

S PERSEI
A SEMI-REGULAR VARIABLE WITH TWO PERIODS

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The Variable star S Persei (021558) was discovered by A. Krueger in 1872 (Krueger, 1874) and it has been the object of regular observation for more than ninety years, beginning in 1880. S Persei is listed in the 1969 GCVS as an SRC variable of spectral and luminosity types M3eIa-M4eIa, which is to say, a red supergiant with semi-regular variability. Figures 1(a) and 1(c) show the visual light curve of S Persei from 1880 until early 1974, after 1911 being based on AAVSO observations for the most part. Inspection of the light curve indicates that S Persei has distinctly cyclic light oscillations of greatly varying amplitude. Also noticeable is a decline in the star's median magnitude from $\sim 9^m1$ to $\sim 9^m7$ near JD 2,416,000, with a subsequent gradual decline to around 10^m1 .

The first serious attempt to interpret the unusual light fluctuations of S Persei was made by H. H. Turner (1904). Turner explained the observed light variations by positing the existence of three periodicities of length 840^d , 1120^d , and 3360^d with respective semi-amplitudes 0^m6 , 0^m4 , and 0^m4 , the superposition of which made up the observed light curve. Thirty-five years after Turner's study, T. E. Sterne (Campbell, 1939) took a fresh look at the light curve of S Persei. He found that the hypothesis that the observed light curve resulted from the interference of two periodicities provided a better explanation of the light variations than did Turner's three periodicity model. Sterne gave the lengths of these periodicities as 810^d and 916^d and found both to be of semi-amplitudes 1^m1 , which are the values still given in the GCVS. Leon Campbell agreed with Sterne's interpretation, calling S Persei "...a long period variable with two periods..." and remarking that Sterne's formulation still left certain aspects of the light curve unexplained. In particular, the flattening of the light curve near JD 2,418,000 and the fall in the median brightness were not accounted for. In this paper a new analysis of the light curve of S Persei is presented: one which is based upon a power spectrum analysis of the curve and which takes advantage of the additional observations obtained since 1939.

Power spectrum (or periodogram) analysis is a useful technique by which periodicities in a series of data can be examined. First developed by A. Schuster in 1898, the method has been subsequently described in a number of mathematics texts and scientific papers (see, for example, Sharpless, Riegel, and Williams, 1966). Basically, it involves the computation of a power, $S(K)$, for a given trial period, K , using a series of N observations equally spaced in time. The power is computed according to the expression

$$S(K) = \frac{\left[\sum_{j=1}^N f_j \sin \frac{2\pi j}{K} \right]^2 + \left[\sum_{j=1}^N f_j \cos \frac{2\pi j}{K} \right]^2}{\sum_{j=1}^N (f_j)^2}$$

In the above expression K is the trial period in units of the spacing between observations, while the f_j 's are the observations themselves, expressed as a deviation from the mean. $S(K)$ must be calculated for many different values of K , thus effectively limiting use of this method to those having access to a high speed computer. A graph, or periodogram, can then be constructed of $S(K)$ vs. K . When K corresponds to a period in the observations, a maximum appears in the periodogram. Should a maximum of $S(K)$ exceed the mean, $\bar{S}(K)$, by a factor of four, we can say with some confidence that the K corresponding to that maximum is a real period in the observations, and not simply a statistical fluctuation. Caution must be used in interpreting a periodogram, however, as non-periodic elements in the observations can lead to spurious maxima. In general, the power spectrum method should be used only in those cases where there is reason to suspect a periodic element in the observations to be tested.

The light curve of S Persei shows clear signs of a cyclic variation and is thus a candidate for power spectrum analysis. There is an exception to this statement, however, in that there is no evidence that the flattening of the light curve near JD 2,418,000 is anything but irregular. The inclusion of this flattening in the power spectrum analysis leads to a very complex periodogram. This complexity is probably misleading, and a periodogram constructed from data excluding the flattening is probably more truly indicative of the periodic elements in the light curve of S Persei.

In light of the above reasoning, a periodogram was constructed from 153 magnitudes obtained at 100^d intervals from the light curve between JD 2,421,900 and JD 2,437,200. Computations were carried out on the Digital Dec-10 System computing facilities of Wesleyan University. Trial periods K were tested at 5^d intervals from 5^d to 3000^d and at 20^d intervals from 3000^d to 15000^d . The portion of the resulting periodogram with peaks easily exceeding $4\bar{S}(K)$ is shown in Figure 2.

The two largest peaks confirm Sterne's view that the light curve of S Persei can be at least partially explained in terms of two interfering oscillations. However, a revision of Sterne's 810^d and 916^d periods to 825^d and 940^d is indicated by the periodogram. The ratio of the two interfering periods is thus 1:1.14. Also, the unequal sizes of the two peaks indicate unequal amplitudes for the two periodicities.

A synthetic light curve based upon the interference of 825^d and 940^d periods with respective semi-amplitudes 0^m9 and 0^m65 and sinusoidal forms is presented in Figures 1(b) and 1(d). A pronounced "beat" phenomenon is evident in these figures. Compensating for the fall in the median brightness, it is seen that the synthetic curve follows the run of the observed curve well for certain time intervals and poorly at other times. The flattening of the curve near JD 2,418,000 remains unexplained as in Sterne's model. In addition, the interference, or beat, phenomenon seems much weaker in the observed curve than in the synthetic curve after JD 2,433,000, although a weak beat effect still seems present in the observed curve after that date. This weakening of the beat phenomenon and the lesser amplitude of S Persei in recent years can be explained if the amplitude of one of the two periodicities, probably the 940^d periodicity, has greatly lessened since JD 2,433,000. In fact, a similar decrease in

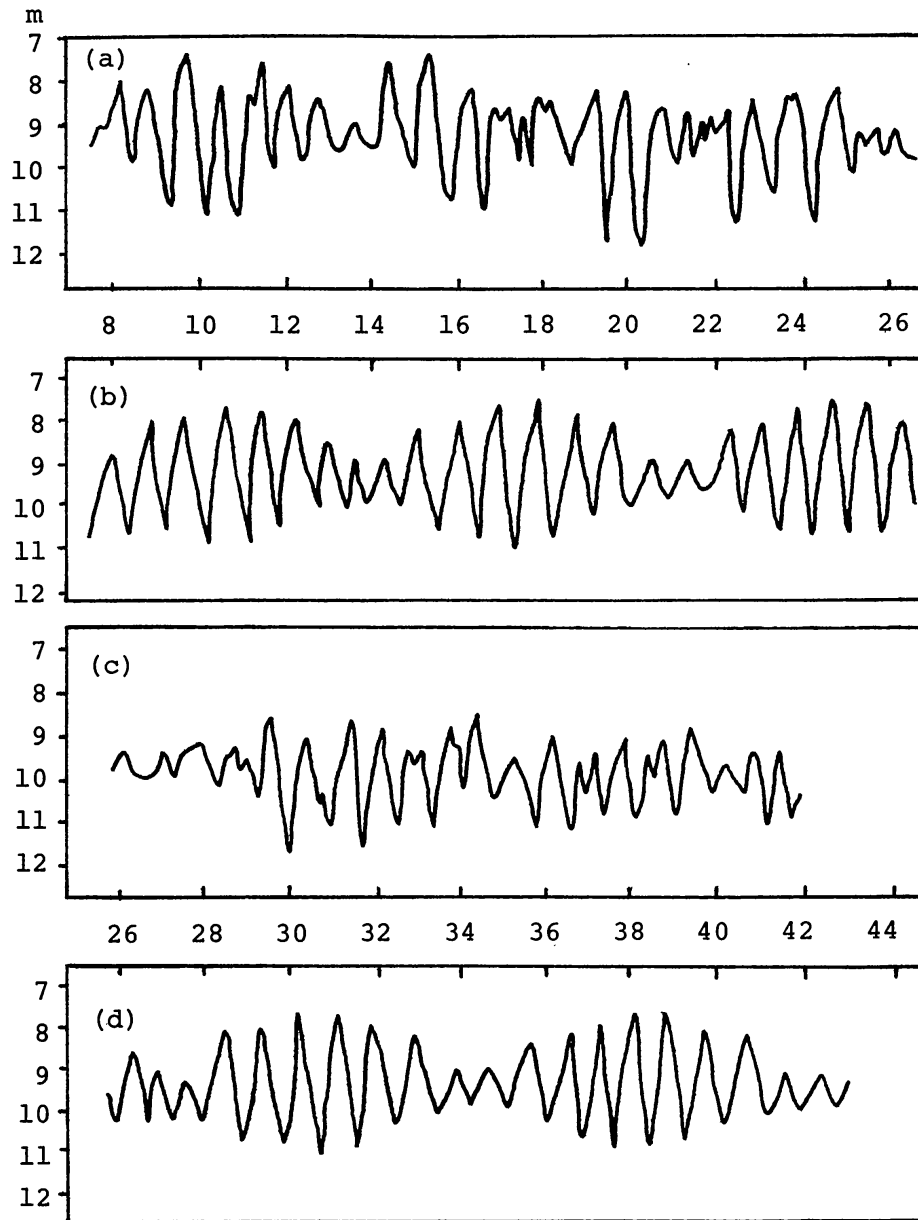


Figure 1. Parts (a) and (c) show the observed AAVSO visual light curve for S Persei. Parts (b) and (d) show the corresponding synthetic curve. The abscissa scale is in thousands of days after JD 240000. The ordinate is in magnitudes.

the amplitude of the 940^d periodicity may account for the decline in the amplitude range of S Persei between JD 2,410,000 and JD 2,422,000. Sterne's model which gives the amplitudes of the two periodicities as equal and of size 2^m provides a good fit for the curve during the interval JD 2,408,000 to JD 2,414,000.

Thus, it is seen that, while there is some reason to term S Persei a semi-regular variable with two periods, a number of questions remain unanswered. The question is raised of the extent to which the two period model represents real physical processes in the star. The relation of S Persei to other stars showing beat phenomena, such as VX Sagittarii (Dinerstein, 1973), remains unclear. The nature of the variations in the strength of the beat phenomenon in S Persei is unsettled. It will be interesting to see whether in the future the beat phenomenon in S Persei entirely disappears or whether it will become re-established. Certainly continued AAVSO observation of this star would seem worthwhile.

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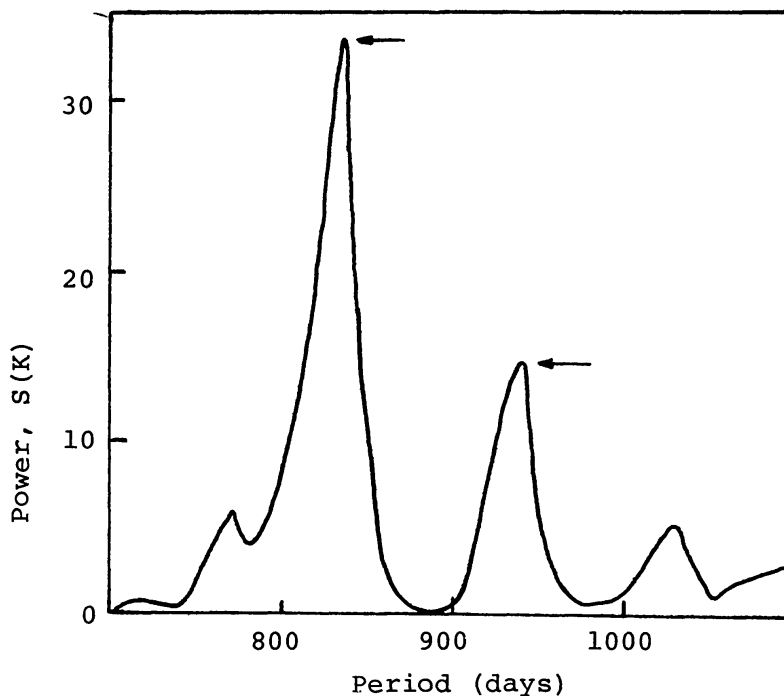


Figure 2. Periodogram for S Persei for trial periods of 700^d to 1100^d . Both marked peaks easily exceed $4 \bar{S}(K)$.