Daylight Photometry of Bright Stars—Observations of Betelgeuse at Solar Conjunction

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Abstract   Betelgeuse is an important variable star with many observations in the AAVSO International Database, but there is an annual gap of about four months where Betelgeuse is close to the sun and not observable at night. This gap could be filled with daylight observations. The star is bright enough to be imaged with small telescopes during the day, so photometry is possible when the sun is up. We present V-band photometry of α Ori taken with an amateur telescope equipped with an interline-transfer CCD camera and neutral density filter. These data compare favorably with contemporaneous nighttime photometry. The method used is a variation on ensemble photometry (using other bright daytime stars), and involves large stacks of very short exposures. The ensemble method provided V magnitudes of Betelgeuse with calculated errors of (0.020 ± 0.008) mag from February to April 2021. From May to July, at the closest distances to the sun, the photometry of Betelgeuse could be continued with mean errors of (0.040 ± 0.013) mag.

1. Introduction

Betelgeuse has been a subject of great astrophysical interest. Ground-based telescopes can acquire reliable photometry from roughly early September to late April. That leaves four months without data in each year's light curve. Given the extraordinary recent behavior of this star, that break in coverage is most unfortunate. While space-based photometry of Betelgeuse is now practiced during solar conjunctions (Dupree *et al.* 2020), the technique is difficult and cannot be performed in a standard photometric passband.

With care, it is possible to gather daytime aperture photometry of Betelgeuse. Such data were collected from February to July of 2021 with a small telescope located in Central Europe at only 200 m elevation, where the atmospheric clarity is far from exceptional.

2. Observations and data acquisition

The telescope used is of Newtonian configuration with aperture 250 mm and focal length 1250 mm, carried on a computer-controlled German equatorial mount. The optical tube is fitted with a cylindrical sun shade extending 50 cm beyond the aperture. Together with a secondary shade around the focuser aperture, the shielding permits pointings as close as 10 degrees to the sun.

The camera is an ATIK model 460exm CCD, equipped with a wheel of photometric filters. For daylight work an additional neutral density (ND) filter of 1 percent transmission (as proposed by Miles 2007) is mounted ahead of the wheel. The camera is cooled to 0°C for daytime work and has its own sun shade. Pointing of the mount is accurate enough to place a target star directly in the camera field of 20'×30'.

During photometry only a 687×550-pixel region of the CCD is read out. Image frames were typically integrated for 0.1 sec, though sometimes for 0.2 sec. This kept the daytime sky to 50 percent or less of the sensor full-well depth. On each target, consecutive frames were taken to accumulate 10 sec of total exposure time. This process takes approximately 150 sec when using 0.1 sec exposures. Since guiding is not possible in daylight, the pointing is subject to drift. To mitigate this effect, it is helpful to use groups of ten exposures co-added and saved and later re-registered to make the 10-sec stack (this was done for about half of the presented photometry). Flat-field images were acquired by exposing against the daytime sky through a 3 mm-thick white polystyrene foam board. The benefit of the 10-sec stack is illustrated in Figure 1. Depending upon the surrounding sky brightness, stars as dim as V=6.5 can be successfully sampled this way.

3. Sky background

The brightness of the sky can be measured (in mag/arcsec²) if a bright star with known magnitude is included in the field. The value is calculated by the formula:

$$m_{sky} = -2.5 \log \left(\frac{N_{sky}}{p^2 N_{star}} \right) + m_{star} \qquad (1)$$

where N_{sky} = mean counts/pixel within sky annulus; N_{star} = summed pixel counts within star aperture after subtraction of sky background; and p = pixel scale in arcsec/pixel.

This method was first tested on the nighttime sky and gave values comparable to those of the "Sky Quality Meter" (SQM; Cinzano 2005), which is not usable in daylight. The individual measurements were repeatable within ± 0.1 mag/arcsec².

The daylight sky brightness depends on several factors, the prime one being angular distance of the field from the sun. As a test, the sky brightness was measured on 17 cloudless days (July to September 2020 and February to April 2021) with selections from 22 stars at different distances from the sun. The altitude of the sun ranged from 10° to 52°. The results are seen in Figure 2. As expected, the smaller the angular distance from the sun, the higher the sky brightness.

Other factors are the altitude of the sun and the level of water vapor in the atmosphere. The latter is correlated to the extinction coefficient, so there is also a correlation between sky brightness and extinction, which can be seen in Figure 3. This figure shows the measured sky brightness around β Aur versus the measured extinction coefficient during 9 days between May 31 and July 22, 2021, around local solar noon.

Additional factors, e.g. airmass or the strong polarization of light at daytime can influence the sky brightness as well.

4. Photometry methods

4.1. Instrumental and standardized magnitude

Instrumental magnitudes m_{inst} were calculated using rectangular apertures for both star and sky background. The instrumental magnitude is given by:

$$m_{inst} = -2.5 \log \left(\frac{N_{star}}{t} \right) \qquad (2)$$

where N_{star} = background-subtracted pixel counts within star aperture; t = exposure time in s.

The processing of the images was done with the Fitsmag software package (Nickel 2021).

The standardized V magnitude of a star can be calculated from the instrumental magnitude by (Da Costa 1992):

$$V = m_{inst} + a_0 + a_1 (B-V) + a_2 X + a_3 X (B-V) + ... \qquad (3)$$

where $(B-V)$ = color index of star; X = airmass of the observation; a_0 = the zero point magnitude (in the following written as m_0); a_1 = the color-term (in the following called Transformation coefficient T_v; a_2 = primary extinction coefficient (in the following replaced by $-k_v$); a_3 = second order extinction coefficient. The second order extinction coefficient is very small for the V band and can be neglected.

Having in hand values for the transform coefficient and the target star's airmass, color, and instrumental magnitude, a standard magnitude can be calculated if the zero point and primary extinction coefficient can be established. This is done with observations of an ensemble of reference stars at a range of airmasses. For each of these stars, we have:

$$V = m_{inst} + m_0 + T_v (B-V) - k_v X \qquad (4)$$

and

$$V - (m_{inst} + T_v (B-V)) = m_0 - k_v X \qquad (5)$$

where k_v and m_0 are unknowns. If the differences $V-(m_{inst} + T_v(B-V))$ of several stars are plotted against their airmasses, then k_v and m_0 can be calculated by linear regression (LR) as shown in Figure 4.

With known m_0 and k_v the standard magnitude can be calculated by:

$$V_{var} = m_{inst} + T_v (B-V)_{var} + m_0 - kvX_{var} \qquad (6)$$

It can be shown (Appendix A) that the result of this calculation is

mathematically equivalent to differential ensemble photometry, where a mean value of differential magnitudes against an ensemble of comparison stars is calculated.

T_v for the combined V and ND filters was -0.0026 ± 0.0057. For the V filter alone it was $+0.024$, so the ND filter shifts the color significantly. The spectral transmission of the ND filter (Vendor: Antares) was analyzed by the author using a slit spectrograph measuring the fraction of transmitted light of a LED light source: it has a local peak at 550 nm, as shown from the transmission curve in Figure 5.

4.2. Signal to noise ratio

The well known sources of noise in CCD photometry are shot noise, thermal electronic noise, and readout noise. The quantum signal-to-noise-ratio (SNR) of an observation can be calculated using the "CCD equation" (Merline and Howell 1995). In a high-count regime, this equation is approximated by:

$$SNR_q = \frac{N_{star}}{\sqrt{N_{star} + n_{pix} \left(1 + \frac{n_{pix}}{n_{sky}} \right) N_{sky}}} \qquad (7)$$

where N_{star} = number of collected photo electrons from star; n_{pix}, n_{sky} = number of pixels in star and sky apertures; N_{sky} = number of collected electrons/pixel from sky background. The inverse of SNR_q may be called "normalized quantum noise" σ_q:

$$\sigma_q = \frac{1}{SNR_q}$$

In short exposures scintillation noise can be significant. The normalized scintillation noise σ_s is the standard deviation of a series of star intensities divided by the mean value of the intensities (if no other noise would be involved). The total noise σ_{total} and the SNR from both CCD noise σ_q and scintillation σ_s are:

$$\sigma_{total} = \sqrt{\sigma_q^2 + \sigma_s^2}, \qquad SNR = \sqrt{\frac{1}{\sigma_q^2 + \sigma_s^2}} \qquad (8)$$

Scintillation noise can be estimated with the modified "Young formula" (Young 1967 and Osborn *et al.* 2015):

$$\sigma_s = 0.003953 \times X^{3/2} D^{-2/3} e^{-H/8000} \sqrt{t^{-1}} \qquad (9)$$

where D = aperture of telescope in m; H = local height above sea level in m; X = airmass; t = exposure time in s. For the observing station ($D = 0.25$ m, $H = 200$ m) this gives the following values of σ_s:

	X = 1.0	X = 1.5	X = 2.0
t = 1s	0.00976	0.01793	0.0276
t = 10s	0.00309	0.00567	0.00874

Actual scintillation measurements (at daylight) were tested against the formula and demonstrate its reliability in this application.
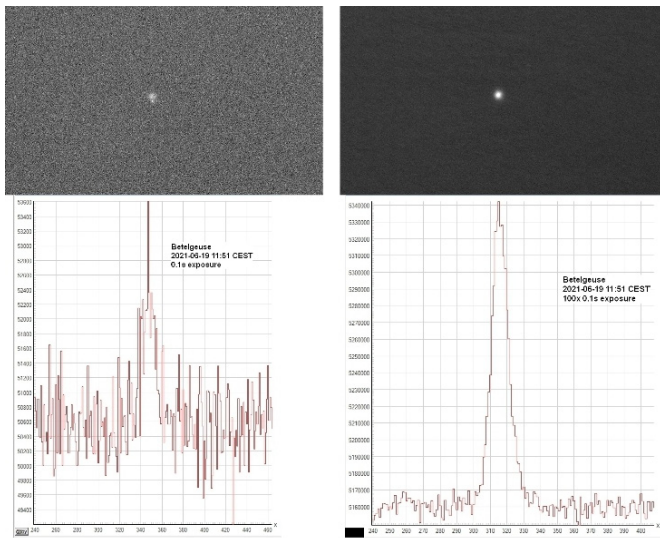
Figure 1. Images and profiles of Betelgeuse near to sun, left: single image, right: stack of 100 images.
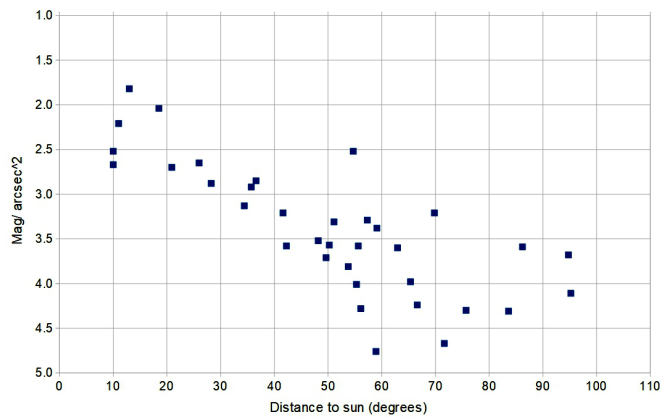

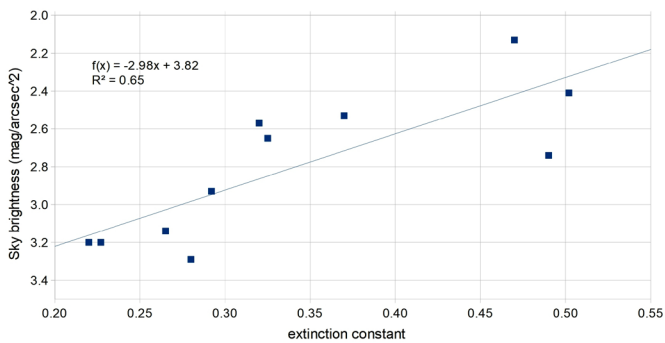Figure 2. Measured brightness of sky background in mag/arcsec² versus distance from sun.

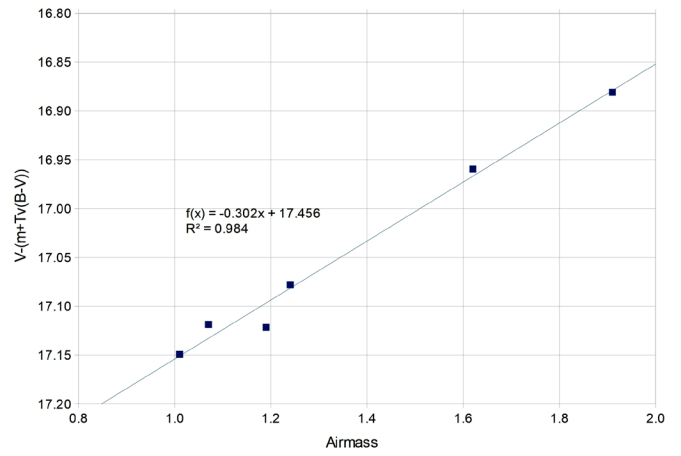Figure 3. Sky brightness around β Aur at noon versus extinction coefficient.

Figure 4. Plot of $V - (m_{inst} + T_v (B-V))$ vs. airmass of 6 bright stars with linear regression line.
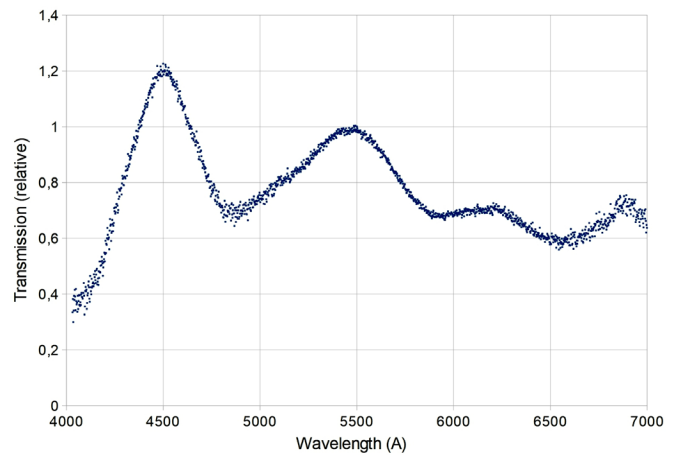

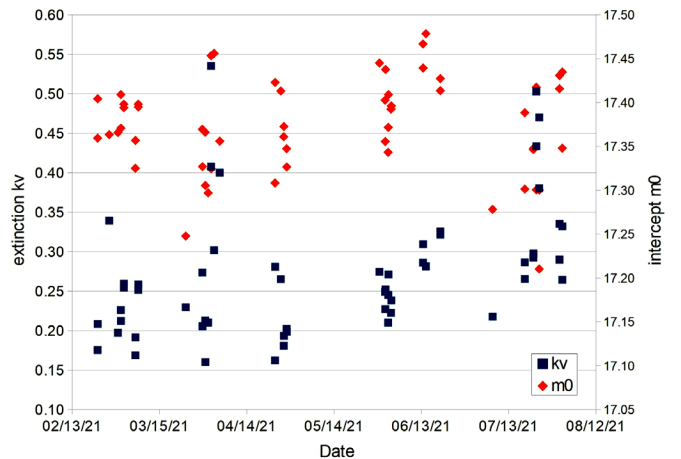Figure 5. Transmission curve of ND filter (units relative to 550 nm).

Figure 6. Values for extinction coefficient k_v (squares) and intercept m_0 (diamonds).

4.3. Total photometric error

The statistical error of ordinary differential photometry can be estimated as follows:

$$\text{error[mag]} = \sqrt{\text{error}^2\,(\text{variable}) + \text{error}^2\,(\text{comparison})} \quad (10)$$

In the ensemble approach used here, the zero point is the reference magnitude, and its error is a function of the LR:

$$\text{error[mag]} = \sqrt{\text{error}^2(\text{var}) + (X_{var} - \overline{X})^2\,\sigma^2\,(k) + \text{SER}^2} \quad (11)$$

where error(star) = $1.0857\,(1/\text{SNR(star)})$; X_{var} = airmass of variable; $\overline{X}$ = mean value of the airmass of the reference stars; σ_k = error of extinction coefficient; SER = standard error of the LR (Equation 5). This formula is derived in Appendix A.

4.4. Reference stars for Betelgeuse

The reference stars must be bright enough in comparison to the sky. The SNR should be not much below 50; this is the case if the star magnitude is nearly equal to the sky brightness (in mag/arcsec$_2$). From Figure 2, it follows that down to a distance of 30° to the sun the star magnitude should be at least V=2.8; between 15° and 30° it should be brighter than 2.

Most bright stars have a slight variability, therefore only those with a magnitude range of less than 0.1 mag (from GCVS (Samus *et al.* 2017) were selected as comparison stars. V magnitudes were taken from the Extended Hipparcos catalogue (Anderson and Francis 2012). Table 1 shows the selected stars.

Depending on their distance from the sun, not all of these stars can always be used. Betelgeuse is closest to the sun on June 20, 2021, therefore the distance of the stars to the sun are listed for this date in the table. Around this date β Tau and γ Gem are too close to the sun; from June 30, α and γ Gem cannot be used.

α Gem has two components (A, B), separated by 5.5", which have to be measured together. If it is used in the ensemble, the aperture must be large; that could increase the error. ζ Ori is a double star with 2.2" separation, which may be no problem. β Aur is an EA variable with known period; it should be used only outside of the eclipses.

5. Observation results

5.1. Results of reference stars and extinction

From February 21 to July 31, 2021, daytime observations of Betelgeuse, γ Ori (as check star) and 4 to 7 reference stars were performed during 61 runs on 33 days. The reference stars were selected from among the following: α Gem, α Ari, α Cas, β Ori, α CMi, β Gem, β Tau, ζ Ori, βAur, γ Gem.

The SNR of the star measurements used was typically above or around 100. Results with SNR below 50 (2 of 200) were discarded. The remaining range was between 53 and 290. The LR statistics for Betelgeuse observations are shown in Table 2. The first group represents data from February to April 2021, the second, May to July 2021, where the stars are closer to the sun. The errors of the second group are significantly higher, as can be expected from the brighter sky and from the worse seeing during this time. The extinction was also higher due to many hazy days. All values of k_v and m_0 are shown in Figure 6.

The whole range of extinction values was between 0.16 and 0.53; values above 0.3 corresponded to a very hazy sky. The standard error did not correlate significantly to the extinction value.

For the LR, up to one star was excluded if the deviation of the star magnitude from the regression line was significant ($>2\times\text{std.error}$) and greater than 0.05 mag. In 4 of 65 regression calculations this rule was used.

5.2. Observations of Betelgeuse (February to July 2021)

The daylight observations of Betelgeuse from February 21 to July 31, 2021, together with nighttime photoelectric photometry (PEP) are shown in Figure 7. The PEP data were collected with

Table 1. Reference stars for Betelgeuse.

Star Name	V mag. (XHip)	B–V mag.	δ mag. (GCVS)	Dist. to α Ori (°)	Sun Dist. (°) 2021-06-20	Comment
β Ori	0.18	–0.03	0.05	19	33	
α CMi	0.40	0.43	0.07	26	32	
β Gem	1.16	0.99	0.07	33	25	
α Gem	1.58	0.03	0	34	24	double (5.5")
γ Ori	1.64	–0.22	0.05	7.5	18	
β Tau	1.65	–0.13	0	22	7	
ζ Ori	1.74	–0.20	0.07	10	25	double (2.2")
β Aur	1.90	0.08	0.09	37	21.5	EA (P = 3.96 d)
γ Gem	1.93	0.00	0	14	12	
α Ari	2.01	1.15	0.06	57	51	
α Cas	2.24	1.17	0.07	78	64	

Table 2. Statistics of linear regression from February to July 2021.

	February to April Mean Value (std. dev.)	May to July Mean Value (std. dev.)
Standard error	0.017 (0.008)	0.036 (0.011)
Extinction constant (k_v)	0.247 (0.084)	0.299 (0.073)
Error of (k_v)	0.027 (0.010)	0.045 (0.016)
Intercept (m_0)	17.365 (0.047)	17.381 (0.064)
(R^2)	0.962 (0.036)	0.917 (0.056)

Table 3. The mean values of the differences between day and night magnitudes.

	Difference (day – night) (mag.)	Error (day) (mag.)	Error (night) (mag.)
Mean value (n = 6)	0.006	0.018	0.020
Std. dev.	0.026	0.007	0.012
Error of mean	0.011	0.003	0.005

an Optec SSP-3 photometer mounted on a 235-mm telescope in North America. The PEP V data were taken in concert with B data, and the Δ(B–V) with respect to the comparison star (HD 37160) was established from the instrumental magnitudes and transformation coefficients. During the period 2459255 (Feb 9, 2021) to 2459327 (Apr. 22, 2021) the mean Δ(B–V) was 0.910 with a standard deviation of 0.010. The comparison (B–V) is taken as 0.958, implying that the mean (B–V) of Betelgeuse was 1.868. The color transformation of the daylight measurements were based on a constant (B–V) of Betelgeuse of 1.85 (from Hipparcos).
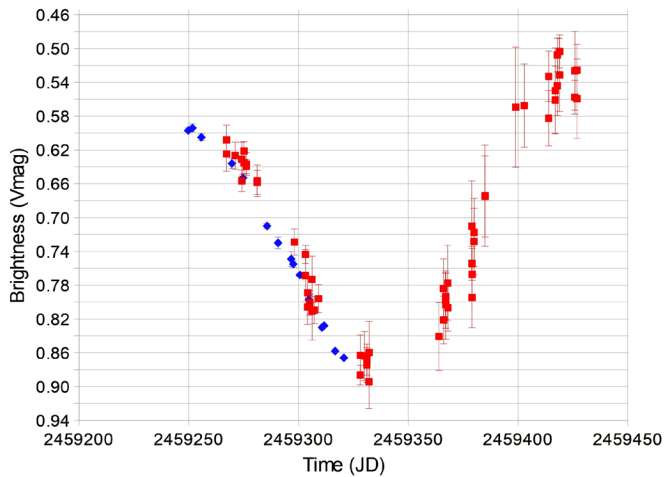
Figure 7. Light curve (V-magnitudes) of Betelgeuse February to July 2021. Blue diamonds: PEP data (night), Red squares: CCD data (daylight). PEP error bars are too small to see.
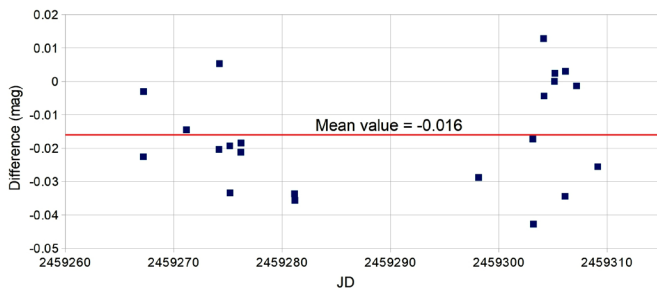

Figure 8. Difference between daylight and interpolated PEP magnitudes of Betelgeuse. Horizontal line: mean value.
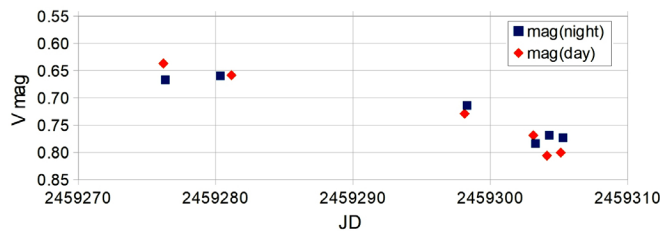

Figure 9. Comparison of daytime and nighttime magnitudes of Betelgeuse resulting from the same method. Blue squares: Nighttime results; Red diamonds: Daytime results.

The PEP magnitudes are in most cases slightly dimmer, and the difference between the daylight results and interpolated PEP magnitudes is shown in Figure 8. The mean value of the differences is -0.016 ± 0.003 mag ($n = 22$). The angular distance from the sun was between 16° and 114° and the sky brightness around Betelgeuse was between 1.8 and 6.0 mag/arcsec².

The mean errors of the daylight results were 0.02 mag (February–April) and 0.04 mag (May–July), which is only slightly higher than the LR standard errors of the linear regression of the reference stars (Table 2); this is a consequence of the instrumental magnitude error of Betelgeuse in the range 0.006–0.008 mag, which was much smaller than the standard error in all cases, as well as the extinction error (airmass term in Equation 11), which was in the range 0.001–0.015 mag.

On 6 nights, from March 02 to March 31, 2021, Betelgeuse was observed also at night with the same CCD equipment and method as in the daytime, and the magnitudes were calculated using the same comparison star ensemble as in the daytime. The results are shown in Figure 9. The mean values of the differences between day and night magnitudes are shown in Table 3.

Day and night magnitudes are not significantly different; the mean values of the calculated errors are also in the same range.

All daylight observations were uploaded to the AAVSO International Database, where the PEP data may also be found; the data may be visualized using the AAVSO Light Curve Generator (www.aavso.org/LCGv2). If more than one daylight observation was available per day, only the mean value of the magnitudes was recorded.

6. Discussion

The accuracy of the magnitude results depends partly on the selection of reference stars. For consistency, the magnitudes of the Betelgeuse ensemble stars were all taken from the Hipparcos catalogue. There are other catalogues, e.g. the *General Catalog of Photometric Data* (GCPD; Mermilliod *et al.* 1997). For some stars used as references, there are differences between Hipparcos and GCPD of up to 0.042 mag. In a sample of three cases, where β Ori, α CMi, and β Gem were used, the V mag result with GCPD values differed by about –0.01 mag and the error was slightly smaller. Therefore, future projects should use the GCPD magnitudes of the reference stars, if available.

The systematic differences of –0.016 (± 0.003) between daylight and PEP magnitudes cannot result from differences in the (B–V) values used for Betelgeuse in transformation: the PEP and daylight (B–V) differ by only about 0.018. This would result in an offset of less than 1 mmag in the daylight photometry. But the difference can be explained by the error of the color transformation constant T_v. Because of the high color index of Betelgeuse of around 1.85 this difference can be induced by a T_v error of –0.0097 (mean value of color indices of the reference stars was around 0.2 mag). The calculated T_v error was only slightly lower (0.0057), therefore this explanation is plausible. If the GCPD magnitudes were used, the systematic error would be on the order of –0.026 and the error of T_v would be on the order of 0.016. Therefore, in future projects the transformation constant should be adjusted in some way.

7. Conclusions

We have shown that CCD photometry in daylight with amateur equipment can be performed with an accuracy on the order of 0.02–0.05 mag. This is possible even within 10° of the sun, at least for stars of brightness on the order of $V \leq 2$. Daylight photometry of Betelgeuse was compared to nighttime PEP photometry obtained over a period of eight weeks; systematic differences between the light curves were less than 0.02 mag. Observations of bright variable stars could be collected in this way nearly uninterrupted by conjunction with the sun.

The photometry could be improved in three ways:

(1) A camera with higher framing rates could enable stacking of more than 1000 0.1-sec frames. The additional frames would increase the SNR, reduce the effect of scintillation, and allow the observation of stars with magnitudes in the range up to $V = 3$.

(2) Observing regions with altitudes above 1,000 m and with low humidity may provide a darker sky and would also enlarge the range of observable stars.

(3) Because the sky brightness is lower at longer wavelengths, it could be better to use redder filters to reduce the background noise, but in this case the transformation to the V magnitude would be more difficult. In any case an additional measurement with photometric R filter would give more information.

References

Anderson, E., and Francis, C. 2012, *Astron. Lett.*, **38**, 331.

Cinzano, P. 2005, *ISTIL Internal Rep. No. 9, v.1.4*, 1.

Da Costa, G. 1992, in *Astronomical CCD Observing and Reduction Techniques*, ed. S. B. Howell, ASP 23, Astronomical Society of the Pacific, San Francisco, 90.

Dupree, A., Guinan, E., and Thompson, W. T. 2020, *Astron. Telegram*, No. 13901, 1.

Merline, W. J., and Howell, S. B. 1995, *Exp. Astron.*, **6**, 163.

Mermilliod, J.-C., Mermilliod, M., and Hauck, B. 1997, *Astron. Astrophys., Suppl. Ser.*, **124**, 349.

Miles, R. 2007, *J. Br. Astron. Assoc.*, **117**, 278.

Nickel, O. 2021, FITSMAG photometry software (https://www.staff.uni-mainz.de/nickel/fitsmag.html).

Osborn, J., Föhring, D., Dhillon, V. S., and Wilson, R. W. 2015, *Mon. Not. Roy. Astron. Soc.*, **452**, 1707.

Samus, N. N., Kazarovets, E. V., Durlevich, O. V., Kireeva, N. N., and Pastukhova, E. N. 2017, *Astron. Rep.*, **61**, 80.

Young, A. T. 1967, *Astron. J.*, **72**, 747.

Appendix A: Ensemble comparison and linear regression

With LR, the values of $V_i - (m_i + T_v Y)$ of a sample of n reference stars with catalogue magnitudes V_i, color index Y_i and instrumental magnitudes m_i are correlated to their airmass X_i via the linear function:

$$V_i - (m_i + T_v Y_i) = m_0 - k_v X_i \qquad (12)$$

LR of this formula provides the regression parameters m_0 and k_v. Incorporating the residual ϵ_i the magnitude of each reference star is given by the equation:

$$V_i = (m_i + T_v Y_i) + \epsilon_i + m_0 - k_v X_i \qquad (13)$$

The differential magnitude V_{var} of a variable with instrumental magnitude m_{var}, with respect to a reference star (with instrumental magnitude m_i) would be:

$$V_{var} = m_{var} - m_i + T_v(Y_{var} - Y_i) - k_v(X_{var} - X_i) + V_i \qquad (14)$$

The ensemble method uses the arithmetic mean value of all reference stars, therefore:

$$V_{var} = \frac{1}{n} \sum_{i=1}^{n} (m_{var} - m_i + T_v(Y_{var} - Y_i) - k_v(X_{var} - X_i) + V_i) \qquad (15)$$

which simplifies to:

$$V_{var} = m_{var} + T_v Y_{var} - k_v X_{var} - \frac{1}{n} \sum_{i=1}^{n} (m_i + T_v Y_i - k_v X_i - V_i) \qquad (16)$$

Now V_i is replaced by the right side of Equation 13:

$$V_{var} = m_{var} + T_v Y_{var} - k_v X_{var}$$
$$- \frac{1}{n} \sum_{i=1}^{n} (m_i + T_v Y_i - k_v X_i - (m_i + T_v Y_i) - \epsilon_i - m_0 + k_v X_i) \qquad (17)$$

which becomes:

$$V_{var} = m_{var} + T_v Y_{var} - k_v X_{var} - \frac{1}{n} \sum_{i=1}^{n} (\epsilon_i - m_0) \qquad (18)$$

Because the sum of the residuals \epsiloni is zero, this yields:

$$V_{var} = m_{var} + T_v Y_{var} - k_v X_{var} + m_0 \qquad (19)$$

This is exactly the same as Equation 6. This means that the LR method is equivalent to a differential comparison method with an ensemble of comparison stars. Therefore the error calculation can be based on the mean value of differential magnitudes.

For the evaluation of the resulting error one works with Equation 16. This can be transformed to:

$$V_{var} = m_{var} + T_v(Y_{var} - \frac{1}{n}\sum_{i=1}^{n}Y_i) - k_v(X_{var} - \frac{1}{n}\sum_{i=1}^{n}X_i) - \frac{1}{n}\sum_{i=1}^{n}(m_i - V_i) \qquad (20)$$

$\bar{X}$, $\bar{Y}$, and $\bar{V}$ are mean values of X_i, Y_i, and V_i)

$$V_{var} = m_{var} + T_v(Y_{var} - \bar{Y}) - k_v(X_{var} - \bar{X}) - \frac{1}{n}\sum_{i=1}^{n}(m_i + \bar{X}) \qquad (21)$$

In this formula $\bar{V}$ is constant, m_{var}, T_v, Y_{var}, k_v, and m_i have statistical errors, which are independent of one another. Therefore $\sigma^2(V_{var})$, the squared error of V_{var}, can now be estimated from a sum of the variances of these errors:

$$\sigma^2(V_{var}) = \sigma^2(m_{var}) + T_v^2\sigma^2(Y_{var}) + (Y_{var} - \bar{Y})^2 \sigma^2(T_v)$$
$$+ (X_{var} - \bar{X})^2 \sigma^2(k_v + \frac{1}{n} \sum_{i=1}^{n} \sigma^2(m_i) \qquad (22)$$

The variance of $m_i + T_v Y_i$ is obtained from the sum of the residuals of the linear regression divided by $1/(n-2)$:

$$\sigma^2(m_i + T_v Y_i) = \frac{1}{n-2} \sum_{i=1}^{n} \epsilon_i^2 \qquad (23)$$

The right side of the above equation is the square of the standard error of the regression (SER). It follows:

$$\sigma^2(m_i) = SER^2 - \sigma^2(T_v Y_i) \qquad (24)$$

and

$$\frac{1}{n}\sum_{i=1}^{n}\sigma^2 m_i = SER^2 - \frac{1}{n}\sum_{i=1}^{n}Y_i^2\sigma^2(T_v) = SER^2 - \sigma^2(T_v)\frac{1}{n}\sum_{i=1}^{n}Y_i^2 \qquad (25)$$

From Equation 22 follows:

$$\sigma^2(V_{var}) = \sigma^2(m_{var}) + T_v^2\sigma^2(Y_{var}) + (Y_{var} - \bar{Y})^2 \sigma^2(T_v)$$
$$+ (X_{var} - \bar{X})^2 \sigma^2(k_v) + SER^2 - \sigma^2(T_v)\frac{1}{n}\sum_{i=1}^{n}Y_i^2 \qquad (26)$$

If the error of T_v is neglected, then the approximate error of V(ensemble) can be calculated by:

$$\sigma(V_{var}) = \sqrt{\sigma^2(m_{var}) + (X_{var} - \bar{X})^2 \sigma^2(k) + SER^2} \qquad (27)$$