O-C BY COMPUTER

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Abstract

Two methods are introduced in which O-C (Observed minus Calculated) is calculated automatically by allowing the computer to find residuals of observations from an adopted standard light curve. In one method the residuals are directly in the time coordinate, one for each magnitude. The other is a non-linear least squares method, minimizing the sum of the squares of the residuals in magnitude, to give a mean O-C for the group.

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Two methods of finding O-C by computer have been developed at the Maria Mitchell Observatory. The general idea of both is illustrated in Figure 1, in which the (unknown) true variation of a periodic variable is supposed to be illustrated by the continuous curve. The dots outline a computed curve, which will have been based on previous studies of the star. Its maxima occur at a series of calculated times, C, given by whatever elements are to be tested and refined in the O-C analysis.

The quantity required is the size of the correction to be applied to the computed maxima (or any points along the computed curve) to bring them into coincidence with the true curve. This (unknown and unknowable) correction is illustrated by the horizontal arrows at the top of the figure, labeled "True-Comp." An approximation of "O-C" to "True-Comp" is to be found from observations.

The squares in Figure 1 represent the set of observations, generally following the "true" curve but differing from it because of observational errors in the magnitudes. It is from these observations that we will find O-C, our best approximation to "True-Comp." Many methods have been used over the years, ranging upwards from simple lists of single observed maxima. At the Maria Mitchell Observatory we have been favoring a graphical method in which a plot of a "Comp" curve is superimposed on a folded light curve, so that a group of many observations can define O-C to high precision. Now we have automated the procedure for use on a microcomputer. Two rather different methods are illustrated by arrows in Figure 1 leading from the "Comp" curve to a few observed points.

The horizontal arrows toward the left of the figure illustrate a method programmed in BASIC by Holliman (1986). For each magnitude a pair of possible values of O-C is calculated (unless the star is near minimum or maximum at the time of the observation). The choice between the two is based on the consistency of the values of several observations close together in time. It is clear by inspection of Figure 1 that the shorter arrows in each set are consistent with each other while the longer ones are not. The process is what variable star workers frequently do graphically when they are testing trial periods by plotting magnitude against phase, perhaps using different symbols for observations made within different time intervals. In Holliman's method the residuals are calculated by solving a Fourier series approximation to the light curve for the time (or phase) at which each observed magnitude is expected to occur. The choice between the two possibilities is also made automatically.

The vertical arrows leading from the Computed curve to three of the observations near the right of Figure 1 illustrate the other method. The sum of the squares of a group of residuals like these is calculated. The sum will be large if the calculated curve is not a close approximation to the true one, and it is from the size of the sum that the method finds its own approximation to the desired "True-Comp." Various trial values are added to the date (or phase) along the "Comp" curve, to give trial approximations to the "True" curve, such as the one shown in Figure 2 by crosses. The residuals of the same three observations from the trial curve are drawn on Figure 2, and it is clear that there has been an improvement.

By trial and error a value of "Trial-Comp" is found which will make the sum of the squares of the residuals a minimum. This procedure, which I have programmed in FORTRAN, is an application of the principle of least squares. The usual **method** of least squares cannot be applied to the case, since the method requires that the observed quantity be the dependent variable (y) in a simple linear equation, such as $y = mx + b$, in which the unknown or unknowns occur as the independent variable or variables (x). Magnitude is the dependent variable in our case; O-C (or "Trial-Comp") is the independent variable; the relation is far from linear.

With a computer, however, the **principle** of least squares can be used by brute force, in cases like this where the quicker linear method does not apply. The computer finds not only the best trial, which is then adopted as O-C for the group, but also finds the way the sum of the squares of the residuals increases for poorer trials on either side. The rate of this increase gives the standard error of the derived O-C and therefore the so-called "one-sigma" confidence region, i.e., the best value plus or minus one standard deviation. There is about a 68% probability that the true value lies within this confidence region.

The non-linear least squares method was applied by Maria Mitchell staff members this summer (Barton 1986; Lysaght 1986; Wilner 1986). The Fourier series method was applied by its author to the RR Lyrae star, V Com (Holliman 1986b).

This program development and the microcomputer on which it was carried out are part of a project supported by the National Science Foundation under grant AST 83-20491. Earlier versions of the nonlinear least squares method were developed on main-frame computers at Columbia University, the Pennsylvania State University, and the Harvard-Smithsonian Center for Astrophysics. I am grateful for this support and for the grants of computer time.

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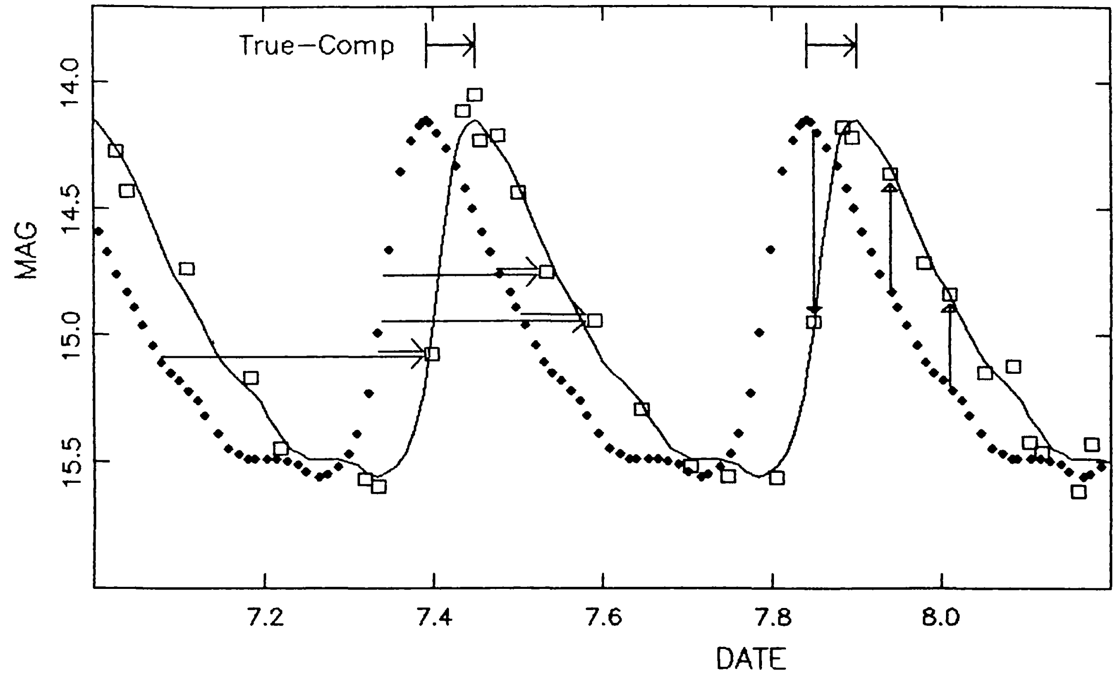

Figure 1. The true light curve of a variable star (continuous curve); an approximation to it (dots) computed from previous work; and a sample of observations (squares). Arrows pointing to some of the observations are their residuals, in time or in magnitude, from the computed curve. The methods described in the text use the residuals to find approximations to the shift "True-Comp" required to bring the curves into coincidence.

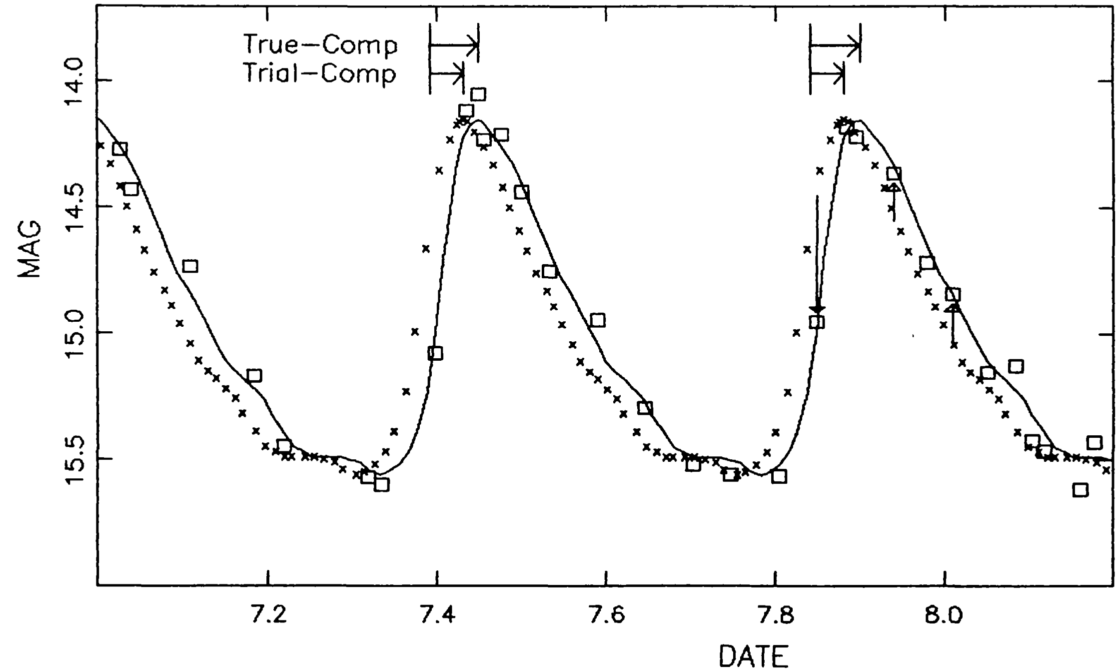

Figure 2. The same situation as in Figure 1, but with the standard curve replaced by a trial curve (crosses) that is a closer approximation to the true situation. The trial is formed by shifting the computed curve by an amount, "Trial-Comp," indicated at the top of the figure.

THE CHANGING PERIOD OF V1303 SAGITTARII

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Abstract

V1303 Sgr, an 18-day Population II Cepheid, was studied for period changes. The deviation from a constant period was found to be statistically significant, and the data indicate that the period is decreasing at the rate of 0.0025 cycle per century. The new elements are:

$$JD_{(max)} = 2439011.426 + 18.4527\ E - 0.0000117\ E^2. \quad (1)$$

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Data from V1303 Sgr were gathered from three sources: the photographic plates at the Maria Mitchell Observatory, research by P. Th. Oosteroff and J. A. Horikx (1954), and research by K. K. Kwee and L. D. Braun (1967). The reference elements were taken from the **Second Supplement to the General Catalogue of Variable Stars** (Kukarkin et al. 1974):

$$JD_{(max)} = 2438241.176 + 18.46812\ E. \quad (2)$$

An average light curve of magnitude vs. phase was made from selected photographic data (1978-1982) and is illustrated in Figure 1. For the Maria Mitchell and Kwee data, light curves of magnitude vs. phase for intervals of less than two years were plotted. To find the O-C values, each of these plots was compared to the average light curve by a non-linear least squares analysis on the Maria Mitchell Observatory's computer. Additional O-C values were computed solely from the observed times of maximum brightness in the Oosteroff and Horikx paper.

The best fitting parabola for all of the O-C data was computed by least squares and is shown in Figure 2. The significance of the curvature was verified by a statistical F-test (Pringle 1975). Fear that the 15 Oosteroff O-C values, although given lower weight, would mold the curve into a parabola forced me to fit a parabola to the other points alone.

The new elements from all of the data are:

$$JD_{(max)} = \underset{\pm 0.077}{2439011.426} + \underset{\pm 0.0004}{18.4527}\ E - \underset{\pm 0.0000009}{0.0000117}\ E^2. \quad (3)$$

The new elements from the data without the Oosteroff points are:

$$JD_{(max)} = \underset{\pm 0.09}{2439011.46} + \underset{\pm 0.0009}{18.4533}\ E - \underset{\pm 0.0000038}{0.0000146}\ E^2. \quad (4)$$

Figure 2 shows the modern curve and its confidence limits plotted with all of the data points. The confidence interval was calculated according to an extension of the method given by Brandt (1978) for regression lines. The least squares parabola gives the best estimated O-C curve. The confidence curves, surrounding the parabola, mark an interval of plus and minus one standard deviation. Thus, the probability that a true O-C value lies within the pictured interval is 68%. Most of the older data fell directly within the derived confidence limits of the newer curve. In fact, the two curves are consistent within the expected error. This suggests that the curve is a true representation of the recent period behavior of the star., because, as illustrated, an extrapolation of the modern curve would give the approximate position of the older points. The rate of change of period, based on all of the data, is -0.0025 cycle per century.

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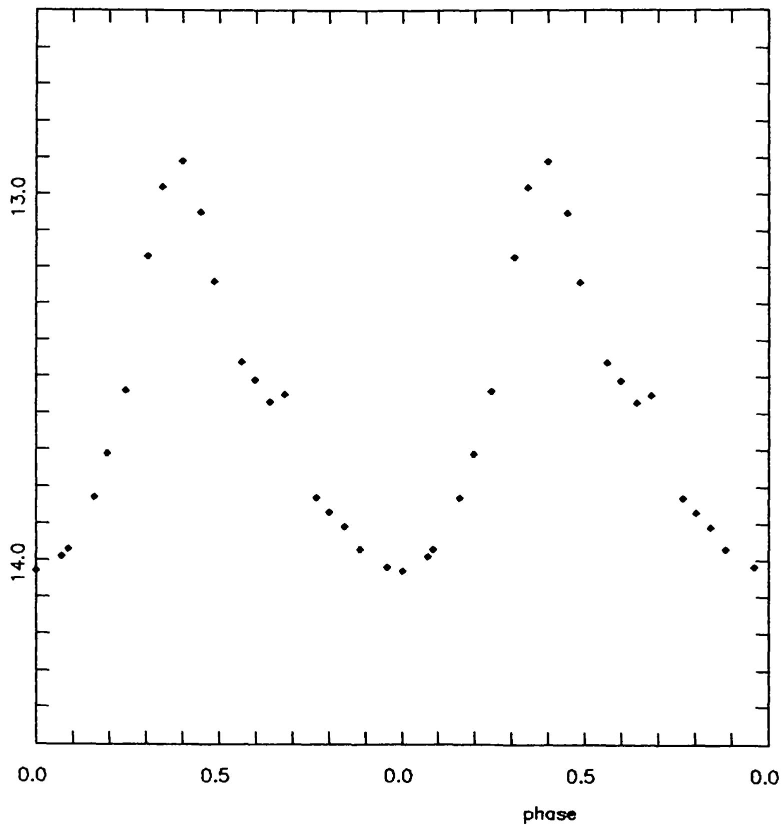

Figure 1. Average photographic light curve for V1303 Sgr. The magnitude vs. phase points were calculated by averaging 1978-1982 data in 20 overlapping intervals of width 0.01 in phase.

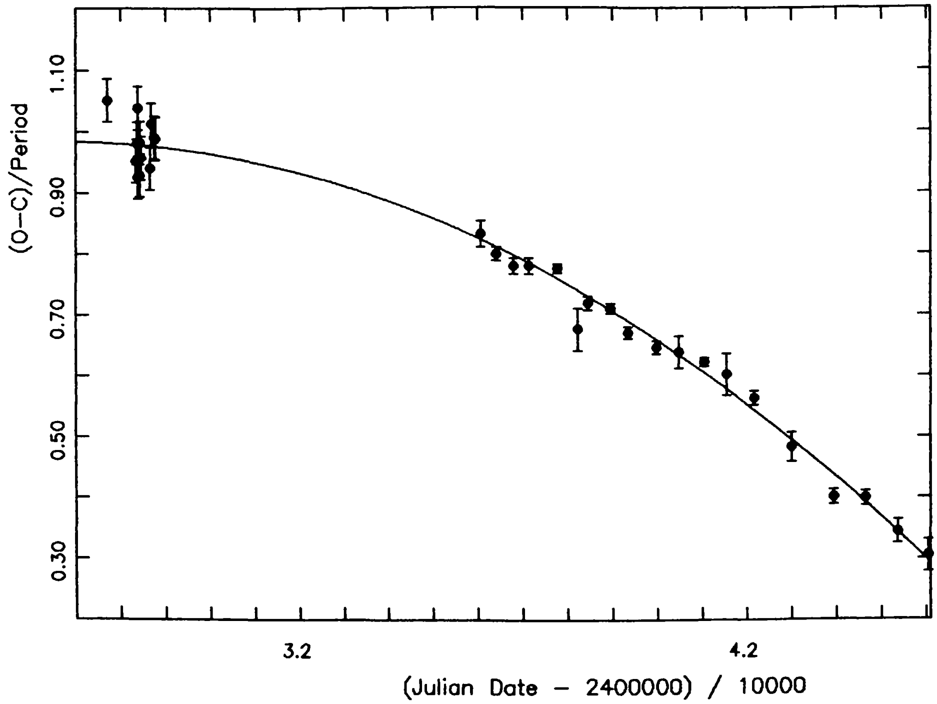

Figure 2. O-C graph for V1303 Sgr with least squares parabola, using all of the available data. The data within the range JD 2427000-2429000 are by Oosteroff and Horikx (1954). The point at JD 2438230 is from Kwee and Braun. The rest are from Maria Mitchell Observatory plates.

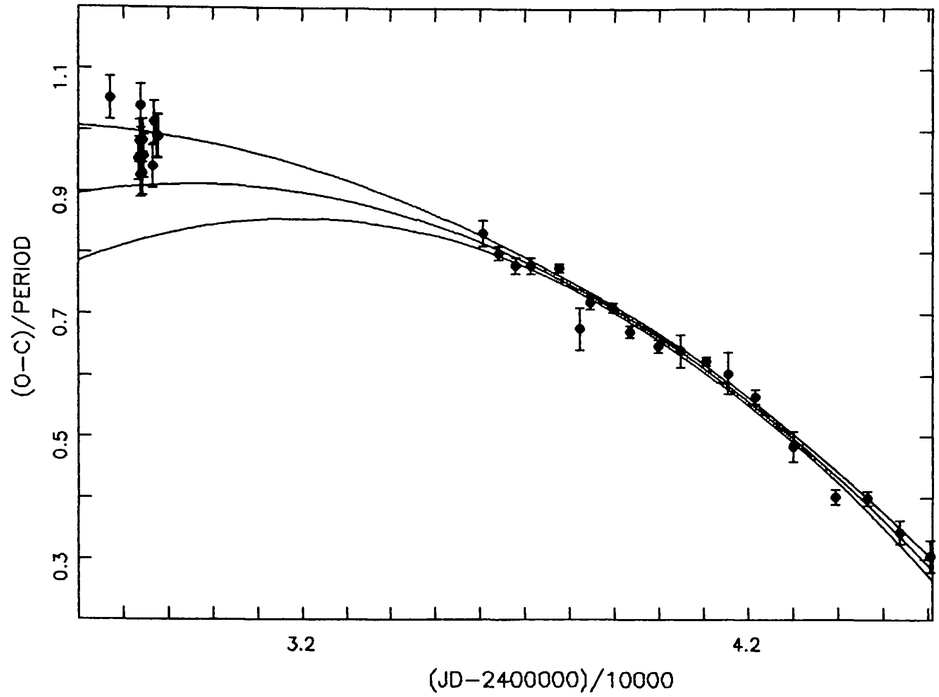

Figure 3. O-C graph for V1303 Sgr, phase shift vs. Julian Date, containing the same data points as Figure 2. The three curves are the least squares fit to the modern data (JD 2434000-2446100) and its upper and lower confidence limits.