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SEMIREGULAR VARIABLES:
ARE THEY CHAOTIC OR RINGING?

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Abstract

Two types of irregular behavior in semiregular variables, labeled randomness and chaos, are described and the results of recent theoretical explorations are illustrated. The need for precise long-term observations to distinguish between multi-periodic and irregular pulsations in semiregular variables is emphasized.

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1. Introduction

Like the categories of cats and dogs, the semiregular variable stars encompass a wide variety of types. For the most part, they are easy to recognize but not so easy to describe or to define in words alone. Their light varies continuously in a way that may appear to be regular for a while but soon strays from the predicted pattern. The *General Catalogue of Variable Stars* (Kukarkin *et al*. 1969) describes the class as consisting of:

> "...giants or supergiants possessing an appreciable periodicity, accompanied, or at times disturbed, by various irregularities in the light variations. The periods of the semiregular variables range in extremely wide limits - from about 30 to 1000 days or more. The forms of the light curves are extremely diverse; the amplitudes usually do not exceed 1 - 2 magnitudes."

Probably the best known of the semiregulars are Alpha Orionis and Mu Cephei, light curves of which are shown in Figure 1. Mu Cep has been under surveillance since 1848. Its amplitude is fairly small (typically 0.4 magnitude visually) and for reasons that I will describe, this small amplitude creates a major uncertainty in our understanding of such stars. A long-term photoelectric program on semiregular stars - stretching over several years or a decade - would pay large dividends.

For a given observational error, a star with a smaller amplitude will, of course, have a larger *relative* error in its light curve than will a star with a larger amplitude. The shape of its light curve and its periodicity, or lack of periodicity, will be less well defined. If the star were known to be regular, as is the case with many eclipsing variables, it would be possible to take the average of observations spread over many cycles and derive a light curve whose precision is far greater than the precision of the individual observations. However, most of the cooler intrinsic variables do not behave in this regular manner; each cycle is unique. Improved curves will only come with improved precision in each measurement. In the case of the semiregulars, this improved accuracy is crucial because the choice of an explanation for the variability depends on what is known about the detailed nature of its irregularity.

This situation is rather unusual in variable star work. In the past, improved accuracy has usually done little beyond providing a more precise quantitative model. For example, an improved period for a Cepheid will give better estimates of its density and its brightness, and an improved light curve for an eclipsing binary will give a better determination of its radius. The nature of the model is not affected, merely its dimensions. However, in the case of the semiregulars, improved accuracy in the observations may determine the nature of the model, and it may swing the balance between two competing physical theories. In other words, it may change our point of view.

2. Types of Pulsational Behavior

Two types of behavior may be found among the pulsating stars: periodic and irregular.

The classical Cepheids are, with a few interesting exceptions, well-behaved examples of singly-periodic stars whose periods may change slightly over the course of time but whose futures are highly predictable. The theory of stellar pulsation has explained these stars as self-excited oscillators behaving very much like pendulum clocks. In a clock, gravitational energy from the descending weight is converted to oscillation energy by the action of an escapement. In a Cepheid, heat energy leaking outward from the region of nuclear burning is converted to pulsation energy by an "escapement" in the outer envelope. The star swings in and out with a well-defined period, much as a pendulum's period is determined by its length. Unlike a simple pendulum, a star can vibrate in more than one pattern at a given time, and some stars show two superimposed periods. For example, many of the RR Lyrae stars show "beats" that can be described as the interaction of two simultaneous modes of oscillation.

The ringing of a bell is a more useful analogy than a pendulum. When a bell is struck by its clapper, a variety of "overtones" is excited and instead of oscillating in a simple pattern with a single frequency, the bell usually vibrates in several superimposed patterns. Each pattern is a "mode," and each mode has a characteristic frequency. (This analogy was popular in the early days of spectroscopy, when the emission of colored light by atoms was often compared with the radiation of musical notes by a bell.) The bell's pitch is determined by the weight and shape of the bell as well as the stiffness of the metal. The musical quality of the sound is determined by the amount of sound energy that is radiated in the overtones, and the strength of the overtones depends on the detailed shape of the bell and the position of the clapper.

The spectrum of sound can be used to analyze the bell, and in the same way the frequencies of multi-periodic stars are clues to interior structure. Thus the light curves of such stars are particularly interesting. They are just as predictable as the light curves of singly-periodic stars, although the prediction is more complicated (but not essentially so), because more terms must be added together to represent the light curve. If the periods are incommensurate and not related as in a rational fraction (such as 22/7), the curve may never repeat.

The light curves of irregular variables, on the other hand, cannot be represented by a finite equation, and they are as unpredictable as, say, next month's weather. The question is not simply one of not having enough data. Some phenomena are inherently unpredictable over finite time intervals, even with the most sophisticated methods of modern mathematics and with the largest computer that can be imagined. The flow of smoke from a chimney and the rippling of water on the surface of a stream are two such phenomena. No matter now many measurements are made, their patterns will be elusive. The fact that even with global satellites weather forecasts made more than three days ahead seem no more reliable now than they were several decades ago is a result of this inherent irregularity. All the data in the world will not prevent our being misled by a disturbance that may have been too small to be measured on one day but grows until it dominates the weather a few days later.

3. Causes of Irregular Variations

There are two types of physical explanations for irregular behavior: randomness and chaos. As an example of "randomness," a random process is one in which the events are largely independent of each other, such as the flipping of a coin. The only consistent properties of such events are statistical ones, such as the tendency for equal numbers of heads and tails and the absence of long runs of either heads or tails. If a periodic pattern seems to be emerging, it will soon disappear, and it is impossible to make detailed predictions of long-range or short-range patterns with complete certainty.

A physical example of this randomness is the pattern of sound generated by raindrops on a roof. Each drop falls independently of the others and the resulting pattern has local regions in which more than the average number accumulate and other areas which are hit by fewer than average. The random pattern of the impacts generates a random array of sound waves that is irregular and virtually impossible to predict. A crucial point is that the pattern of noise generated by rain does not tell us much about the roof; it tells us about the pattern of the raindrops. In a similar way, the variations of certain types of stars may be the response of their atmospheres to random hits from below, such as would be produced by the boiling motion known as convection. If this is the case, then the patterns may not tell us much about the star as a whole, but only about the nature of the convection.

The second type of irregular behavior is called "chaos," and during the past two decades the discussions of chaos in physics and mathematics have shown that it is a remarkably common process. The physical origin of chaos is difficult to describe, although the result is not difficult to recognize. The turbulent pattern of smoke rising from a chimney and the eddies in a trout stream are examples of chaotic motion. They are described not as the results of random hits but rather as the result of internal processes that occur when motions become exaggerated - when things get out of hand, so to speak. Epidemics of a disease or fads in clothing are examples of exaggerated behavior, and erratic swings in population among competing species in nature is another. These changes are not simply the summation of many small changes; they reflect a collective and cooperative behavior. This behavior is called "non-linear" in the jargon of mathematics because the forces acting in such systems are highly curved, rather than linear, functions of the state of the system. The study of chaos is focused on the study of non-linear equations.

A further example of chaotic motion may be obtained by suspending a pendulum on a swivel that permits it to swing in a complete circle. If we tap the pendulum gently it will oscillate back and forth periodically, and once we have measured the period we can predict its path quite accurately for a long time. The motions will damp out after a while, and our prediction will gradually become less and less accurate unless we take the damping into account. We can do this by measuring the amplitude from time to time and exploiting these measurements in our prediction.

Suppose, however, that we kick the pendulum so that it swings higher and higher until, finally, it swings almost to the top of the circle, hesitates a moment, and then swings down again. The length of the pause at the top of the swing is sensitively dependent on the speed at the bottom of the swing and the strength of the kick. If the pendulum persists in swinging nearly to the top, we will find it extremely difficult to predict its behavior. That is, we will have a hard time guessing when the pendulum will reach the bottom, say, two or three swings later. This difficulty is produced by the forces acting on the pendulum near the top - they are non-linear and produce erratic behavior. Notice that this erratic behavior is not generated by the randomness of the kicks; it can occur even when the kicks are as uniform as we can make them. The circular pendulum swinging with just the right amplitude is inherently unstable. Slight differences in the conditions in one cycle can make a large difference in the next cycle, just as a slight difference in the weather one day can be multiplied into a large difference in a day or two.

Returning to the semiregular stars, the irregularity of their oscillations may be like that of the pendulum - not a consequence of random jostles but of forces acting inside the star to produce unstable motions. This is the distinction between the sources of irregular behavior.

L. Perdang and S. Blacher (1982) carried out an important theoretical study of chaotic behavior in simplified models for pulsating stars. They pointed out that astronomers have been preoccupied with regular, periodic behavior, while many variables do not fit this mold, and they showed that it may be possible to explain erratic behavior without resorting to randomness. Figure 2, taken from their paper, shows the irregular oscillations of one of their models. Such chaos is common when the amplitude of the motion exceeds a critical value, as it does for the circular pendulum.

Before developing a theory for a variable star, it is essential to decide which of these behaviors describes the star.

4. The Status of Mu Cephei

The first question is, are the semiregular stars multi-periodic or irregular and truly unpredictable? If they are found to be irregular we would proceed to the next question, namely, is this behavior produced by a random pattern of hits or is it caused by an internal chaos?

For Mu Cep, the first question has been answered both ways in the past several decades, and that is why more data are needed now. Thirty years ago a study by J. Ashbrook, R. Duncombe, and A. J. J. van Woerkom (1954) concluded that the light curve of Mu Cep could be constructed by a process of random jolts followed by quickly damped oscillations, as indicated in Figure 3. Without providing a detailed physical picture, they decided that the light variation is "not explained by a simple pulsation; instead it may be interpreted as arising from temporary, random surface disturbances on the star." The brightness fluctuations of the solar atmosphere, observable as "granulation," may provide a model for such disturbances. These fluctuations are generally thought to be produced by turbulent motions in the outer layers of the sun and may look like the rising of a plume of hot air above an open fire or a chimney. Most stars are expected to show such fluctuations, some more strongly than others.

On the basis of the early study by Ashbrook and his collaborators, it seemed for many years that Mu Cep was an irregular star, responding to random disturbances. However, in 1966 this interpretation was questioned, and a recent study has in fact reversed the earlier conclusion. L. Mantegazza has shown (1982) that, rather than following a completely chaotic behavior, Mu Cep may be multi-periodic, with two basic frequencies and about a dozen harmonics and combination tones. The periods of the basic frequencies are not related in any simple way (that is, they are incommensurate) so that the curve does not repeat, but is predictable and can be extended into the future as in a Fourier analysis. A sample of the light curve and the quality of fitting it with two curves of different complexity are shown in Figure 4 (Mantegazza 1982). The prediction seems quite good in the lower example but this test alone does not prove that Mu Cep is multi-periodic, because random or chaotic behavior can imitate multi-periodic behavior "roughened" by observational errors.

The observational distinction is a subtle one even though the physical difference is large, and until the data are adequate to decide whether Mu Cep is truly irregular or is multi-periodic no theory for such stars can be considered proven. Regular observations at intervals of a week and with a precision of 0.01 magnitude over a period of a decade or so would probably answer this intriguing question.

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Payne-Gaposchkin, C. and Gaposchkin, S. 1938, Variable Stars, Harvard Observatory Monographs, Cambridge, MA.

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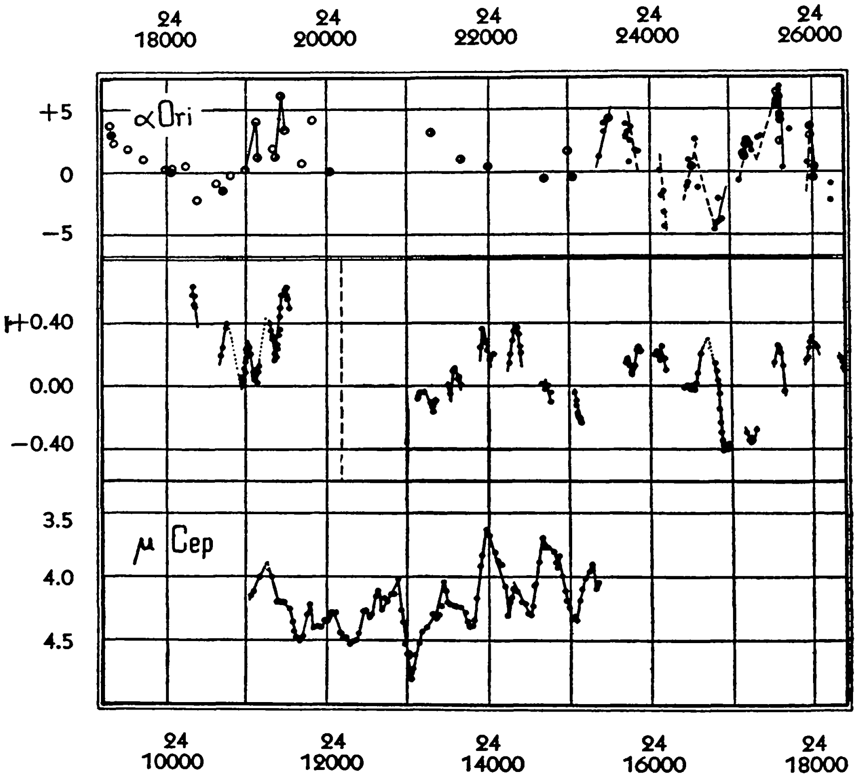

Figure 1. Observed light curves of Alpha Orionis and Mu Cephei from Variable Stars, Payne-Gaposchkin and Gaposchkin.

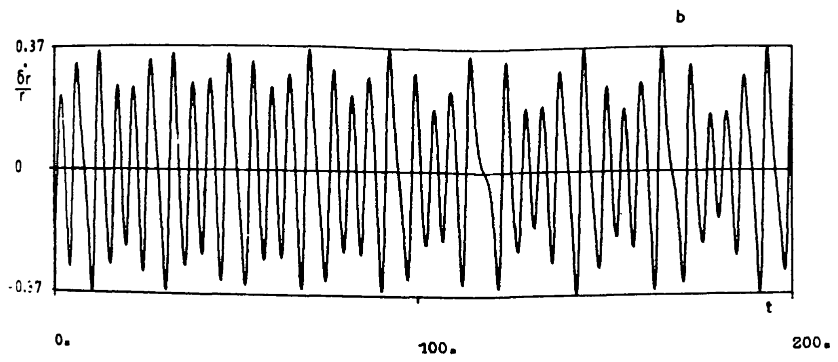

Figure 2. Chaotic velocity curve generated by a simple non-linear oscillator model for stellar pulsation from Perdang and Blacher (1982).

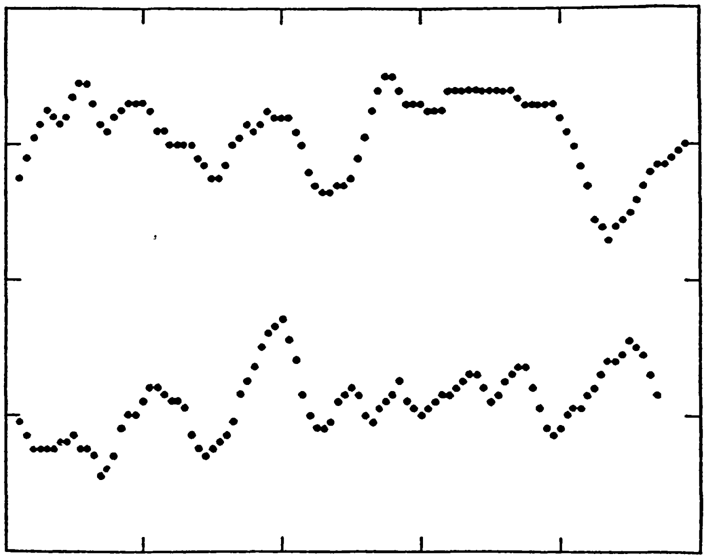

Figure 3. Typical light curve generated by the random process described by Ashbrook *et al.* (1954). Each point on the curve is computed from the preceding two points and a random pulse, e, according to the relationship: $m(t) = 1.664m(t-1) - 0.759m(t-2) + e$. In this expression, $m(t-1)$ is the magnitude at the previous time-step.

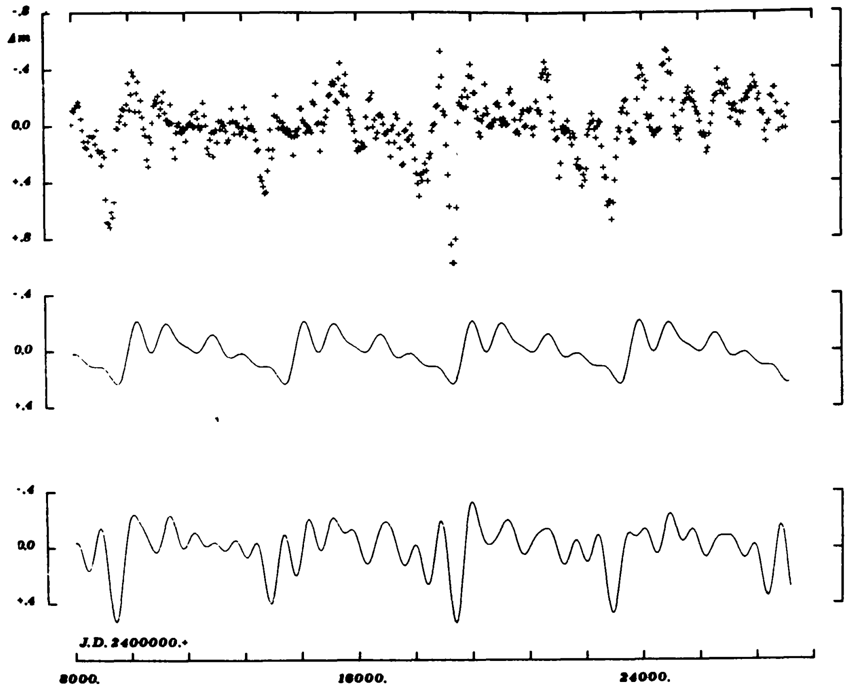

Figure 4. Top - brightness data for Mu Cephei; middle - synthetic light curve based on superimposition of the fundamental and six harmonics; bottom - synthesis using the entire spectrum. From Mantegazza (1982).