Variability Properties of Red Giants and Supergiants in the AAVSO Binocular Observing Program

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Abstract The AAVSO Binocular Observing Program contains 153 stars, almost all of them red giants (127) or red supergiants (10). In this paper, we use Fourier and light curve analysis of visual and photoelectric observations in the AAVSO International Database to determine periods and amplitudes of these stars. Of the stars analyzed, 110 stars had sufficient data and periodicity to yield results. The stars pulsate in the fundamental and/or first overtone (at least 24 are bimodal); more luminous stars tend to pulsate in the fundamental. In addition, at least 61 of the stars had a "long secondary period" (LSP) 5 to 10 times the pulsation period. We determine and discuss the pulsation and LSP amplitudes. These are known to be variable with time. The variability properties of our stars are determined, to some extent, by the way in which the program stars were originally selected. The results are also affected by the limitations of the data, including limited accuracy of the visual data, seasonal and other gaps in the data, and the complexity of the stars' variability, including time-variable periods and amplitudes.

1. Introduction

There are 153 stars in the AAVSO Binocular Observing Program, for visual and other observers. The majority of them are red giant or supergiant stars. These stars are unstable to radial pulsation, in one or more modes, usually the fundamental and/or first overtone mode.

About a third of such stars also have a "long secondary period" (LSP), 5 to 10 times the pulsation period. The cause of these LSPs was unknown for almost a century. Recently, Soszyński *et al.* (2021) have ascribed them to eclipses by dust-enshrouded companions which were originally planets, but which have subsequently accreted gas and dust from the star, and become brown dwarfs or low-mass stars.

The variability of these stars is complicated in other ways. The periods "wander" by a percent or two, on long time scales. The pulsation amplitudes vary significantly, on time scales of 20 to 30 pulsation periods (Percy and Abachi 2013). The maximum and minimum brightnesses of the large-amplitude Mira stars also vary randomly. Some of these complications may be due to the fact that the outer layers of these stars are dominated by large convection cells.

Despite the success of the Soszyński *et al.* mechanism for explaining the LSP phenomenon, there are still some puzzling aspects. One is the existence of so-called "LSP stars" in the All-Sky Automated Search for Supernovae (ASAS-SN) variable star catalog (https://asas-sn.osu.edu/variables): there are 185 red giants in which the LSP amplitude was much larger than that corresponding to the pulsation period, so the period given in the catalog is the LSP, not the pulsation period, and the star is classified as an LSP star. (The ASAS-SN catalog contains only the dominant period, not multiple periods.) We began a study of some of these LSP stars (Percy and Shenoy 2023). But we realized that there were much more extensive data on red giants in the AAVSO International Database (AID), which would help to put the LSP stars in context. We had previously used some of these data in several studies of LSPs in red giants (e.g. Percy and Diebert 2016; Percy and Leung 2017), but we realized that there was even more information which could be derived.

Most of the stars in the AAVSO Binocular Observing Program have periods given in the program star list, which we refer to as "catalog periods." We decided to use our analyses to re-examine the periods, because some are missing or incorrect. Many stars have LSPs or multiple pulsation modes, so they cannot be described by a single period.

Another important motivation was to provide feedback to the hundreds of AAVSO observers who have contributed the tens of thousands of observations of these stars. What science can be derived from their data? This project also provided an important research experience for co-author Zhitkova, an undergraduate astronomy major.

2. Data and analysis

Visual and photoelectric Johnson V observations were downloaded from the AAVSO website (Kloppenborg 2023) and analyzed carefully using light-curve and time-series routines in the AAVSO VSTAR software package (Benn 2013). All data were used; the visual and V data were analyzed separately. The V data tend to be recent—from the last decade or two—whereas the visual data may stretch back for many decades. Because of the wandering of the periods, and the different densities of the two datasets, the derived periods from the visual and V data could be slightly different. We generally adopted a weighted mean of the two. The visual and V amplitudes often differed significantly, for various reasons; we have listed both of them separately in Tables 1–3. Note that they are semi-amplitudes, not full amplitudes or max-to-min ranges. Because the pulsation periods "wander" in time, we generally rounded off the pulsation periods to the nearest day.

We occasionally consulted the light curves and other data in the variable star catalog of the ASAS-SN survey (Jayasinghe *et al.* 2018; Jayasinghe *et al.* 2019; Shappee *et al.* 2014).

Absolute K magnitudes, MK, were determined from the GAIA parallaxes and K magnitudes, and corrected for interstellar absorption by converting E(B–V) reddening to K absorption. These data were generally taken from the ASAS-SN variable star catalog. A few stars had highly anomalous E(B–V); MKs are not listed for these stars.

We recognize that our results are affected by the sparseness of the data for some stars, the limited accuracy of the visual observations, especially for red variables (Cadmus 2020), the seasonal and other gaps in the data, and whatever selection effects are present in choosing the stars for the Binocular Observing Program, as well as by the complexity of the stars' variability.

We also suspect that some of the low-amplitude "periods" near one year may be due to the Ceraski effect (Percy 2015), which is a physiological phenomenon which affects visual observations.

3. Results

The results of the analyses are given in Tables 1–3, which list: the star name, the variable star type, the absolute K-magnitude MK, the adopted dominant pulsation period PP, its amplitude in visual and Johnson V, the adopted LSP, and its amplitude in visual and Johnson V. As mentioned, the amplitudes are actually semi-amplitudes, as determined by Fourier analysis, not full max-to-min ranges. There are a total of 110 stars in Tables 1–3.

Some stars showed evidence of bimodal pulsation, generally in the fundamental and first overtone modes. These stars are listed in Table 4, which gives: the star name, the longer pulsation period Pa, the shorter pulsation period Pb, and the ratio Pb/Pa. There are 24 stars in this table.

Figure 1 shows the period-luminosity diagram (MK vs. log P(d)) for the stars in Tables 1–3. Three sequences are seen; from the left, they are: first overtone pulsation period, fundamental pulsation period, and LSP. The sequences are approximately parallel, but the results in Figure 2 indicate that the two pulsation sequences are not exactly parallel. Note that the lower-luminosity stars are more likely to pulsate in the first overtone, whereas higher-luminosity stars are more likely to pulsate in the fundamental.

Figure 2 shows the so-called Petersen diagram for the stars in Table 4: Pb/Pa vs. log Pa. There is some scatter, but the figure is very similar, in placement and slope, to that for a different sample of red giants (Percy and Huang 2015). Note that the analysis of bimodal pulsators has to be done with great care. The Fourier spectra contain alias periods, separated from the true period by multiples of 0.00274 cycle/day, and caused by the seasonal gaps in the data. They may also contain harmonics, with periods of 1/2 or sometimes 1/3 of the true period, and caused by a non-sinusoidal phase curve. For red giants, the first-overtone period is close to 1/2 the fundamental period, so period ratios of 0.50 must be treated with some suspicion. But Figure 2 slopes; it is not horizontal. And, as mentioned, it is consistent with other studies, and with theoretical pulsation models.

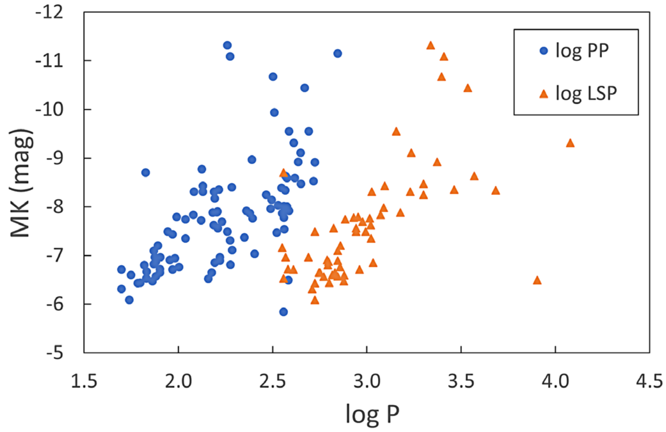

Figure 1: The relationship between the absolute K-magnitude MK and log period. Blue circles correspond to pulsation periods, orange triangles to LSPs. The outliers in the right side are BM Sco and Y Pav.

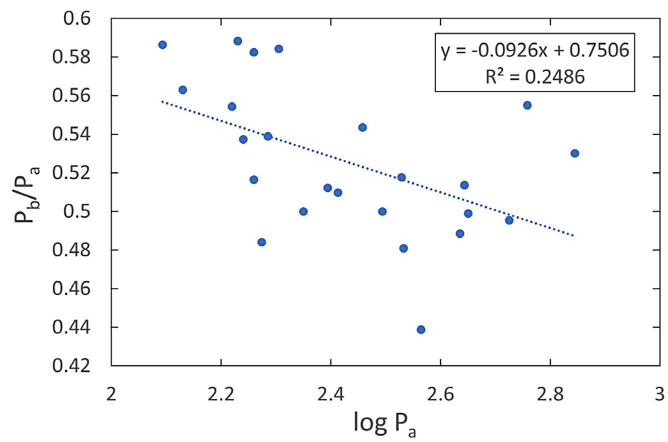

Figure 2: For bimodal pulsators, the relationship between the ratio of the shorter (first overtone) period to the longer (fundamental) period and the logarithm of the latter. The star with the smallest ratio is T Ind.

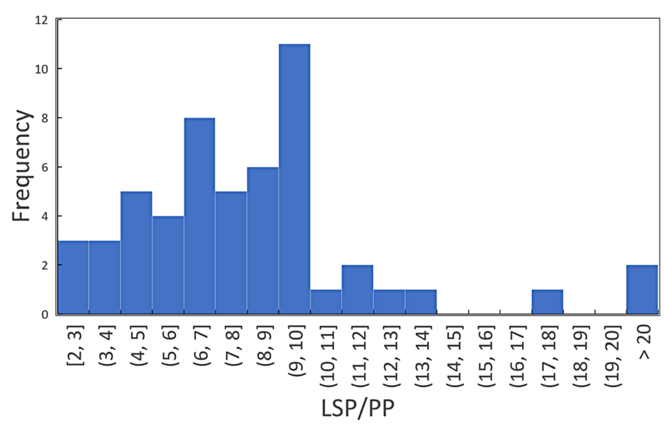

Figure 3: Histogram showing the distribution of the ratios of the LSP to the dominant pulsation period (LSP/PP).

Figure 3 shows a histogram of LSP/PP values. They peak at about 6 and 9, as previous studies have shown, presumably depending on whether the PP is the fundamental or the first overtone. The peak at LSP/PP = 6 is rather flat.

We were curious as to whether these ratios were constant, or a function of the size and luminosity of the star, but we found no such clear trend.

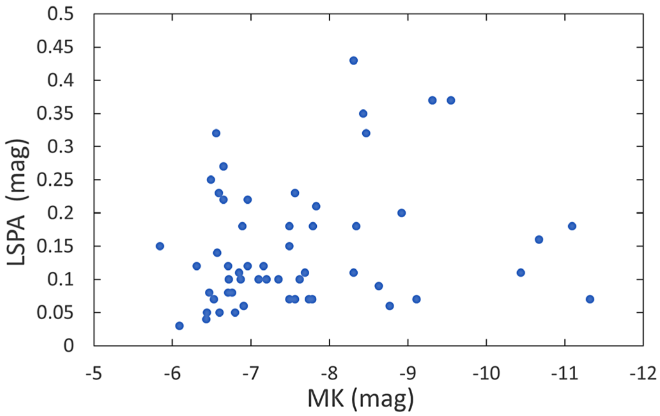

Figure 4: The relationship between the visual LSP semi-amplitude LSPA and the absolute K-magnitude MK.

Figure 4 shows the relation between the LSP semi-amplitude and MK. The data are sparse, but it appears that, on average, amplitudes are smaller in low and high-luminosity stars, and larger for moderate-luminosity ones. This is consistent with results from our parallel study of "LSP stars" in the ASAS-SN database (Percy and Shenoy 2023).

It is generally true that red giant pulsation amplitudes are greater in higher-luminosity stars. In our case, that relation is not clear, but is at least partly a result of the selection of stars in the Binocular Observer Program, which is a mixture of SR, M, and supergiant stars.

The following program stars had insufficient data, or had no obvious periods in the Fourier spectrum or light curves: X Cnc, TU CVn, AG Car, BZ Car, CK Car, EV Car, IX Car, ASAS J110135-6102.9, T Cen, V766 Cen, AR Cep, μ Cap, RR CrB, SV Crv, CH Cyg, AY Dor, CL Hyi, R Lep, CF Mic, SX Mon, X Oph, V407 Pup, WX Ret, UX Sgr, V905 Sco, τ^4 Ser, RX Tel, RW Vir, BK Vir.

3.1 Notes on individual stars

RW Boo There is an ~50-day period in the light curve, so the 407-day period is an LSP.

R Dor This star is possibly bimodal, with periods of 172 and 323 days.

UX Dra This star was initially considered "unsolved." There were three comparable peaks in the Fourier spectrum, separated by approximately 0.00274 cycle/day, suggesting that one or two peaks might be aliases. But the light curve shows evidence of the periods given in Table 1, so those periods may be correct.

U Hya The catalog period is 183 days. The 360-day period that we have derived is also present in the AAVSO light curve. This star may possibly be bimodal.

CE Lyn There appears to be an 50-day period in the ASAS-SN light curve, so the 512-day period that we have derived may be an LSP.

HK Lyr This star may possibly be bimodal, but the evidence is weak.

RV Mon There is some evidence in the Fourier spectrum for an 80-day period, and the catalog period is 121.3 days, so the 355-day period that we have derived may be an LSP. There is also some evidence for this period in the ASAS-SN light curve.

BO Mus The catalog period is 132.4 days, and there is a 134-day period in the light curve. There may be an LSP in the range 1800–2500 days.

GO Peg The catalog period is 79.3 days, and there is a period of this order in the ASAS-SN light curve, so the 382-day period that we have derived may be an LSP.

PV Peg The ASAS-SN light curve shows a time scale of 50–70 days, so the 700-day period that we have derived may be an LSP. The catalog period is 520: days.

BM Sco It is not clear whether the derived period is a pulsation period or an LSP; we think it is the former.

RY UMa There is a 30-day period in the ASAS-SN light curve, so the 284-day period that we have derived may be an LSP. The catalog period is 310 days.

FP Vir There is a 70- to 80-day period in the light curve, so the 373-day period that we have derived may be an LSP.

4. Discussion

Figure 1 can be compared with the P–L relations given by Wood (2000) and Soszyński *et al.* (2021) for stars in the Magellanic Clouds. There are sequences for first overtone (1O) and fundamental (F) mode pulsators, and for LSPs. The 1O and F sequences may appear to be parallel, indicating a constant period ratio, but Figure 2 shows that the period ratio varies slightly. Figure 2 is consistent with period ratios from a different sample of red giants (Percy and Huang 2015) and with theoretical pulsation models (Xiong and Deng 2007).

In Figure 3, the lack of LSP/PP values greater than about 12 is presumably because red giants pulsate primarily in the fundamental and first overtone, with very few in the second overtone, and hardly any in higher overtones. This is consistent with the period-luminosity diagrams given by Wood (2000) and Soszyński *et al.* (2021).

The correlation between the pulsation period and the luminosity in Figure 1 is easy to understand; both depend primarily on the radius of the star. The correlation between the LSP and the luminosity is less easy to understand. The luminosity depends on the radius of the star; the LSP depends on the radius of the dusty companion's orbit. Specifically, the actual relation implies that the latter radius is about two stellar radii, on average (Kim and Percy 2022).

There are stars in Tables 1–3 in which the LSP amplitude is significantly larger than the pulsation amplitudes. The ASAS-SN variable star catalog would have probably determined the dominant period to be the LSP, and classified these as "LSP stars." There are also stars in Tables 1–3 for which the period and the MK suggest that the catalog period is actually the LSP, not a pulsation period. These include the following stars: RW Boo, BM Eri, PV Peg, CI Phe, RY UMa, VW UMa, and GO Vel. There are also stars for which the catalog period differs from ours by a factor of approximately two. In these cases, our analyses and previous ones may have picked up different pulsation modes, the amplitudes of which vary with time.

Tables 1–3 include a few variables which are classified as supergiant (SRc, Lc) variables. In most cases, the MK and

Table 1. Variability properties of red giants and supergiants in the AAVSO Binocular Observing Program.

Star Name	*Type*	*MK*	*PP(d)*	*SA(PP)v*	*SA(PP)V*	*LSP(d)*	*SA(LSP)v*	*LSP(SA)V*
V373 And	SRB	—	202	0.11	—	1067	—	0.12
θ Aps	SRB	–7.35	109	0.14	0.26	1054	0.10	0.15
R Aqr	M + Z And	–9.55	387	1.72	1.36	—	—	—
R Aql	M	—	276	0.79	1.54	—	—	—
UU Aur	SRB	–9.11	445	0.13	—	1721	0.07	—
ψ^1 Aur	Lc	–11.32	182	0.07	—	2176	0.07	—
R Boo	M	–7.37	224	2.43	2.25	—	—	—
RV Boo	SRB	–7.92	230	0.06	—	—	—	
RW Boo	SRB	–6.71	407	0.12	—	—	—	
RX Boo	SRB	–7.89	160	0.07	—	—	—	
FG Boo	SRB	–6.65	—	—	—	556	0.22	0.20
RS Cnc	SRB	–7.87	239	0.13	0.24	—	—	—
RT Cnc	SRB	–6.94	96	0.06	—	—	—	—
V CVn	SRA	–7.31	188	0.25	0.37 —	—	—	
Y CVn	SRB	–8.25	292	—	—	2000	—	—
W CMa	SR	–8.63	370	0.09	—	3717	0.09	—
VY CMa	Lc	–9.55	492	0.18	—	1433	0.37	—
RT Cap	SRB	–8.40	192	0.11	—	—	—	—
R Car	M	–7.96	308	2.30	2.22	—	—	—
S Car	M	–6.65	150	1.20	1.27	—	—	—
BO Car	SRc	—	367	0.09	0.13	—	—	—
R Cen	M	—	546	0.80	—	—	—	—
RV Cen	M	–8.47	447	0.91	—	1996	0.32	—
V744 Cen	SRB	–6.96	166	0.10	—	—	—	—
W Cep	SRc	—	408	0.07	—	—	—	—
SS Cep	SRB	–7.69	170	0.02	0.17	947	0.11	0.28
T Cet	SRB	–7.90	161	0.16	0.25	—	—	—
o Cet	M	—	333	2.38	2.34	—	—	—
RU Crt	SRB	–6.56	—	—	—	590	0.32	0.28
BH Cru	M	–8.53	522	1.10	—	—	—	—
W Cyg	SRB	–7.72	132	0.14	0.19	—	—	—
RS Cyg	SRA	–8.59	414	0.48	0.58	—	—	—
AF Cyg	SRB	–6.71	93	0.11	0.18	913	0.08	—
V460 Cyg	SRB	–8.35	164	0.05	0.13	2906	—	0.14
V1070 Cyg	SRB	–6.43	61	0.03	0.07	529	0.04	0.12
U Del	SRB	–7.83	120	0.05	0.08	1180	0.21	0.29
CT Del	SRB	–6.96	80	0.06	0.11	370	0.22	0.16
EU Del	SRB	–6.44	63	0.05	0.10	630	0.05	0.08
R Dor	SRB	–9.94	323	0.14	0.31	—	—	—
RY Dra	SRB	–7.62	154	0.04	0.09	1050	0.10	0.10

Note: The columns are: star name, variability type, absolute K magnitude, pulsation period (PP), PP semi-amplitude in v, PP semi-amplitude in V, long secondary period (LSP), LSP semi-amplitude in v, and LSP semi-amplitude in V.

Table 2. Variability properties of red giants and supergiants in the AAVSO Binocular Observing Program.

Star Name	*Type*	*MK*	*PP(d)*	*SA(PP)v*	*SA(PP)V*	*LSP(d)*	*SA(LSP)v*	*LSP(SA)V*
TX Dra	SRB	–6.57	76	0.08	0.12	700	0.14	0.16
UX Dra	SRB	–8.02	177	0.06	0.13	720	—	0.07
AH Dra	SRB	–7.11	193	0.23	0.20	—	—	—
Z Eri	SRB	–7.20	78	0.05	0.10	725	0.10	0.12
RR Eri	SRB	–7.43	93	0.06	0.11	—	—	—
BM Eri	SR	–6.65	—	—	—	565	0.27	—
BR Eri	SRB	–6.65	80	—	0.17	677	—	0.12
TV Gem	Lc	–11.09	188	0.04	0.22	2564	0.18	—
BU Gem	SRc	–10.67	318	0.09	—	2493	0.16	0.24
π^1 Gru	SRB	—	195	0.10	0.40	—	—	—
X Her	SRB	–6.76	101	0.06	0.18	722	0.08	0.22
ST Her	SRB	–7.88	152	0.08	0.20	1504	—	0.11
UW Her	SRB	–7.49	182	—	0.17	986	0.07	—
IQ Her	SRB	–6.87	76	0.05	—	625	0.10	—
OP Her	SRB	–7.10	74	0.05	0.06	699	0.10	0.10
V939 Her	LB	–6.67	68	0.06	—	—	—	—
g Her	SRB	–7.49	88	0.03	0.07	877	0.15	0.15
R Hya	M	–8.59	377	1.06	1.18	—	—	—
U Hya	SRB	–6.93	360	0.08	0.50	—	—	—
V Hya	SRA	–8.91	530	0.53	—	—	—	—
Y Hya	SRB	–7.78	363	0.13	0.23	852	0.07	—
RT Hya	SRB	–7.03	254	0.09	0.25	—	—	—
RV Hya	SRB	–7.54	365	0.15	—	—	—	—
T Ind	SRB	–8.34	369	0.14	0.17	4830	0.18	—
R Leo	M	–8.14	312	1.72	0.18	—	—	—
S Lep	SRB	–7.56	—	—	—	878	0.23	0.38
RX Lep	SRB	–7.56	162	0.05	—	667	0.07	0.22
Y Lyn	SRc	–8.43	135	0.08	—	1245	0.35	0.52
SV Lyn	SRB	–6.53	68	0.06	—	361	0.07	—
CE Lyn	SRB	–6.31	50	—	—	512	0.12	—
XY Lyr	SRC	–8.31	121	0.03	0.10	—	—	—
HK Lyr	SR	–8.00	376	0.09	0.13	—	—	—
T Mic	SRB	–8.01	363	0.34	—	—	—	—
X Mon	SRB	–6.85	156	0.50	0.55	1080	0.11	0.30
RV Mon	SRB	–7.16	—	—	—	355	0.12	0.31
BO Mus	SRB	–8.77	133	0.06	—	1700	0.06	—
X Oph	M	–7.47	333	0.51	0.83	—	—	—
W Ori	SRB	–8.92	432	0.13	—	2358	0.20	—
BL Ori	SRB	—	156	0.05	0.10	1290	0.05	—
BQ Ori	SRB	–7.76	248	0.18	0.50	1035	—	0.28

Note: Columns as in Table 1.

Table 3. Variability properties of red giants and supergiants in the AAVSO Binocular Observing Program.

Star Name	*Type*	*MK*	*PP(d)*	*SA(PP)v*	*SA(PP)V*	*LSP(d)*	*SA(LSP)v*	*LSP(SA)V*
S Pav	SRA	–7.92	387	0.57	—	—	—	—
Y Pav	SRB	–9.31	410	0.14	0.22	12000	0.37	—
GO Peg	SRB	–6.72	80	—	—	382	0.10	0.15
PV Peg	SRB	–6.89	—	—	—	700	0.18	0.25
SU Per	SRc	–10.44	467	0.07	—	3424	0.11	—
CI Phe	SRB	–6.59	—	—	—	758	0.23	—
R Pic	SR	–6.89	166	0.48	—	—	—	—
Z Psc	SRB	–8.17	156	0.13	0.19	—	—	—
TV Psc	SR	–6.09	55	0.02	0.05	530	0.03	0.06
L^2 Pup	SRB	–5.84	361	0.30	1.09	—	—	—
V1943 Sgr	SRB	–8.03	338	0.41	—	—	—	—
AH Sco	SRc	–11.15	700	0.31	—	—	—	—
BM Sco	L	—	383	0.19	0.23	8000	0.25	—
SW Scl	SRc	–6.52	144	0.30	—	—	—	—
Y Tau	SRB	–8.97	245	0.10	0.18	—	—	—
W Tri	SR	–7.74	109	0.04	—	768	0.07	—
X TrA	SR	–8.39	353	0.12	0.19	—	—	—
DM Tuc	SRB	–6.82	74	0.20	—	—	—	—
Z UMa	SRB	–6.81	189	0.40	0.90	—	—	—
RY UMa	SRA	–5.84	30	—	—	284	0.15	0.38
ST UMa	SRB	–6.91	90	0.04	0.13	615	0.06	0.16
TV UMa	SRB	–6.60	56	0.02	0.08	654	0.05	0.07
VW UMa	SRB	–6.80	66	—	—	620	0.05	0.07
V UMi	SRB	–6.47	73	0.10	0.19	755	0.08	0.20
GO Vel	SRB	–7.49	—	—	—	530	0.18	—
MN Vel	SRA	–8.31	135	0.16	—	1060	0.43	0.60
RT Vir	SRB	–7.98	371	0.17	—	1227	—	0.26
SS Vir	SRA	–7.87	355	0.70	1.10	—	—	—
SW Vir	SRB	–8.31	155	0.18	0.41	1700	0.11	0.20
FP Vir	SRB	–8.70	67	0.05	0.09	362	—	0.19
FI Vir	SR	-6.96	75	0.11	0.20	490	0.12	0.13

Note: Columns as in Table 1.

Table 4. For bimodal pulsators, the star name, the longer (fundamental) period Pa, the shorter (first overtone) period, and the ratio Pb/Pa.

Star Name	*Pa(d)*	*Pb(d)*	*Pb/Pa*
V373 And	202	118	0.584
ψ^1 Aur	182	94	0.516
R Boo	224	112	0.500
V CVn	188	91	0.484
R Cen	531	263	0.495
RV Cen	447	223	0.499
V744 Cen	166	92	0.554
SS Cep	170	100	0.588
W Cyg	259	132	0.510
AF Cyg	174	94	0.537
V460 Cyg	341	164	0.481
TX Dra	135	76	0.563
AH Dra	193	104	0.539
BU Gem	573	318	0.555
UW Her	182	106	0.582
T Ind	367	161	0.439
R Leo	312	156	0.555
W Ori	432	211	0.488
BL Ori	287	156	0.544
BQ Ori	248	127	0.512
Y Pav	440	226	0.514
V1943 Sgr	338	175	0.518
AH Sco	700	371	0.530
V UMi	124	73	0.586

pulsation period are consistent with this class but, for V939 Her, the MK (–6.67) and the pulsation period (68 days) are more consistent with an SR classification.

Figure 4 relates to the amplitude of the LSP. This amplitude would be expected to depend on the ratio of the effective area of the dust-enshrouded companion to the area of the red giant, and also on the angle of the companion's orbit place to the line of sight. It is not surprising, then, that the amplitude decreases for larger, more luminous stars, assuming that the size of the companion is not larger in more luminous stars. It is less clear why it also decreases for smaller, less luminous stars. Perhaps, since these have weaker winds, the companion has not accreted much matter, and is therefore smaller. Also, at each value of MK, there is a range of amplitudes, depending on the inclination of the companion's orbit.

5. Conclusions

This analysis of the variability properties of the red giant and supergiant stars in the AAVSO Binocular Observing Program has displayed the full gamut of phenomena which are expected to occur in these stars. They pulsate in the fundamental (F) and/or first overtone (1O) mode; 24 are bimodal pulsators. 58 show long secondary periods which are about five times the fundamental pulsation period, or about ten times the first overtone period. For the bimodal pulsators, the period ratio varies slowly with the luminosity of the star. The period-luminosity graph shows approximately parallel sequences for the 1O or F pulsation and for the LSPs. The pulsation amplitude increases with increasing pulsation period in a complex way, which may be partly due to the way in which the program stars were selected. The LSP amplitude seems to be greatest for moderate-luminosity stars.

We hope that the observers in the AAVSO Binocular Program will read this paper, and derive some satisfaction from knowing that they are contributing to our understanding of these very complex stars.

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