. The data for all other stars are represented by gray points. The U Per distribution is partially obscured by the RU Cyg points but the data for these three stars also appear in Figure 2. The data for a few of the carbon stars are identified in panel b. The solid lines are linear fits to the m_v – V data vs. either V or B–V. A color version of this plot in which all the stars are differentiated is shown in Cadmus (2021). Figure adapted from Cadmus (2021), ©2021. The American Astronomical Society. All rights reserved.

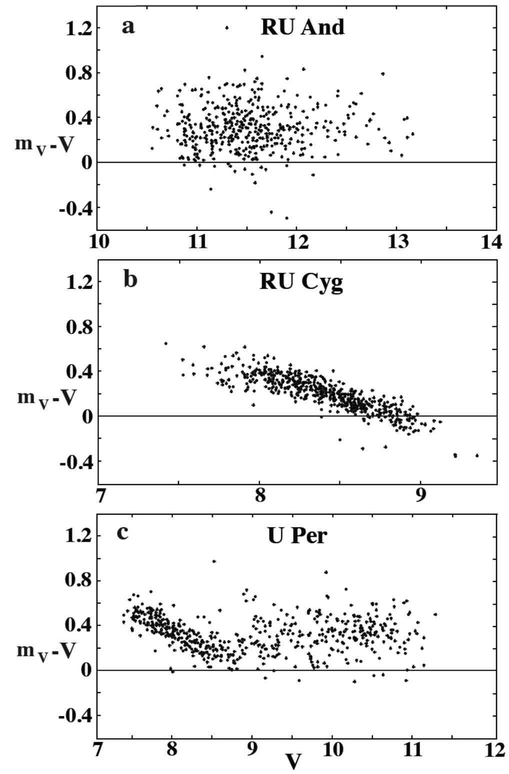

Figure 2. The visual/photometric difference m_v – V plotted vs. V for RU And (panel a), RU Cyg (panel b), and U Per (panel c). Figure adapted from Cadmus (2021), ©2021. The American Astronomical Society. All rights reserved.

varying transformations when the data for the stars are fit individually (Cadmus 2021). The primary source of uncertainty is therefore the choice of stars to include in the collective fit. To estimate the size of this effect the overall transformation was calculated by fitting 11 randomly-constructed groups of stars. The standard deviations in the resulting coefficients suggested that the constant is uncertain by roughly 0.2, the V coefficient by roughly 0.02, and the B–V coefficient by roughly 0.05, so the V slope is consistent with zero but the B–V slope is not. The more important point, however, is that this overall transformation is only a very rough representation of the general trend of a diverse set of data.

Figure 2 (adapted from Cadmus 2021) shows the contribution of three individual stars to these transformation distributions. The motivation for the present work lies in understanding these details.

The RU And distribution in panel a of Figure 2 is a case that looks like a part of the overall distribution. The data for RU Cyg in panel b illustrate the behavior of the distribution for a star with a strong V-dependence. The U Per distribution in panel c is an example of a more complex case with two regions of different character that will be discussed in more detail later.

As shown in Figure 1b the situation is neater for the B–V distribution except for the existence of a lower branch populated by two carbon stars. The emphasis here is on the variation of m_v – V with V for individual stars but this lower branch deserves a few comments. It is populated primarily by RS Cyg, with some contribution at the left end by WZ Cas. This might suggest that visual observers provide estimates for RS Cyg that are less affected by the overall color dependence than are the estimates for other very red stars, but this seems unlikely. Another possibility is that there is some other process that tends to cancel the color dependence for WZ Cas and RS Cyg. These two stars do not differ substantially from the stars in the upper branch in V, B–V, period, or difference in color relative to the comparison stars on the AAVSO charts. The presence of other stars close to these two variables might suggest the existence of very easy-to-use comparison stars, but the information available in this study does not indicate that they were heavily used and they are probably far enough from the variables to be resolved by most observers. In addition, a limited and crude laboratory experiment did not reveal any dramatic tendency for estimates to be affected by the presence of a nearby star. RS Cyg does have a larger amplitude and simpler light curve than the other carbon stars in this sample. Further investigation of this phenomenon is planned.

The distribution of the measurements of m_v – V vs. V for each star forms a clump that is often inclined relative to the nearly-flat overall trend. The tilt of these individual distributions is almost always with greater values of m_v – V at smaller values of V. This means that as the star gets brighter visual observers tend to underestimate the brightness to a greater extent so they report an amplitude of variation that is too small: an amplitude deficit. The possibility of an amplitude deficit was previously noted for EU Del by Percy *et al.* (1993) and may be present in the data of Lebzelter and Kiss (2001). As a further check on the reality of this phenomenon binned AAVSO visual data were compared with AAVSO photometric V data. Several other investigations reported in Cadmus (2021) demonstrated that this effect is real and originates in the visual rather than in the photometric data. It is different for different stars but similar for different observers. The purpose of this paper is to investigate the nature of this unexpected dependence of $m_v - V$ on V and the resultant amplitude deficit. The amplitude deficits presented here were calculated as the negative of the slopes of the corresponding $m_v - V$ vs. V distributions, not from the amplitudes themselves, but that corresponds to a definition of the amplitude deficit as:

$$\text{Deficit} = \frac{(\text{photometric amplitude} - \text{visual amplitude})}{\text{photometric amplitude}} \quad (2)$$

The deficits for all the stars, in order of increasing deficit, are given in Table 1, which also includes mean V and B–V values, approximate typical V amplitudes, spectral types, and variability types. The photometric data were estimated from the Grinnell light curves; the spectral types and variability types were obtained from www.aavso.org/vsx. These are semiregular stars whose light varies on multiple time scales so the amplitude estimates are very approximate.

The deficit effect is more pronounced for stars with smaller amplitudes (Figure 3). The amplitudes reported here are estimates of the typical full amplitudes—the total ranges of variation in brightness—and do not reflect the most extreme variations.

The potential causes for the deficits, and for the shapes of the individual $m_v - V$ vs. V distributions in general, can be grouped into those that seem not to be involved (section 4), those that apparently affect the nature of the distributions but are not their underlying cause (section 5), and those that might cause these phenomena in general (section 6). A viable explanation for the deficit effect must account for the fact that the $m_v - V$ vs. V distributions are very different for different stars but each is well defined because different observers generate similar observations for each star. It is therefore helpful to look at sets of data that have been restricted in particular ways.

3. Insight from restricted sets of data

The multiplicity of possible causes for the strange behavior of the $m_v - V$ vs. V distributions can be simplified by considering only sets of data that eliminate some variables. For example, if the deficit persists when the $m_v - V$ vs. V distribution for an individual star is calculated using estimates associated with a single known observer, a single known chart, and a single known pair of comparison stars (an "OCC" set), variations in those factors can be eliminated as possible causes of the effect. This was tested in 99 OCC cases and the deficits usually persisted. The distributions for individual stars generated from the data for different OCC sets are generally a bit different, but

Table 1. Amplitude deficits and stellar characteristics, ordered by size of amplitude deficit.

Star	*Mean V*	*Mean B–V*	*Typical V Amplitude*	*Deficit*	*Spectral Type*	*Variability Type*
V778 Cyg	10.2	3.4	0.4	–0.244	C4,5J	SRA
S Aql	10.2	1.6	2.0	–0.157	M3e-M5.5 e	SRA
RZ UMa	9.3	1.6	0.7	–0.052	M5-M6	SRB
S Per	10.2	2.7	3.2	–0.047	M3Iae-M7	SRC
S Aur	10.8	4.8	2.3	–0.041	C4-5,4-5(N3)	SR
RU And	11.5	1.5	2.0	–0.012	M5e-M6e	SRA
Z UMa	7.5	1.6	1.6	–0.012	M5IIIe	SRB
R UMi	9.4	1.7	1.5	0.001	M7IIIe	SRB
U Boo	11.1	1.6	1.6	0.015	M4e	SRB
U Per	9.0	1.8	3.5	0.018	M5e-M7 e	Mira
RS Cyg	7.8	3.3	1.8	0.033	C8,2e(N0pe)	SRA
W Tau	10.1	2.2	1.3	0.039	M4-M6.5	SRB
RX Boo	7.6	1.8	0.6	0.071	M6.5e-M8IIIe	SRB
RY Dra	6.5	3.3	0.5	0.080	C4,5J(N4p)	SRB
RS Aqr*	11.7	1.6	3.0	0.081	M2e	Mira
X Her	6.2	1.5	0.6	0.097	M6e	SRB
W Cyg	5.9	1.6	1.0	0.106	M4e-M6eIII	SRB
V Boo	8.4	1.6	1.1	0.108	M6e	SRA
U LMi	11.2	1.4	1.4	0.127	M6e	SRA
RS Cnc	5.9	1.7	0.6	0.153	M6S	SRB
RV And	9.9	1.8	1.8	0.154	M4e	SRA
V CVn	7.3	1.6	1.3	0.158	M4e-M6eIIIa:	SRA
SW Vir	7.2	1.7	1.1	0.159	M7III	SRB
X Mon	8.0	1.5	1.5	0.171	M1eIII-M6ep	SRA
RS Lac	11.2	1.0	1.7	0.171	K0	SRD
RV Peg*	11.7	1.9	5.0	0.177	M6e	Mira
SX Her	8.3	1.6	0.9	0.202	G3ep-K0(M3)	SRD
U Del	6.7	1.7	1.0	0.209	M5II-III	SRB
U Cam	7.5	4.0	0.9	0.217	C3,9-C6,4e(N5)	SRB
X Lib*	11.7	1.7	3.0	0.259	M4e	Mira
UX Dra	6.2	2.8	0.5	0.267	C7,3(N0)	SRB
ST UMa	6.6	1.7	0.6	0.295	M4-M7III	SRB
WZ Cas	7.1	2.9	0.6	0.302	C9,2Jli(N1p)	SRB
TT Cyg	7.5	2.6	0.4	0.317	C5,4e(N3e)	SRB
RW Boo	7.9	1.5	0.6	0.348	M5-M7III	SRB
RT Hya*	8.2	1.6	1.3	0.363	M6e-M8e	SRB
RU Cyg	8.4	1.9	1.0	0.407	M6e-M8e	SRA
RW Sgr*	9.7	1.9	1.0	0.506	M4II/IIIe-M6III:e	SRA

** Only limited data are available for these stars.*
Note: The amplitude values are very approximate and represent the typical range of variation.

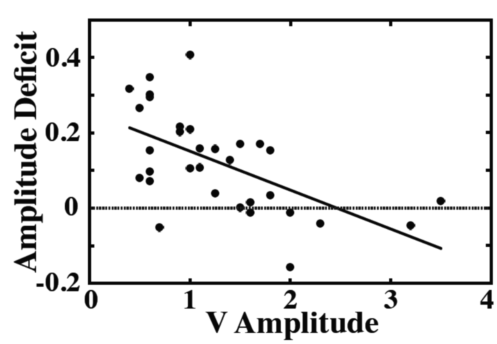

Figure 3. The relationship between the amplitude deficit and the approximate full amplitude of variation in V. The solid line is a linear fit to the data.

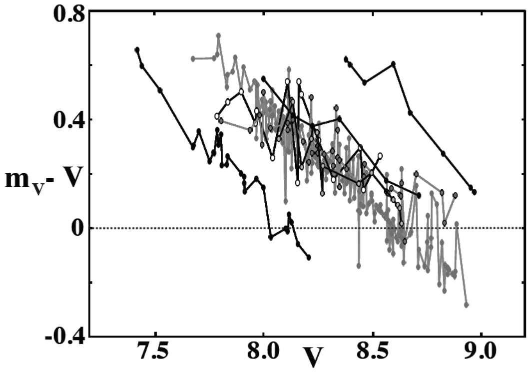

Figure 4. The variation of m_v – V with V for several OCC restricted data sets for RU Cyg. The points in each set are connected by lines to make the groups more apparent.

each shows the deficit effect. An example of this behavior is shown in Figure 4.

Figure 4 shows a sample of the OCC data sets that contribute to the complete m_v – V vs. V distribution for RU Cyg shown in Figure 2. RU Cyg was chosen as an example because the effects described here are so clear in that case, but they occur for most of the other stars in our program as well. If the visual/photometric differences had no dependence on V, each of these OCC patterns would be horizontal. However, in each case the distribution for each OCC set has a slope that arises from systematic effects in the observation process.

The nature of the m_v – V vs. V distributions involves a hierarchy of processes. At the most fundamental level, if an observer were always to report the same brightness, resulting in a severe underestimate of the range of variation, the distribution would have a slope of –1. The predominantly negative slopes of the real distributions are diluted versions of this extreme case. The chart and comparison star information is not available for most of the estimates in the AAVSO database so these restricted OCC sets of estimates are small and may be subject to systematic effects associated with particular observers. Nevertheless, the m_v – V vs. V distributions for the OCC sets are usually approximately linear with consistent negative slopes for each variable star.

The overall m_v – V vs. V distribution for a particular star is then the aggregate of all of its OCC sets of estimates. For a single variable star the primary difference in the OCC sets is the choice of comparison stars, which changes with the variable star's brightness, so the various OCC sets are offset from one another in V as shown in Figure 4. The effects of the observer and chart are usually less dramatic. If the range of the variable is small then the OCC sets substantially overlap and the resulting m_v – V vs. V distribution has a pronounced slope. If the amplitude of the variable is large, however, the individual diagonal OCC distributions are spread out over a substantial range of V and the overall distributions for such variables have a much smaller slopes, resulting in smaller deficits at larger amplitudes as seen in Figure 3. This sensitivity to the amplitude of variation at least partially explains why the m_v – V vs. V distributions for different stars look different. Unfortunately, it is not possible to reconstruct the entire observed distributions from the individual OCC distributions because essential information is not available for most estimates. While the numerous and varied OCC data sets discussed here provide useful information, it is important to remember that these cases are only a sample and may not be representative of all possible situations.

The examination of the OCC data sets strongly suggests that the deficits do not arise from differences in observers, charts, or comparison stars. With that background the following sections explore a variety of potential effects that may or may not be responsible for the deficit effect.

4. Possible factors that probably do not influence the shapes of the m_v – V vs. V distributions

The investigation of the cause of the star-dependent amplitude deficits is complicated by the large number of factors that can influence visual magnitude estimates. The effects described in this section were examined but found not to be primary causes of this phenomenon, but this information narrows the possible remaining options and has some relevance to understanding the visual estimating process generally.

4.1. The photometric data

The reliability of the photometric V data is discussed in detail in Cadmus (2021).

4.2. Stellar characteristics

No significant correlation was found between the size of the deficit for a given star and its values of mean B–V, period, declination, galactic latitude (which might be related to the number of comparison stars near a variable), or any other obvious stellar characteristic except V. The effect is apparently not caused in any direct way by any of these factors.

4.3. Angular orientation of the variable and the comparison stars

The angular positions of the comparison stars relative to the variables on the sky (the position angle effect) is known to affect observers' brightness estimates (Roberts 1897; Isles 1970; Williams 1987). However, a casual survey of AAVSO charts indicates that the comparison stars are reasonably well distributed around the variables and not likely to be the cause of major systematic effects in the present case. In addition, the orientation of the stars in the field is dependent on the optics used and the circumstances of the observations so any position angle effect should average out over the large number of observers and observations considered here.

4.4. Length of time spent viewing the stars

The color response of the human eye can vary on time scales ranging from hours to years (Sterken and Manfroid 1992) and Whiting (2012) has reported that red stars are perceived to be brighter if they are observed for a longer period of time. Both of these effects are difficult to investigate and are unlikely to operate in a manner that is systematic enough to produce deficits for many stars and many observers.

4.5. Observing conditions

In some cases there are effects associated with adverse observing conditions and small numbers of visual observers at the ends of the observing seasons but these effects are not generally correlated with V.

4.6. Time dependence

There are detectable variations in the distributions over time, but these are not large enough or systematic enough to be responsible for the deficit effect in general.

5. Possible factors that may have some effect on the shapes of the m_v – V vs. V distributions but are probably not the underlying cause of the deficits

5.1. Binning of the visual data

Tests with various binning intervals showed that binning of the visual data may cause a small reduction in amplitude, especially if the period is short, but that it is not the cause of the differences of interest here.

5.2. Color differences between the variables and the comparison stars

The origin of the overall transformation between visual estimates and photometric V measurements may be related to a systematic, and generally unavoidable, difference of about 1.4 magnitude between the colors of the variable stars in this project and the less-red colors of the comparison stars on AAVSO charts (Cadmus 2021). The wavelength response of human vision is different for different observers and different situations, but generally lies a bit to the blue of the V passband (Hallett 1998) so a visual observer will perceive a red variable star as slightly dimmer relative to its comparison stars than would be measured photometrically. This is roughly consistent with the overall amount of offset between the photometric and the visual data but the m_v – V vs. V distributions for individual stars depend on factors that are unique to each star making detailed comparisons difficult. This is further complicated by the Purkinje effect (Thackeray 1935; Grouiller 1936; Percy 2007), which describes the dependence of the human wavelength response on light level. While the color difference between variable and comparison stars probably explains the overall offset for each star, most of the stars considered here have very little color change around their cycles so there is little phase-dependence in the color difference that would result in a variation of m_v – V with V.

5.3. Angular separation between the variable star and the comparison stars

For 72% of the program stars there is a clear correlation between increasing angular separation between the variable star and a comparison star and increasing brightness of the comparison star. With the color difference between variables and comparison stars that exists for these red stars it is possible that observations made with brighter, more distant comparison stars might be systematically different from those made with fainter comparison stars, leading to a variation of m_v – V with V. However this would not affect the OCC data sets, which involve only a single pair of comparison stars but still show the deficit effect, suggesting that angular separation is not a primary cause of the deficits.

5.4. Human observers

Although there is enough agreement among different observers to produce clear patterns, individual observers do generate different results, primarily in the overall offsets of the brightness estimates, which were found to be at least 0.6 magnitude in some cases that were investigated. The effect of these differences does not always average out but the fact that the deficit effect is present in individual OCC data sets shows that variation among observers is not a primary cause.

There is still the issue of phenomena that affect the estimates of a single observer. This will be addressed further in section 6, but one possibility that falls in the present "possible but unlikely" category is "anchoring": the well-established tendency for peoples' previous experience to inappropriately skew their judgments. This specific mechanism seems unlikely because the deviations between visual estimates and photometry generally scale gradually with V and are not enhanced near the extrema.

5.5. The comparison star charts

The comparison star charts themselves are, of course, attractive suspects for the cause of the deficits because they have the property of being different for different stars but similar, if not identical, for different observers. Previous investigators (Stanton 1978, Stanton 1981, and Zissell 2003, for example) have investigated the accuracy of the charts' comparison star sequences. This was also thoroughly explored in the present project and while the charts probably play a role, several investigations suggest that the nature of the charts, as opposed to the way that they are used, is not the primary culprit.

A number of the stars in the comparison star sequences for U Per and RU Cyg, which are particularly problematic cases, were measured at Grinnell and no systematic differences with the current AAVSO charts were found. In addition, distributions generated using OCC data sets that include only estimates that were known to have been made with specific charts usually show the deficit effect so it does not arise from chart-to-chart differences. There are some chart-dependent effects but no clear evidence that there is anything seriously wrong with any specific charts. This conclusion is blurred a bit by the tendency for the charts that were used to be correlated with observers and by the use of some non-AAVSO charts.

Inspection of the photometry tables associated with the AAVSO charts for the stars in this project revealed that, to a greater or lesser extent depending on the star, different catalogs or sources were used for comparison stars of different brightness, as one would expect, but no obvious widespread problems were found and the AAVSO comparison star values for the large-deficit star RU Cyg came almost entirely from one catalog, ruling out this process as a primary cause in this significant case.

U Per and S Per are interesting in this context because their m_v – V vs. V distributions have two components (see Figure 2c for U Per). When U Per is fainter than about V = 8.8 the distribution looks flat, although with a great deal of scatter, but when the star is brighter than this the distribution has a well-defined slope with much less scatter. The comparison star magnitudes that are associated with the bright, sloped regime are from the Tycho-2 catalog and these were confirmed at Grinnell as discussed earlier. The slope does not appear to originate in a comparison star problem; a more likely explanation is offered in section 6. The sloped part of the S Per distribution apparently arises because this star was brighter earlier in this project when there was a systematic error in the comparison star chart, explaining the deficit in this specific situation. All this suggests that V-dependent effects in the charts resulting from the use of different source catalogs are involved in some cases, but that it not the underlying cause of the widespread deficits.

The overall offset between visual estimates and photometric measurements apparently occurs because the comparison stars are almost always less red than the variables of interest here, but a possible complication that might affect the shape of an individual star's m_v – V vs. V distribution is a variation of B–V with V within the comparison star sequence on a chart. This would cause the observer's estimate for a particular star to shift relative to the photometric value by an amount that depends on the star's brightness and produce the deficit effect. To investigate this possibility the comparison star V and B–V values were extracted from the AAVSO web site for the C-scale charts for each of the stars, except in a few cases for which that scale yielded only a few values. Linear fits to plots of B–V vs. V for the comparison stars revealed that there is a significant variation of B–V with V for the comparison stars on many of the charts, but not in a consistent direction and not correlated with the deficits for each of the stars, suggesting that whatever tendencies there might be for the colors of the comparison stars to depend systematically on V might affect the deficits of individual stars but are not the origin of the (primarily positive) deficits overall.

For chart problems to produce the kinds of m_v – V vs. V distributions that are reported here those problems would have to have a systematic variation with V, and that seems unlikely. Several of the processes discussed above might lead to systematic effects for individual variable stars but none provides a comprehensive explanation for the deficits generally.

5.6. Wavelength dependence of the light curve amplitude

The amplitudes of Miras, and presumably those of semiregular variables as well, are much greater in the visible than in the infrared (Reid and Goldston 2002). The typical visual response is more blue-sensitive than is the V passband (Hallett 1998) so on this basis one might expect a larger amplitude for the visual estimates than for V measurements, which is the opposite of what is reported here. This systematic variation in amplitude is undoubtedly in play but is apparently not a primary cause of the deficits.

5.7. Spectral variations over the star's cycle

The spectra of these stars change over their cycles and it is conceivable that those changes could affect the m_v – V vs. V distributions if they interacted with the difference in the visual and photometric wavelength responses. This is a difficult hypothesis to test because it requires a detailed comparison of phase-dependent spectra with both the V passband and the less-well-defined visual passband. That process is beyond the scope of this investigation but some general observations are possible.

The influence of spectral variations could take two forms (or a combination of both): the effect of the overall shape of the spectrum and the nature of specific spectral features. There is no evidence of a correlation between the deficits and either the mean B–V values or the spectral types of the stars. An ongoing project at Grinnell has provided a large collection of spectra of these stars but those data are not yet in a form that is appropriate for detailed analysis. However, a casual look at preliminary versions of the spectra revealed that their overall shapes fall into several categories. There is some systematic variation in the mean deficits among the categories but the variations of the deficit values within each category are much larger. The lack of a clear association of deficit with overall spectral shape suggests that the shape, and therefore its variation with phase, is probably not a primary cause of the deficits.

Exploring the possible role of most individual spectral features is beyond the scope of this investigation but the information is available to pursue one case. The behavior of the very strong Na D absorption feature in the spectrum of RS Cyg has been investigated as part of a separate project. Although the strength of this feature varies with phase, RS Cyg has a relatively small deficit, suggesting that variation of the Na absorption is not, by itself, sufficient to cause a deficit.

While these results on the possible effect of spectral variations are tentative and incomplete, they do not show clear signs that they might be important and do not offer any explanation for the shapes of the distributions shown in Figure 2. This question will be easier to address when fully processed spectra become available.

6. Possible explanations for the amplitude deficits

The final category of possible causes for the deficits—those that might be responsible for the existence of these effects as opposed to altering their details—is the most difficult to address in spite of the conspicuous, consistent, and widespread nature of the phenomenon. If the deficits do not appear to be caused by stellar characteristics or the identities of the observers, charts or comparison stars, then the cause probably lies in the observing process.

One possible cause of the slopes in the m_v – V vs. V distributions and the resulting amplitude deficits involves the relationship between scatter in the visual estimates and the use of comparison stars. If an observer's m_v estimates always scatter symmetrically around the photometric V value then as V changes the m_v – V vs. V distribution will be flat and there will be no deficit. However, if the observer's estimate options are constrained to lie between the magnitudes of the comparison stars, as is the case for a single OCC set, at some point as the star brightens the estimates that would be the brighter than V by the greatest amount exceed the limit of the brighter comparison star and are no longer available, possibly eroding the scatter of the estimates corresponding to negative m_v – V values. The same sort of process occurs at the faint end, so the m_v – V values might be systematically high when the star is bright and low when the star is dim and the m_v – V vs. V distribution would no longer be flat but have an overall negative slope. This corresponds to a deficit for the OCC set and ultimately for the star overall.

The scatter is a natural characteristic of the measurement process, but the use of discrete pairs of comparison stars might be involved in the amplitude deficit phenomenon.

Another particularly interesting possibility is revealed by plotting histograms of individual estimates for single OCC sets as shown in Figure 5. The nine examples in that figure are representative of the behavior of the entire set of 97 histograms (the data were too sparse in two cases) but are not always the best examples of each category. These were chosen to represent nine different observers and eight different stars, and because the numbers of estimates were large enough to make the patterns clear.

These examples show that for a given OCC case there is often a tendency for the estimates to fall near the center of the comparison star interval and for observers to avoid estimating the variable star's brightness to be the same as that of either comparison star: "central clustering." Roughly speaking, of the full set of OCC cases about 38% showed a very clear clustering of the estimates near the center of the comparison star interval, about 46% showed some tendency to cluster, and about 15% did not show any obvious clustering. Although these categories are only approximate the tendency to cluster was seen in roughly 84% of the OCC sets. Figure 6 illustrates this effect for the entire sample of OCC restricted data sets.

For each OCC set the central clustering fraction shown in Figure 6 was calculated as the fraction of all estimates falling within an interval of one third of the magnitude difference between the two comparison stars after adjusting the visual estimates to remove overall offsets relative to the V data. To better capture peaks this interval was centered on the mean of all estimates, which is not always ideal. Centering the interval at the midpoint of the two comparison stars significantly changed the results in individual cases but the overall distribution shown in Figure 6b was essentially unchanged. The deficits shown in Figure 6b were calculated using binned visual data but using the raw estimates does not change the overall appearance of the plot. In a few cases these results are confused by observers reporting estimates that fall outside the comparison star range and the usual "quantization" of the estimates to 0.1 magnitude occasionally compromises individual results, but the character of the overall result is not altered by these anomalies.

The distribution of central clustering fractions shown in Figure 6a demonstrates the strong tendency of the estimates to be more concentrated within the comparison star interval than would be expected for a uniform distribution. This tendency to report "middle" estimates produces the sort of underestimate of the range of the variable star that is the amplitude deficit for individual OCC sets. Figure 6b shows that there is a tendency for more concentrated OCC distributions to be associated with larger amplitude deficits, although the large amount of scatter suggests that some additional process (discussed below) is also involved.

This effect is not surprising. Observers are often instructed to estimate where the variable lies in brightness between the two comparison stars (AAVSO 2013), but it is difficult to make brightness comparisons between stars at the 0.1 magnitude level. Some experienced and skillful observers may well achieve this precision, but in practice many observers might decide among "like the fainter comparison star," "like the brighter comparison star," or "between the two stars," but with the "between" option the most common. Another way to say this is that they might not so much estimate where the variable falls in brightness between the two comparison stars as choose two comparison stars for which the variable falls in the middle. This tendency to concentrate estimates toward the center of the comparison star interval enhances the truncation effect described above and acts to increase the deficits.

If the variation of amplitude deficit with clustering fraction shown in Figure 6b demonstrates that the deficits arise from the tendency of observers to pick middle values, one might wonder why the amount of scatter is so large. There are several reasons for this. First, the estimating process involves human perception and judgment so scatter is inevitable. Second, the origin of much of the scatter is revealed by plotting deficit vs. clustering fraction for individual observers as in Figure 6b, which includes distinct symbols to identify the OCC data sets associated with two specific observers and shows that the trends for individual observers are often much better defined than is the case for the entire data set, supporting the idea that clustering of estimates is a primary cause of the deficits. However, although the ranges of clustering fractions are approximately the same for these two observers the deficits are systematically different, and the deficits for two other observers are similar in spite of substantially different degrees of clustering. This suggests that there is another process at work beyond central clustering.

A third reason for the scatter in Figure 6b is that the simple interpretation of these central clustering calculations is based on the assumption that the star's actual brightness variation is not concentrated in the middle of the interval. This is certainly not true in general. To explore the effect of the star's behavior on these estimate histograms the corresponding V histograms were constructed for each OCC data set after shifting the V data to remove the offset relative to the visual data. The shifts required varied dramatically for different OCC sets. A comparison of the distributions of estimates and V values over the range between the comparison star magnitudes showed that in approximately half of the cases the star's behavior did not resemble the central clustering of the visual estimates. In roughly 20% of the cases there was a clear correspondence, and in the remaining cases the situation was less clear. The visual estimates are observations of the star's behavior so some relationship between these two distributions is expected, but these results show that the central clustering of the estimates is not usually a simple reflection of the star's variations.

This tendency to pick middle values almost certainly contributes to the strange shapes, V-dependence, and amplitude deficits that are observed, but does not appear to be sufficient to explain the strength of the phenomenon. However, observers' choices of comparison stars can further amplify the impact of the central clustering effect. If the star's amplitude is large and the brightness of the comparison stars change to follow those variations the resulting estimates, even in the presence of some central clustering, will reasonably represent the actual behavior of V and there will be no significant deficit. However, if the star's amplitude is small and the choices of comparison star pairs are similar to each other, then estimates that are subject to the central clustering effect will tend to be concentrated near the center of the star's variation, resulting in a deficit. The effect is

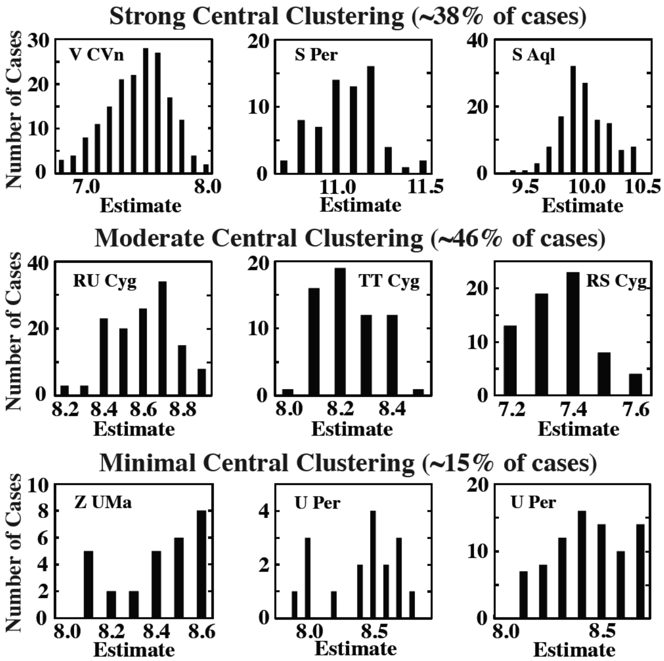

Figure 5. Examples of histograms of visual estimates for individual OCC sets of observations showing the general characteristics of those with different degrees of central clustering. In each case the limits of the "Estimate" axis are at or very close to the two comparison star magnitudes. The two U Per examples show estimates from different observers.

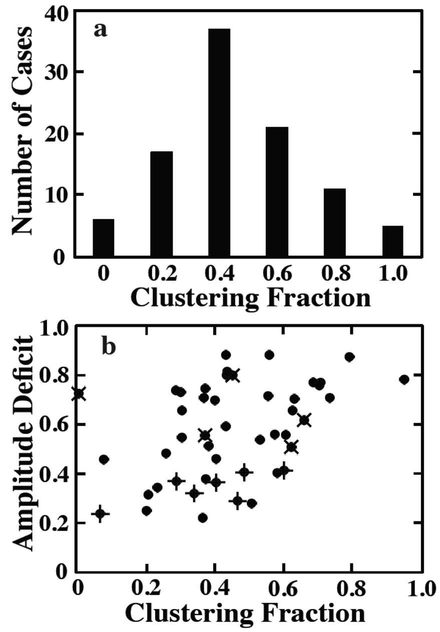

Figure 6. (a) The distribution of central clustering fractions for all 97 OCC data sets. (b) The dependence of the amplitude deficit on the central clustering fraction for OCC data sets with at least 30 estimates. The symbols that include an "x" or a "+" are for two selected observers.

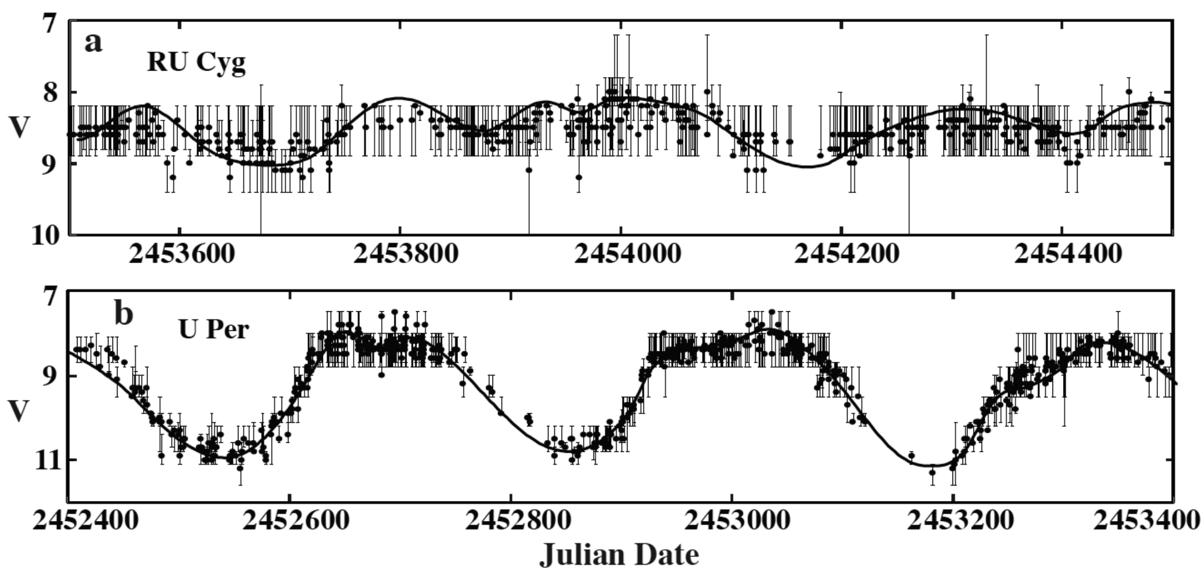

Figure 7. The effect of the choice of comparison stars on the size of the amplitude deficit. The curves are the V data, offset to match the visual data, the points are the individual m_v estimates, and the ends of the "error bars" represent the brightness of the two comparison stars used for each estimate. (a) Light curve data for RU Cyg illustrating how a near-constant set of comparison star choices can lead to a substantial amplitude deficit. (b) Data for U Per showing how agile comparison star choices can minimize the impact of central clustering and lead to a small deficit for the light curve overall.

enhanced if the comparison star pairs do not completely cover the actual range of variation. The effect of comparison star choices is illustrated in Figure 7.

The data shown in Figure 7 represent the bulk of the AAVSO visual estimates for these stars over the selected JD interval and for which comparison star information is available. Only a limited JD range is shown for clarity, but it is representative. RU Cyg is particularly interesting in this context because it has very little variation in B–V, eliminating color effects as a confounding factor. The variation in B–V for U Per is larger, but still small. The comparison star choices for several other stars that were analyzed in this way were more erratic so the deficits, or lack of them, were less visually obvious. In Figure 7a one can see that most of the estimates were made with nearly the same set of comparison stars, which do not always span the variation in V. This is not unexpected for a star with a small amplitude. It is also apparent that most of the estimates are near the middle of the comparison star ranges, as was shown in Figure 5. The result is a substantial underestimate of the amplitude of variation of RU Cyg and a relatively large deficit of 0.41. Much of the data in this case came from two observers. The data for U Per in Figure 7b illustrate the opposite situation: the V amplitude is large, the comparison star pairs follow that variation, and there is little underestimation of the amplitude of the light curve overall in spite of some central clustering. This case is discussed in more detail below.

The identification of minimal adjustment of the comparison star pairs for stars with smaller amplitudes as a contributing factor to the deficits reinforces the conclusion in section 2 that deficits generally decrease with increasing V amplitude as shown in Figure 3.

7. Discussion

Visual estimates of the brightness of red variable stars could be affected by many factors, but the surprising nature of the m_v – V vs. V distributions and their associated amplitude deficits does not appear to be caused by most of them. In particular, the persistence of this phenomenon in the OCC data sets eliminates many possibilities involving charts, comparison stars, and observers. The primary causes seem to be normal observational scatter, effects associated with the limitations of fixed comparison star intervals, and a tendency for observers to report estimates that are in the middle of the range between the comparison stars. These effects can operate in conjunction with a set of comparison star choices that are relatively restricted to amplify the deficits. None of this suggests any misbehavior on the part of the observers. If the amplitude of the star's variation is small then the useful comparison star options are limited; estimates near the middle of a pair of comparison stars, or at least not near the ends, may arise both because it is not easy to interpolate between them and because the pair may have been chosen to straddle the brightness of the variable.

This conclusion requires some qualification. This project involved a large amount of data (one spreadsheet had 1.5 million cells) and the possible involvement of numerous processes, so there is always the chance that something of significance slipped by unnoticed. The details of the variable star estimating process involve the ways in which the observers think and perceive, and that is both complicated and idiosyncratic. It is therefore possible, or likely, that the explanation for the deficits and other effects is different in different situations. There is also the possibility of selection effects. The analysis presented here was done with estimates that were accompanied by the identification of both the comparison stars and the chart that was used. These are limited subsets of the vast AAVSO database and in some cases are dominated by a small number of observers. However, the persistence of the deficit effect over a wide range of stars, observers, and other parameters suggests the likelihood of a dominant process. The fact that central clustering is pervasive and the role of a restricted set of comparison stars is clearly involved in some cases suggests that the combination of these two effects is that dominant process.

With this insight the origins of the strange two-part m_v – V vs. V distribution for U Per shown in Figure 2 becomes more clear. As described in Section 6 the relatively large amplitude of U Per results in a small deficit for the light curve as a whole, but there is a clear V-dependence in the sloped section at the bright end of the distribution. This can be understood by noticing that the corresponding regions around the maxima in the U Per light curve in Figure 7b are roughly flat and therefore similar to the RU Cyg data shown in Figure 7a. Most of the U Per OCC sets that were analyzed showed substantial central clustering (although the two shown in Figure 5 are not among them) so the amount of variation near the maxima is underestimated by the visual observers for the same reasons that were given for RU Cyg in section 6, leading to the slope in the bright section of the m_v – V vs. V distribution. This process is not applicable for the fainter section for which the distribution is flat. Another way to see the difference in the two sections is to realize that there are many more observations when the star is bright, as can be seen in Figure 7b, which means that there are many OCC sets whose m_v – V vs. V distributions are roughly aligned in the way that is illustrated in Figure 4, resulting in a strong, well-defined slope. The distributions of the OCC sets associated with the faint part of the light curve are spread out in V because of the star's more pronounced variation when faint, leading to a broad swath of OCC distributions that are flat overall.

This work was motivated by the need to discover whether the odd star-dependent behavior of the m_v – V vs. V relationship for red stars is a symptom of some underlying problem. It appears that it is not, but only reflects natural processes involved in making the estimates that do not compromise the overall calibration of the relationship between m_v – V and V. In addition, a number of other possible effects have been explored and eliminated as explanations.

8. Conclusions

The accurate visual estimation of the brightness of variable stars—especially red ones—is a tricky business that the members of AAVSO and similar organizations have executed admirably well. Given the difficulty of the task and the complexity of human vision, it is not surprising that some systematic effects occur. The most significant is the overall tendency of visual observers to increasingly underestimate the brightness of stars as they become redder. The analysis presented here addresses the more limited question of why visual observers often underestimate the amplitudes of the variations as well. The primary mechanism appears to involve a pervasive tendency to report estimates that are clumped toward the center of the range between the comparison star magnitudes as well as the observer's choice of comparison stars. When the amplitude of the variable is small there will be little variation in the choice of comparison stars and central clumping of the estimates will have a greater effect than if the star's amplitude is greater. This leads to the observed decrease in amplitude deficit with increasing variable star amplitude. Unlike the overall color-induced error the deficits appear not to be a direct consequence of the stars' red colors. At the most basic level the phenomena discussed here arise because the photometric V measurement process is "seamless" while the visual observing process is fragmented by the use of discrete pairs of comparison stars. This result is based on the investigation of a limited set of stars, the range of possible relevant phenomena is large, and the influence of individual observers may be significant, so the explanations offered here may not be universal and should be treated with appropriate caution, but they do appear to be plausible.

The behavior of the variable leaves the observer with little discretion in the choice of comparison stars but perhaps greater awareness of the tendency to pick central values—sometimes appropriately and sometimes not—might lead to some mitigation of the effect of this phenomenon on visual light curves.

While this investigation considered only a particular group of red stars, further work may show whether the processes described here are relevant for other kinds of stars as well.

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