Computer Simulations of Stellar Pulsation

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Abstract

Pulsational and cataclysmic variables are contrasted theoretically and several aspects of adiabatic pulsations are illustrated by computer simulations based on a star consisting of elastic atoms confined by extensible gravitating shells.

1. Introduction

All stars are intrinsically variable in one way or another. Some variations are too slow to have been detected — except in theory — and others, no doubt, are too rapid. Some types of variation occur only once in a stellar lifetime. Others occur cyclically, and the star seems little changed after each cycle.

Observationally, the cyclic variables can be divided into two classes:

a) *Cataclysmic*, or explosive, variables which show periods of relative quiet between their abrupt outbursts of brightness;

b) *Pulsating* variables, such as the RR Lyrae stars, Cepheids, and long period variables, which continually change in brightness.

Empirically, these two classes have a common characteristic. The stars with longer cycles differ systematically from those with shorter cycles. For example, the cataclysmic variables of longer period have larger amplitudes than those of shorter period. Among the pulsating stars, longer period is associated with lower surface temperature and lower average density of the stellar material.

This parallelism is a result of the fact that both the cataclysmic and pulsational variables reflect the interplay of elasticity and energy in the gas that comprises the stars. Higher temperature produces swifter motions, and higher density produces greater pressure.

But in terms of stellar models, these two sequences of variables are quite distinct. I will first describe the distinction in terms of a simple model, and I will then focus on the pulsational variables.

2. A Simple Model for Cataclysmic Variables

I will not repeat the known facts about cataclysmic variables, except to say that they are binary stars and the eruptions are caused by the mass-exchange between the components.

As a characature of an explosive system, consider a long-handled cup that is balanced on a hinge at the lip of a bucket, as in Fig. 1. When the cup is empty, it is held horizontally against a stop by a weight on its handle. Now imagine water to be poured slowly into the cup, representing the flow of mass from one star to another in the binary star system. This is a model of a cataclysmic system because the cup is motionless until the contained water overbalances the weight. At this point, the cup tips abruptly and the water spills out. Then the cup returns to its temporary equilibrium. This equilibrium would be stable if it were not for the external influence represented by the stream of water.

The length of the cycle of such a system is determined by a number of parameters, most importantly the size of the cup, the weight on the handle, and the rate of pouring water into the cup. The essential feature is the flow of water representing the interaction between two stars. If this flow is irregular, the cycle will be irregular.

In a similar fashion, the period of a particular cataclysmic variable depends on a variety of factors. The variability is the result of the interaction of two stars, and the periods and amplitudes depend on the combined properties of the stars.

3. Simulations of Pulsating Variables

In contrast to cataclysmic variables, pulsating stars, such as the cepheid variables, are isolated individuals whose cycle-lengths depend only on the conditions inside the star. Typically, these stars are more regular than cataclysmic variables, and we can refer to their cycle lengths as true "periods." The pulsation period depends primarily on a single parameter, the mean density of the star.

Three types of simple questions may be asked concerning the process:

a) *What produces the springiness inside a star, and how does it determine the period of variation?*

b) *What is the nature of the possible periodic motions?*

c) *How does the energy inside a star become organized to sustain the oscillation and prevent the motion from damping out?*

Thanks to a century of theoretical studies, answers to these questions are well in hand, as long as we confine ourselves to oscillations of very small amplitude. I will focus on the first two questions and will base my discussion on a highly simplified model. Real stars, of course, are not as well behaved as our simplified theoretical models. They swing wildly and appear to be blowing their skins off, raising at the same time a number of questions can only be treated by more sophisticated models. Some of these will be mentioned by other speakers at this symposium.

Despite their simplicity, the simulations I will describe are physically realistic, and their purpose is to convey the physical nature of pulsations. They consider the star to be made up of a relatively small number of atoms, and the history of each atom is followed in detail. By confining myself to stars consisting of only one or two dozen atoms, I have been able to carry out these computations on a microcomputer. A usefully realistic calculation by this method would require tracing the parallel histories of millions of atoms. Such a calculation will probably wait for super-computers based on parallel processing.

My simulations were performed on an Apple MacIntosh using programs I wrote in *Pascal* to trace the history of the atoms making up a star. The program produces a series of pictures (frames of the subsequent movie) at equal intervals of time. These frames are stored on a disc, and a second commercially available program ("MacMovies") compiles the frames into a sequence and displays them rapidly on the computer screen. The calculations require an hour or two for a 15-second animation, and the display program creates a "film-loop" that repeats the sequence without interuption.

a) First Simulation: Springiness Inside a Star

The gas atoms in the deep interior of a star are confined by the barrier of overlying gas. Energy is also trapped in the interior, and this permits a simplified mathematical treatment. I will start by showing the behavior of atoms inside a spherical cavity whose outer wall is a shell that shrinks according to a prescribed speed independently of the motion of the atoms

The first question is, what happens to the speeds of the atoms as the shell contracts? Suppose an atom bounces from the wall elastically — like a perfect tennis ball bouncing from a perfect racket. If the tennis racket is moving swiftly forward at the moment of collision, the ball's motion is reversed and it is sped up — in a smashing drive. In the same way, if the shell in this simulation is contracting and hence moving toward the atom, the atom will speed up when it collides. Conversely, the atom will slow down if it collides with the shell during its expansion.

Figs. 2a and 2b shows two stages in a computer simulation of a contracting shell containing six atoms that started with equal speeds moving in random directions. Fig. 2a shows the start, when the shell was large, and the speed of each atom is indicated by the length of its dash. Fig. 2b shows the situation after the shell has moved inward, and its radius is about one-third of the original radius. (This is an exaggeration of the motions inside a star, where the typical changes are about 10 percent or less.) By this time, the atoms have made many collisions with the inward moving shell and have consequently been accelerated, as indicated by the longer dashes in Fig. 2b.

Comparison of these two diagrams illustrates why the interior is springy and tends to resist the contraction. Each impact tends to kick the shell outward. When the cavity has been squeezed, the atoms strike the walls more often, because they are moving faster and they have shorter distances to travel between hits on the wall. And, furthermore, each collision gives a greater impact to the wall in the outward direction because the atom is moving faster.

Now, the total force of the gas on the shell is the rate at which the kicks tend to push the shell outward. It increases with the atoms' speed and the frequency of their collisions. A related and more commonly used quantity is the pressure, which is the force on each unit area of the shell. We can describe this state of affairs by saying that the pressure in the gas increases as the shell contracts. The atoms fight back, so to speak. And when the shell expands, the pressure on the shell decreases.

Two features are missing from the simulation. In the first place, it ignores the collisions among the atoms. Instead of travelling in straight lines between collisions with the walls, they ought to travel in zig-zag paths. But these collisions have very little effect on the outcome of this simulation of the deep interior of a star, and we may neglect them without much harm. In the second place, the motion of the wall was forced to follow a prescribed speed, instead of reacting to the influence of the collisions. The next simulation will correct this defficiency.

b) Second Simulation: Oscillations of a One-shell Star

For a more complete simulation of a star, we next let the motion of the shell be determined by the interplay of the inward tug of gravity and the outward kicks of the colliding atoms inside the cavity.

The calculation proceeds in time steps of duration, δt. At the start of each step, the speed and position of each atom is known, and the computer finds the position at a time δt later. This becomes the starting position for the next time step, and the particles are assumed to move with constant velocity between collisions with the shell or the inner core. The shell motion is a little more complicated, because the shell is accelerated inward by gravity (according to the inverse square law) and kicked outward by collisions. So, in addition to computing the new position after each time step, the computer program must evaluate the new speed.

This simulation computes the effects of collisions by requiring the energy and momentum of the system to be conserved, unlike the first simulation, where the shell motion was assumed to be unaffected by collisions.

These additional features are minor complications and they do not slow down the computation significantly. A more serious problem is the need to keep track of several dozen atoms in order to make the simulation sufficiently realistic. This puts a demand on computer time, so the calculation is simplified by confining the atoms to move directly inward and outward along the radius of the star.

Each simulation starts with the shell at a prescribed radius, and moving at a prescribed velocity, v_s. The atoms all move with the speed, v_a, half inward and half outward. The program displays each atom and shell after each time step and it plots the changing radius of the shell and the pressure of the gas (Fig. 3).

Once the mass of the shell and the number of atoms are selected, the motion depends only on the starting values of the atom speeds and the shell motion. Suppose, as a special case, the shell is initially motionless. For each choice of atom speed, there is an equilibrium shell radius such that the shell will remain nearly motionless. This is the radius for which the inward and outward forces on the shell are in balance. The resulting plot is rather dull, showing a constant radius, with small fluctuations produced by individual kicks. This behavior corresponds to a static, non-pulsating star. On the other hand, if the shell is initially moving inward, it will overshoot the equilibrium, then rise up again and continue to oscillate indefinitely. In Fig. 3a the radius and pressure are plotted as functions of time for one such simulation. Note the highest pressure corresponds to the smallest radius, as we expect. It is this increase of pressure that forces the shell outward again.

One of the remarkable aspects of this motion is that, as long as the amplitude of the motion is not large, the period of this oscillation does not depend on the starting conditions. Fig. 3a and 3b show simulations with different amplitudes, and the period is nearly the same in both. It depends only on the number of atoms and the equilibrium radius. (Specifically, the oscillation period of a star for these simple motions depends on the mean density, or the number of atoms divided by the volume.) Astronomers have known for nearly a century that this relationship is a general one for spherical stellar oscillations, and they have ben able to use it to evaluate the mean density of an observed pulsating star from the period of its oscillation.

c) Possible Further Simulations

i) Multiple Shells

One extension to such a simulation might be to add an additional shell so the atoms are contained in two expandable compartments. This adds a new dimension to the possible motions, for the shells can move separately. Their motions are not entirely independent of each other, because the motion of one shell will change the space between the shells and will alter the average energy of the atoms between the two shells. These changes will affect the motion of the other shell.

If we start the shells randomly, the subsequent motion will be very complicated, either multiply-periodic or irregular. But there are patterns of motion that appear relatively simple and periodic. These are the so-called "modes." When the model oscillates in one of its modes, the shells all reach their greatest displacement and come to rest at the same time, so they move in synchronism. Each mode has a different cycle length.

Real stars are capable of such synchronized motion in modes. These modal patterns are often favored over irregular motion because they do not damp out as rapidly as a highly irregular pattern. One of the challenges of variable star theory is to identify the modal pattern that is followed by a particular star.

ii) Energy Transfer among Shells

Stars shine because they release heat and light into space.

The simulations described thus far do not shine, because they do not imitate the transfer of heat energy from one part of the star to another and from the star to space. This transfer is crucial in overcoming internal friction and in maintaining the pulsations in certain types of stars — as well as damping the pulsations in other types.

4. Conclusion

Experiments with highly simplfied simulation involving only several dozen elastic atoms confined by one or two shells show remarkably regular motions that illustrate pulsating stars.

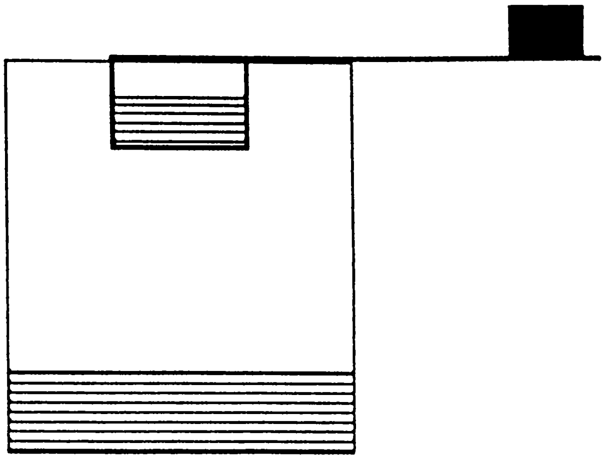


Fig. 1. Schematic diagram of a model for a cataclysmic variable. A cup balanced on a hinge at the edge of a pail is filled with water; the cup tips and the water spills out; the cup returns to its original upright position. As shown here, the cup is nearly filled and is about to tip down.

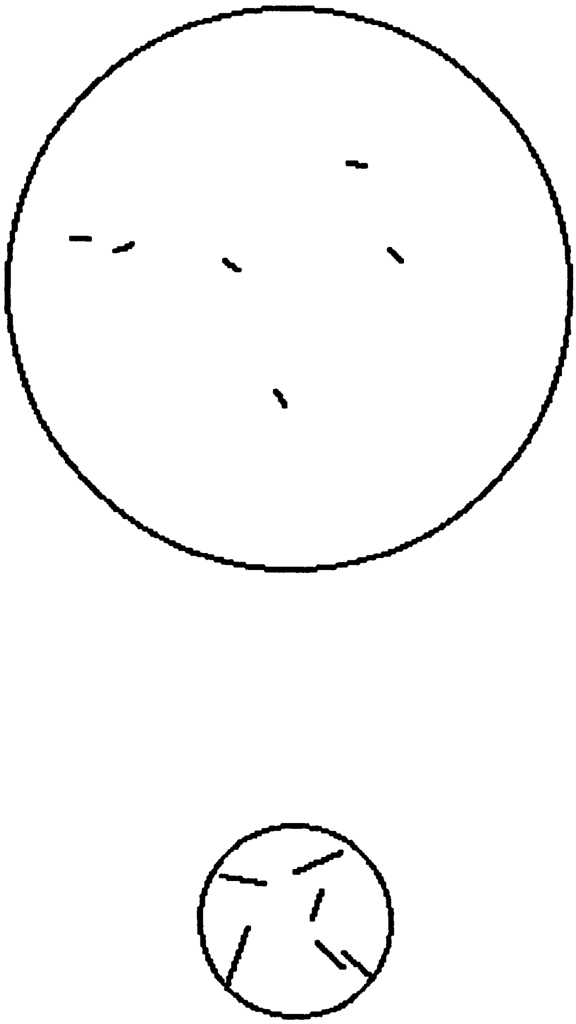
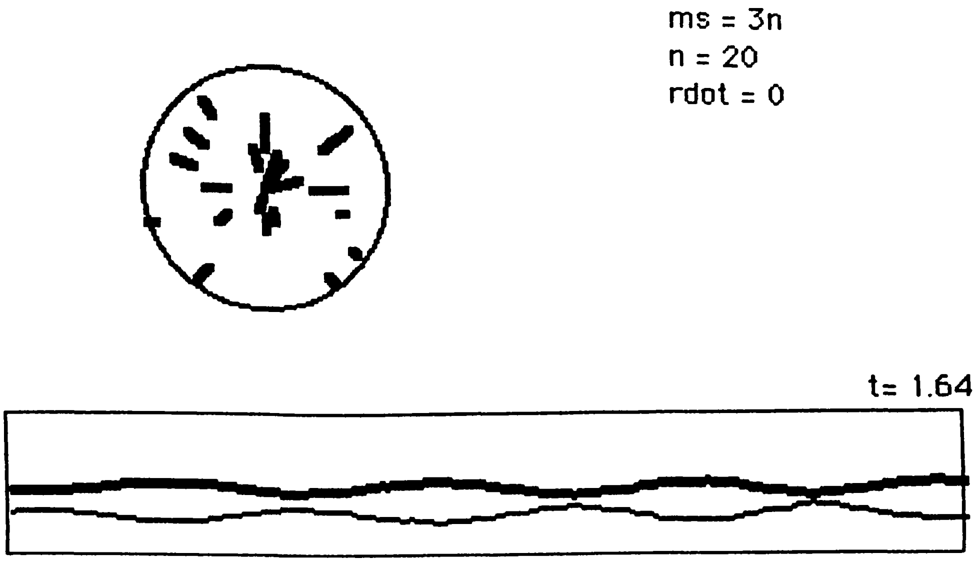


Fig. 2. Two snapshots from a computer simulation of a pulsating star, showing the change of the relative speeds of the atoms. Fig. 2a (*upper*). Six atoms move at random inside a sphere, representing a star. The motion of each atom in a unit time is indicated by the dashes. In the expanded state, the atoms move relatively slowly, indicated by the shortness of the dashes. Fig. 2b (*lower*) In the contracted state, the atoms have acquired speed from their collisions with the contracting walls.



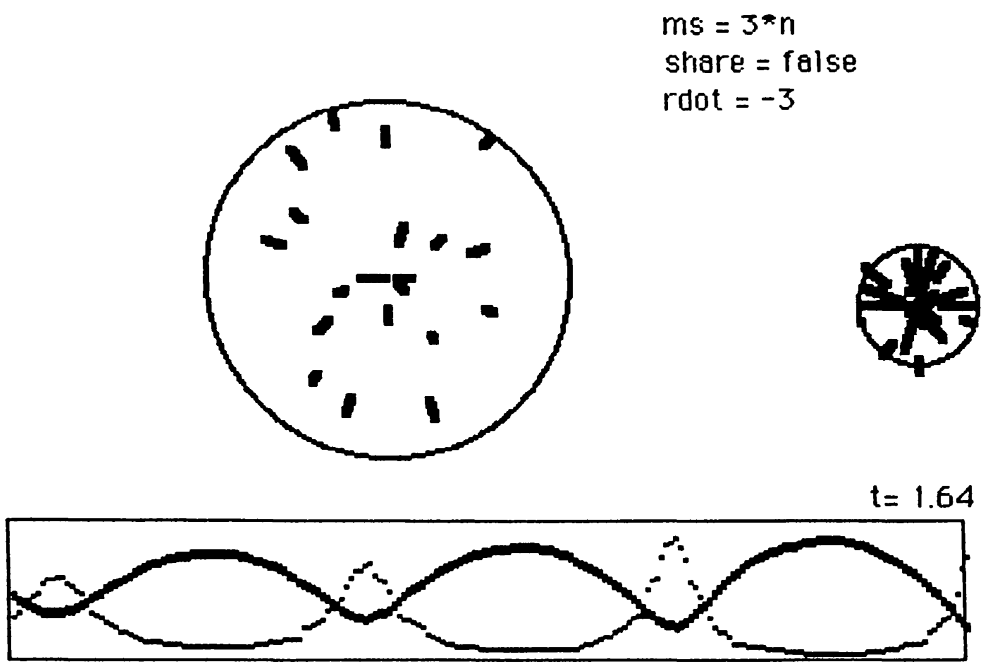


Fig. 3a (*upper*) and 3b (*lower*). Time-histories of two computer simulations of a pulsating star consisting of 20 atoms bouncing elastically inside an extensible shell. In the graphs, time increases to the right. The heavy line is the radius of the shell and the fine line is the energy of the atoms. This energy is related to the pressure inside the star. Note that the highest pressures occur when the radius is smallest during each cycle. These computations are similar except that the lower simulation was started with a more vigorous motion. Note that the period of the larger amplitude pulsation is slightly longer.