

SZ Sculptoris: Light Curve Analyses and Period Study of the Totally Eclipsing, Galactic South Pole, Solar-Type Binary

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Abstract We report here on the first precision *BVRI* observations, analysis and period study of SZ Sculptoris (GSC 6990-0597), a solar type ($T \sim 5040$ K), shallow contact, eclipsing binary. It was observed on 2019 October 05 and 2019 November 05 and 07 in remote mode with the Cerro Tololo InterAmerican Observatory 0.6-m SARA South reflector by R. Samec and R. Hill. The amplitude of the light curves was 0.74, 0.67, 0.65, and 0.61 mag in *B*, *V*, *R*, *I*, respectively. Six times of minimum light were calculated from three primary eclipses and three secondary eclipses with our present observations. Eight times of low light were also taken from ASAS-SN observations. Two additional timings were taken from the *BBSAG Bulletin* No. 39 and from APASS observations. From this 41-year interval orbital period study, it was found that the period is decreasing. This may be due to angular momentum loss (AML) resulting from rotating ion streams leaving along stiff magnetic bipolar field lines from the system. A linear ephemeris was also calculated. A *BVRI* Bessel filtered Wilson-Devinney Program (WD) solution gives a mass ratio, $m_2/m_1 = 0.3680 \pm 0.0007$, a small component temperature difference of 160 K, and a contact fill-out of only 7%. Thus, the system is in good thermal contact with a low fill-out. A mid-latitude spot (colatitude of 70°), radius of 18° , and T-factor of 0.66 ± 0.01 was calculated. The system is a W-type W UMa binary. An eclipse duration of ~ 21 minutes was determined for the primary eclipse and the light curve solution.

1. Introduction

In this paper, we continue our study of precision light curves of neglected, short period solar-type Southern eclipsing binaries (PY Aqr, Samec *et al.* 2019; CW Scl, Samec *et al.* 2017; DD Ind, Samec *et al.* 2016; ZZ Eri, Samec *et al.* 2015; etc.). This particular binary is of solar type. It is near the Southern galactic pole, and appears to have a time of constant light from the plot of the All-Sky Automated Survey for Supernovae (ASAS; Pojmański 2002) curves.

2. History and observations

SZ Scl is given in the *General Catalogue of Variable Stars* (GCVS; Samus *et al.* 2017) as an EW/KW eclipsing binary with a maximum of $V = 12.98$ and a minimum of 13.68 magnitudes. Its ephemeris is given as

$$\text{HJD} = 2444406.8677\text{d} + 0.32082757\text{d} \times E \quad (1)$$

SIMBAD gives $J = 10.283 \pm 0.024$, $K = 9.761 \pm 0.020$, so $J-K = 0.522 \pm 0.044$, and lists the variable as a W UMa binary. Observations of the system are continuously being undertaken by the ASAS-SN program of Ohio State (Shappee *et al.* 2014; Kochanek *et al.* 2017). See the plot in Figure 1. The information included $V = 11.88$, amplitude = 0.59 mag, variable type EW, and an ephemeris:

$$\text{HJD} = 2457547.83478\text{d} + 0.320818\text{d} \times E \quad (2)$$

Our 2019 *BVRI* light curves were taken on 2020 October 05 and 2019 November 05 and 07 at Cerro Tololo InterAmerican Observatory with the 0.6-m SARA South reflector by R. Samec and R. Hill with a thermoelectrically cooled (-50°C) 2KX2K ANDOR Camera with Bessell *BVRI* filters. These were taken in remote mode. Individual observations (differential photometry, V-C) included 598 in *B*, 601 in *V*, 589 in *R*, and 553 in *I*. The standard error of a single observation is 13 mmag in *B* and *V*, 14 mmag in *R*, and 16 mmag in *I*. The nightly C-K values stayed constant throughout the observing run with a precision of about 1%. Exposure times varied from 50s in *B* to 30s in *V* and 20s in *R* and *I*. The *BVRI* observations are given in Table 1. The identity of the photometric targets, V (variable, SZ Scl), C (comparison star), and K (check star) are given in Table 2. A finding chart for the field is given in Figure 2. The UCAC3

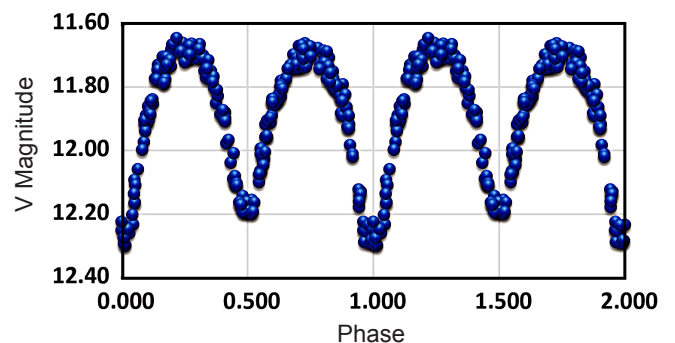


Figure 1. Light curve of ASASSN-V J001546.63-310545.4 (SZ Scl) (Shappee *et al.* 2014; Kochanek *et al.* 2017).

Table 1. Sample of first ten SZ SCL *B, V, R, I* observations.

ΔB	HJD 2458770+	ΔV	HJD 2458770+	ΔR	HJD 2458770+	ΔI	HJD 2458770+
1.108	1.5566	0.887	1.5518	0.779	1.5548	0.667	1.5554
1.127	1.5607	0.922	1.5580	0.788	1.5589	0.677	1.5595
1.142	1.5648	0.941	1.5622	0.829	1.5631	0.709	1.5636
1.191	1.5690	0.972	1.5663	0.861	1.5672	0.712	1.5678
1.237	1.5731	1.016	1.5705	0.899	1.5713	0.771	1.5719
1.312	1.5774	1.069	1.5746	0.957	1.5756	0.832	1.5762
1.444	1.5816	1.152	1.5789	1.015	1.5798	0.904	1.5804
1.550	1.6042	1.250	1.5830	1.191	1.5922	1.045	1.5928
1.467	1.6097	1.273	1.6061	1.191	1.5964	1.037	1.5971
1.410	1.6152	1.218	1.6117	1.175	1.6018	1.053	1.6026

Note: First ten data points of SZ Scl *B, V, R, I* observations.

The full table is available through the AAVSO ftp site at <ftp://ftp.aavso.org/public/datasets/samec482-szscl.txt> (if necessary, copy and paste link into the address bar of a web browser).

Table 2. Photometric targets.

Star	Name	R. A. (2000) h m s	Dec. (2000) ° ' "	V^1	$J-K^2$
V (Variable)	SZ Scl GSC 6990 597 2MASS J00154668-3105449 ASAS 182528-6734.8 Gaia DR2 2319938821297094912	00 15 46.6730013523	-31 05 45.037874181 ³	11.831	0.522 ± 0.044
C (Comparison)	GSC 4515 626 3UC 336-015704	00 15 38.2369562975	-31 08 32. 536651500 ³	10.901	0.312 ± 0.052
K (Check)	GSC 6990 613 3UC118-000630	00 15 49.86144	-31 07 35.8824 ²	12.411 ± 0.017	0.36

¹ APASS (Henden et al. 2009). ² UCAC3 (Zacharias et al. 2010). ³ Gaia DR2 (Gaia Collab. 2016, 2018).

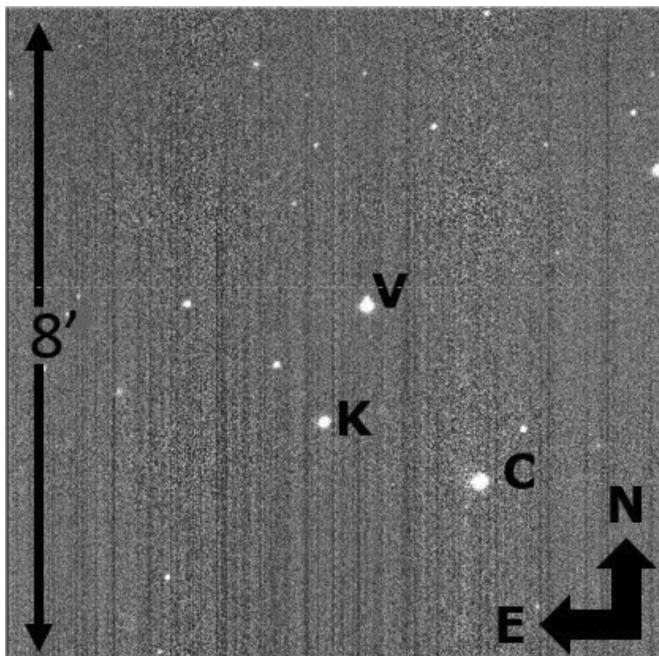


Figure 2. Finding chart of the SZ Scl field; variable star, SZ Scl (V), comparison star (C), check star (K).

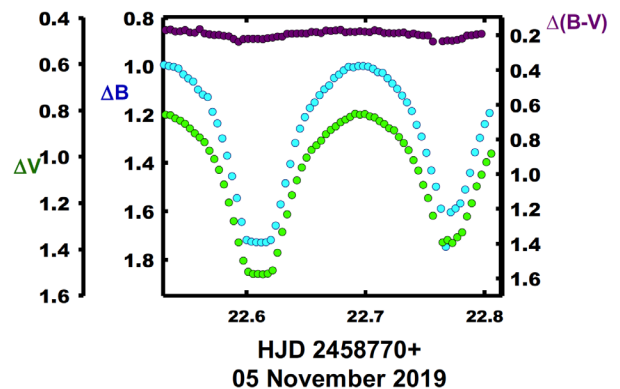


Figure 3. *B, V* light curves and *B-V* color curve from 2019 November 05.

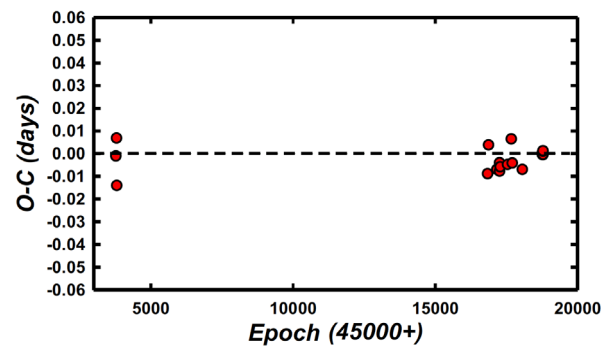


Figure 4. A plot of the quadratic residuals. Note there is a gap in the observations from 3,900 to 16,500 orbits.

is the USNO CCD Astrograph Catalog (Zacharias *et al.* 2010). Sample *B*, *V* light curves of 2019 November 05 are given in Figure 3.

3. Period study

Six times of minimum light were determined from our present *BVRI* observations, which included three primary eclipses and three secondary eclipses:

$$\begin{aligned} \text{HJD I} &= 2458771.75786 \pm 0.00029, \\ &2458792.61086 \pm 0.00010, \\ &2458794.53585 \pm 0.00021 \text{ d}, \end{aligned}$$

$$\begin{aligned} \text{HJD II} &= 2458771.59807 \pm 0.00094, \\ &2458792.77217 \pm 0.00084, \\ &2458794.69767 \pm 0.00079 \text{ d}. \end{aligned}$$

Minima were calculated using a least squares minimization method (Mikulášek *et al.* 2014) and was used to determine the minima for each curve, in *B*, *V*, *R*, and *I*. These were averaged and the standard error was computed.

Eight times of low light were also taken from data taken from ASAS SN observations. (The ASAS observations included ~250 data points spread over a little more than the last four years. Not enough points (maybe only one) were taken on each night to do normal times of minimum light. Minima were obtained from a plot of all the SN ASAS data phased with their period. Next, the data were fit with least square parabolas at the primary and secondary minima. The HJDs of low values of data nearest the times of minima were called times of “low light” (within 0.001 phase unit of the minima). These are not standard times of “minimum light”, but they work well when the period study has gaps. Note their low weights.) Three additional timings of minima were taken from *BBSAG Bulletin* No. 39 (Locher 1978). Two minima were calculated from APASS data (Henden *et al.* 2009) on SZ Scl. Over this 41-year interval, the orbital period appears to be decreasing (at about the 45 sigma level; the errors shown here are standard errors). However, a large gap from HJD 43000 to 57000 is noted. This means the system should be patrolled for minima timings over the next decades to discern the true nature of the orbital period variations. The existence of the early BBSAG minima add an interesting but unsatisfying mystery to the present understanding of this binary.

$$\begin{aligned} \text{JD Hel Min I} &= 2458792.61142(77)\text{d} + 0.3208167(4) \\ &\times E - 0.00000000177(8) \times E^2 \end{aligned} \quad (3)$$

A decreasing period may be due to angular momentum loss (AML) resulting from rotating plasma streams leaving along stiff magnetic bipolar field lines from the system which are expected in such a binary.

A linear ephemeris was also calculated:

$$\text{JD Hel Min I} = 2458792.6187(26)\text{d} + 0.32082509(15) \times E. \quad (4)$$

The quadratic residuals are shown in Figure 4. Table 3

gives the quadratic and linear residuals of the period study. The *B*, *V* and *R*, *I* light curves and *B–V* and *R–I* color curves phased with Equation 3 are given in Figures 5, 6, and 7. In Figure 6, an expanded *B–V* color curve only is given to emphasize the variation in temperature of the system over the month due to spot activity.

The quadratic ephemeris yields a $\dot{P} = 1.09 \times 10^{-7}$ d/yr or a mass exchange rate of

$$\frac{dM}{dt} = \frac{\dot{P} M_1 M_2}{3P (M_1 - M_2)} = \frac{5.289 \times 10^{-7} M_{\odot}}{d}. \quad (5)$$

in a conservative scenario.

4. Light curve characteristics

The curves are of good photometric precision, averaging about 1%. *BVRI* curve averages at quarter phased cycles and key differences are given in Table 4. The amplitude of the light curve varies from 0.74 to 0.61 mag in *B* to *I*. The O’Connell effect, a possible indicator of spot activity, is less than the noise level. The differences in minima is ~0.12 mag, indicating contact light curves. All color curves fall so very slightly at phase 0.0 and phase 0.5, which we have noticed is characteristic of contact binaries. This probably indicates that it is a W UMa contact binary.

5. Temperature

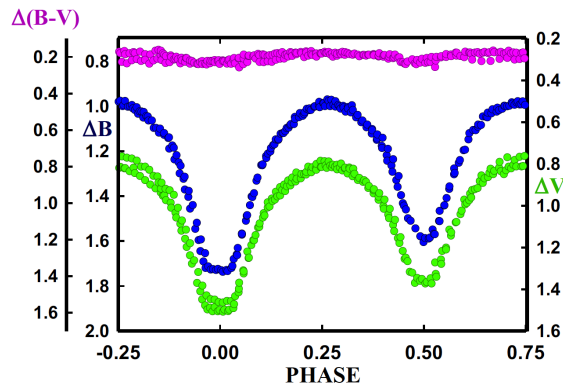
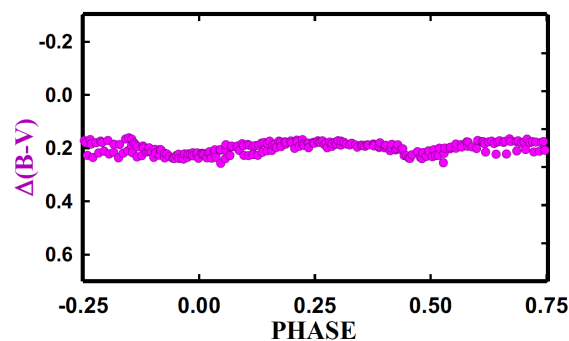
The 2MASS J–K = 0.522 ± 0.044 (SIMBAD) for the binary. This corresponds to a K2 ± 2V eclipsing binary, which yields a temperature of 5040 ± 250 K. Thus the binary is a dwarf, low temperature system. Binary stars of this type are noted for having convective atmospheres, so solar type spots and other magnetic activity are expected. In the case of fast spinning binaries, magnetic activity is especially high.

6. Light curve solution

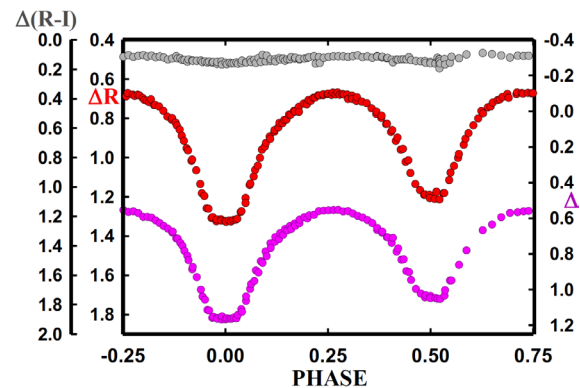
The *B*, *V*, *R*, and *I* curves were pre-modeled with BINARY MAKER 3.0 (Bradstreet and Steelman 2002) and fits were determined in all filter bands. The averaged result of the best fitted residuals was that of a W-type contact binary with a low fill-out of about 11% with one major cool spot, about 0.9 times the temperature of the photosphere and a mass ratio range of 0.34 to 0.39. BINARY MAKER uses black body atmospheres so the first iterations with the Wilson program were the lights with full Kurucz atmospheres (1993). The averaged parameters were input into a 4-color simultaneous light curve calculation using the Wilson-Devinney Program (Wilson and Devinney 1971; Wilson 1990, 1994, 2008, 2012; Van Hamme and Wilson 1998; Van Hamme and Wilson 2007; Wilson *et al.* 2010; Wilson and Van Hamme 2014). The solution was computed in Mode 3 (contact) and converged with similar physical parameters as the binary maker fit. Convective parameters $g=0.32$, $A=0.5$ were used. The third light parameter yielded solid results which may be due to an unseen field star which is only 5.7% the brightness of SZ Scl at phase 0.25 in *V*. The mass ratio was found to be

Table 3. Period Study of SZ Scl.

	Epoch HJD 2400000+	Cycle	Linear Residual	Quadratic Residual	Weight	Error	Reference
1	43812.3150	-46693.0	-0.0180	-0.0141	0.2	—	BBSAG Bull 39 (Locher 1978)
2	43783.4530	-46783.0	-0.0057	-0.0011	0.5	—	BBSAG Bull 39 (Locher 1978)
3	43805.4380	-46714.5	0.0028	0.0068	0.5	—	BBSAG Bull 39 (Locher 1978)
4	56847.8050	-6062.0	0.0279	-0.0089	0.1	—	ASAS (Pojmański 2002)
5	56882.3057	-5954.5	0.0399	0.0038	0.5	0.0064	APASS (Henden <i>et al.</i> 2009)
6	57175.8440	-5039.5	0.0233	-0.0070	0.1	—	ASAS (Pojmański 2002)
7	57267.6010	-4753.5	0.0243	-0.0041	0.1	—	ASAS (Pojmański 2002)
8	57269.8430	-4746.5	0.0205	-0.0078	0.1	—	ASAS (Pojmański 2002)
9	57291.8210	-4678.0	0.0220	-0.0059	0.1	—	ASAS (Pojmański 2002)
10	57547.8350	-3880.0	0.0176	-0.0049	0.1	—	ASAS (Pojmański 2002)
11	57680.0233	-3468.0	0.0259	0.0064	0.5	—	APASS (Henden <i>et al.</i> 2009)
12	57714.6610	-3360.0	0.0145	-0.0042	0.1	—	ASAS (Pojmański 2002)
13	58069.6430	-2253.5	0.0036	-0.0070	0.1	—	ASAS (Pojmański 2002)
14	58771.5981	-65.5	-0.0067	0.0001	1.0	0.0009	Present observation
15	58771.7579	-65.0	-0.0073	-0.0005	1.0	0.0003	Present observation
16	58792.6109	0.0	-0.0079	-0.0006	1.0	0.0001	Present observation
17	58792.7722	0.5	-0.0070	0.0003	1.0	0.0008	Present observation
18	58794.5359	6.0	-0.0079	-0.0005	1.0	0.0002	Present observation
19	58794.6977	6.5	-0.0065	0.0009	1.0	0.0008	Present observation

Figure 5. Phased B , V light curves and $B-V$ color curves. Note the effect of the spots acting over a month's time. The maximum change was ~ 0.06 mags in V .Figure 6. $B-V$ color index curve only, to emphasize the variation in temperature of the system over the month due to spot activity.

0.37 with a fill-out of only 7%. The cool spot did converge with some changes as expected with t-factor of 0.66 and mid-latitude position (colatitude 70°). The secondary eclipse showed an interval of constant light, an eclipse of 21 minutes' duration. The inclination was $88.2 \pm 0.5^\circ$. The difference of component temperatures was low, about 160 K, showing that the system is in good thermal contact despite its shallow contact. The modeled

Figure 7. Phased R , I light curves and $R-I$ color curve.

period was 0.3208185 d. The solution is given in Table 5. The plots of the light curve solutions are shown in Figures 8 and 9. The geometrical surfaces of the binary at phases 0.0, 0.25, 0.5, and 0.75 are displayed in Figure 10a-d.

7. Discussion

SZ Scl is a W-type (the more massive component is cooler) shallow contact W UMa binary. The period decrease is probably due to magnetic braking which causes the binary to lose angular momentum as plasmas leave the system on stiff rotating bipolar magnetic field lines. The cool spot is a firm indication of the magnetic nature of the binary. Both components of this binary are of solar type with a surface temperature of ~ 5203 K (G9V) and ~ 5040 K (K2V) for the components. The mass ratio is 0.37 (M_2/M_1), with an amplitude of 0.74–0.61 mag in B to I , respectively. The inclination is 88.2° , which results in a total eclipse at phase 0.0.

Table 4. Light curve characteristics of SZ Scl.

Filter	Phase 0.000	Magnitude $\pm \sigma$ Min. I	Phase 0.25	Magnitude $\pm \sigma$ Max I
B		1.731 \pm 0.005		0.992 \pm 0.010
V		1.485 \pm 0.023		0.812 \pm 0.015
R		1.327 \pm 0.003		0.675 \pm 0.007
I		1.166 \pm 0.004		0.556 \pm 0.001
Filter	Phase 0.500	Magnitude $\pm \sigma$ Min. II	Phase 0.75	Magnitude $\pm \sigma$ Max II
B		1.591 \pm 0.013		0.988 \pm 0.008
V		1.363 \pm 0.009		0.802 \pm 0.026
R		1.204 \pm 0.011		0.673 \pm 0.002
I		1.047 \pm 0.007		0.562 \pm 0.001
Filter	Min. I – Max. I $\pm \sigma$	Max. I – Max. II $\pm \sigma$	Min. I – Min. II $\pm \sigma$	
B	0.739 \pm 0.015	0.005 \pm 0.018	0.140 \pm 0.019	
V	0.674 \pm 0.038	0.010 \pm 0.042	0.122 \pm 0.032	
R	0.652 \pm 0.010	0.003 \pm 0.008	0.123 \pm 0.014	
I	0.610 \pm 0.005	-0.006 \pm 0.002	0.119 \pm 0.011	
Filter	Min. II – Max. II $\pm \sigma$	Min. II – Max. I $\pm \sigma$		
B	0.604 \pm 0.022	0.599 \pm 0.023		
V	0.561 \pm 0.035	0.552 \pm 0.025		
R	0.531 \pm 0.013	0.529 \pm 0.018		
I	0.485 \pm 0.008	0.491 \pm 0.008		

Table 5. *B, V, R, I* synthetic light curve solution for SZ Scl.

Parameters	Values
$\lambda_B, \lambda_V, \lambda_R, \lambda_I$ (nm)	440, 550, 640, 790
$g_1 = g_2$	0.32
$A_1 = A_2$	0.5
Inclination ($^\circ$)	88.2 \pm 0.5
T_1, T_2 (K)	5040, 5203 \pm 2
Ω	2.5968 \pm 0.0017
$q(m_2/m_1)$	0.3680 \pm 0.0007
Fill-outs: F_1, F_2 (%)	0.07 \pm 0.01
$L_1/(L_1+L_2)_I$	0.6836 \pm 0.0073
$L_1/(L_1+L_2)_R$	0.6804 \pm 0.0072
$L_1/(L_1+L_2)_V$	0.6734 \pm 0.0015
$L_1/(L_1+L_2)_B$	0.6624 \pm 0.0086
JDo (days)	2458792.6113 \pm 0.0001
Period (days)	0.3208185 \pm 0.0000002
L_{3I} at phase 0.25 ¹	0.0736 \pm 0.0007
L_{3R} at phase 0.25	0.0551 \pm 0.0005
L_{3V} at phase 0.25	0.0568 \pm 0.0012
L_{3B} at phase 0.25	0.0618 \pm 0.0007
$r_1/a, r_2/a$ (pole)	0.4424 \pm 0.0009, 0.2792 \pm 0.0015
$r_1/a, r_2/a$ (side)	0.4737 \pm 0.0013, 0.2915 \pm 0.0018
$r_1/a, r_2/a$ (back)	0.5012 \pm 0.0017, 0.3264 \pm 0.0032
Spot I, Primary Component	
Colatitude ($^\circ$)	69.9 \pm 0.3
Longitude ($^\circ$)	174.1 \pm 0.3
Radius ($^\circ$)	18.06 \pm 0.07
T-Factor	0.66 \pm 0.01

¹ Third Light at phase 0.25

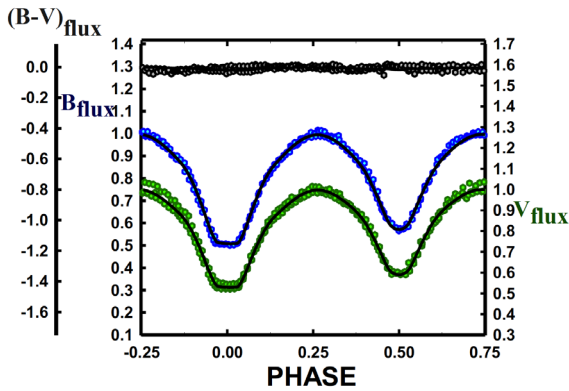


Figure 8. *B, V* and *B–V* normalized flux overlaid by the third light wd solution.

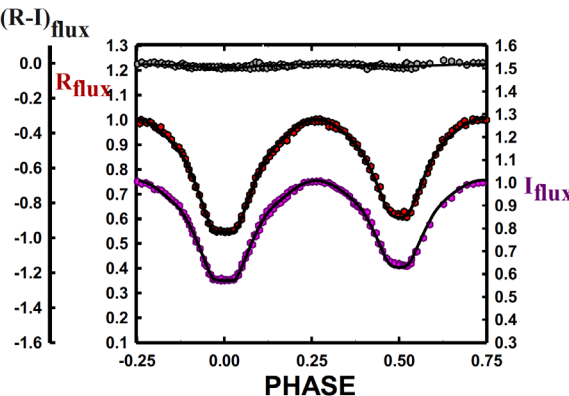


Figure 9. *R, I* and *R–I* normalized flux overlaid by the third light wd solution.

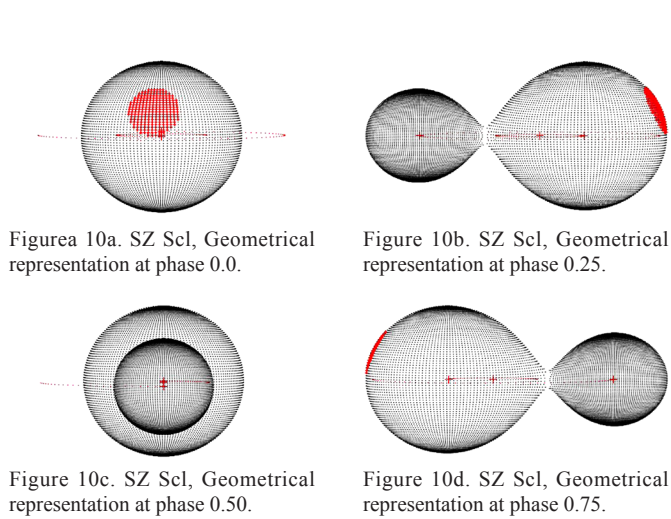


Figure 10a. SZ Scl, Geometrical representation at phase 0.0.

Figure 10b. SZ Scl, Geometrical representation at phase 0.25.

Figure 10c. SZ Scl, Geometrical representation at phase 0.50.

Figure 10d. SZ Scl, Geometrical representation at phase 0.75.

8. Conclusion

The period study of this contact W UMa binary has a 41-year duration. The period was found to be strongly decreasing at about the 45σ level. Albeit, there was a large gap in the observations which begs further study. If this result holds up under more study, the system may merge over time, first resulting in a Red Novae event (Tylenda and Kamiński 2016) and the coalescence of the system into a fast-rotating FK Comae Berenices variable (Sikora *et al.* 2020).

9. Future work

Radial velocity curves are needed to obtain absolute system parameters.

10. Acknowledgements

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